

# How Does *i-Ready Classroom Mathematics* Support NCTM's 5 Practices for Orchestrating Productive Mathematics Discussions?

Complex knowledge and skills are learned through interactions with others, but those interactions don't always come naturally in math lessons. More often, they are the result of careful planning and expertise. Every successful discussion-based math class has two crucial parts:

- Cognitively challenging instructional tasks
- Support for students as they engage with and discuss their thinking on these tasks

*i-Ready Classroom Mathematics* provides rigorous tasks, a discourse-based instructional routine, and helpful supports to simplify planning for mathematical discussions so teachers can focus on interacting with their students.

These examples, drawn from just one session, show how *i-Ready Classroom Mathematics* aligns with the philosophy outlined in NCTM's 5 Practices for Orchestrating Productive Mathematics Discussions.

**LESSON 3 | SESSION 1** ■ ■ ■ ■ ■  
**Explore Sequences of Rigid Transformations and Congruence**

**Purpose**

- Explore the idea that a figure generated by a sequence of rigid transformations is congruent to the original figure.
- Understand that a sequence of rigid transformations including translations, reflections, and rotations does not change the size or shape of a figure.

**START** ■ **CONNECT TO PRIOR KNOWLEDGE**

**Same and Different**

|                             |                             |
|-----------------------------|-----------------------------|
| $(0, 0) \rightarrow (0, 0)$ | $(1, 1) \rightarrow (1, 1)$ |
| $(2, 2) \rightarrow (2, 2)$ | $(3, 3) \rightarrow (3, 3)$ |
| $(4, 4) \rightarrow (4, 4)$ | $(5, 5) \rightarrow (5, 5)$ |

**Possible Solutions**  
All show translations.  
B and C both show translations down.  
B shows a reflection across the x-axis.  
A and D both show mappings from and to Quadrant I.

**WHY?** Support students' facility in identifying and describing single transformations in the coordinate plane.

**TRY IT** ■ SMP 1, 2, 4, 5, 6

**Make Sense of the Problem**  
See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. At each read, focus on a new question, asking: *What is the problem about?* *What are you trying to find out?* and *What are the important quantities and relationships in the problem?*

**DISCUSS IT** ■ SMP 2, 3, 6

**Support Partner Discussion**  
After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that the figures are the same size and shape if:

- one exactly covers the other.
- corresponding sides and corresponding angles have the same measures.

**LESSON 3 | SESSION 1** ■ ■ ■ ■ ■  
**Explore Sequences of Rigid Transformations and Congruence**

Previously you learned how to perform and describe rigid transformations. In this lesson, you will learn what happens when you perform sequences of one or more rigid transformations.

► Use what you know to try to solve the problem below.

**Anchor's Quilt** is an Amish quilt pattern made of polygons. You can use a coordinate plane to plan a quilt. Graph a pentagon with the vertices  $A(0, 0)$ ,  $B(6, -2)$ ,  $C(6, -6)$ ,  $D(4, -6)$ , and  $E(4, 0)$ . Then to graph another pentagon, follow these steps:

- Translate pentagon  $ABCDE$  4 units to the left to form pentagon  $A'B'C'D'E'$ .
- Rotate pentagon  $A'B'C'D'E'$  90° counterclockwise around the origin to form pentagon  $A''B''C''D''E''$ .

Is pentagon  $A''B''C''D''E''$  the same size and shape as pentagon  $ABCDE$ ?

**TRY IT** ■ Math Toolkit: graph paper, protractors, rulers, tracing paper

**Possible work:**

**SAMPLE A**  
See graph in Sample B. Yes, one figure exactly covers the other.

**SAMPLE B**  
See graph. Yes, corresponding sides are equal in length and corresponding angles are equal in measure.

**DISCUSS IT**  
Ask: How did you decide whether pentagons  $ABCDE$  and  $A''B''C''D''E''$  are the same size and shape? Share: The strategy I used was...

**Learning Target** SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7  
Explain how two-dimensional figures are congruent to another figure and how one can be obtained from the first by a sequence of rotations, reflections, and translations given two congruent figures, describe a sequence that establishes the congruence between them.

**Common Misconception** Listen for students who confuse same orientation and same shape. They may say the image is not the same shape but what they may mean is that the orientation makes the shapes look different. Help students understand that shape is maintained even if the orientation is different.

**Select and Sequence Student Strategies**  
Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- cut-out shapes to model the translations
- traced figures to model the translations
- (misconception) drawings show same size and shape but student concludes shape is not the same due to orientation
- drawings in coordinate plane to show the images

45 ■ LESSON 3 Work with Sequences of Transformations and Congruence ■ Curriculum Associates, LLC Copying is not permitted.

## 1 Anticipating

Predict students' approaches to a problem and prepare for common misconceptions.

## 2 Selecting

Choose which students' work to share with the class, with help from the notes accompanying select problems.

## 3 Sequencing

Share students' work in a sequence that guides the class's thinking to achieving the session's goals.

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### Select and Sequence Student Strategies

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**Facilitate Whole Class Discussion**

Call on students to share selected strategies. Remind speakers to project their voices and pause for questions as they explain their strategy. To engage all students, ask them to turn and talk about how they could decide whether the figures are the same size and shape.

Guide students to **Compare and Connect** the representations. To emphasize key mathematical ideas in students' explanations, ask others to repeat and rephrase the statements.

**ASK** How do you know that the figure does not change size or shape when it is transformed?

**LISTEN FOR** I can show that the side lengths of pentagon  $A'B'C'D'E'$  are the same as the side lengths of pentagon  $ABCDE$  using a ruler. I can show that the angle measures of pentagon  $A'B'C'D'E'$  are the same as the angle measures of pentagon  $ABCDE$  using a protractor.

**CONNECT IT**

SMP 2, 4, 5

**1 Look Back** Look for understanding that the original pentagon and transformed pentagon are the same size and shape.

**DIFFERENTIATION | RETEACH or REINFORCE**



**Hands-On Activity**

**Model a sequence of transformations.**

If students are unsure about whether a figure stays the same shape and size when rigid transformations change its orientation, then use this activity to help them visualize how the transformations affect a figure.

**Materials** For each student: large tangram parallelogram, Activity Sheet Graph Paper

- Have students draw axes and then trace the parallelogram on the coordinate plane and label it  $ABCD$ .
- Instruct students to translate the tangram down 2 units, trace it, and label the corresponding vertices  $A'B'C'D'$ .
- Then have students reflect the parallelogram across the  $y$ -axis, trace it, and label the corresponding vertices  $A''B''C''D''$ .
- Ask: Is parallelogram  $A'B'C'D'$  the same size and shape as parallelogram  $ABCD$ ? [yes]
- Ask: Is parallelogram  $A''B''C''D''$  the same size and shape as both parallelograms  $A'B'C'D'$  and  $ABCD$ ? [yes]
- Repeat this activity using other combinations of two transformations.

LESSON 3 | SESSION 1

**CONNECT IT**

**1 Look Back** Is pentagon  $A'B'C'D'E'$  the same size and shape as pentagon  $ABCDE$ ? Explain.  
Yes: Possible explanation: The sides and angles of pentagon  $A'B'C'D'E'$  are the same measure as the corresponding sides and angles of pentagon  $ABCDE$ .

**2 Look Ahead** You already know that you can transform a figure. You can also transform the image of a figure. This is called performing a **sequence of transformations** on a figure.

a. You can add a prime mark to the vertex labels for each transformation you perform to name an image resulting from a sequence of transformations. How many transformations were performed to get image  $A''B''C''D''E''$ ? Explain.  
Three. There are three prime marks after each vertex label.

b. Figures that are the same size and shape are **congruent**. When a figure is translated, reflected, or rotated, the image is always congruent to the original figure. That means pentagons  $ABCDE$  and  $A'B'C'D'E'$  are congruent.

$$\text{pentagon } ABCDE \cong \text{pentagon } A'B'C'D'E'$$

(read "is congruent to")

Write a congruence statement for pentagons  $A'B'C'D'E'$  and  $A''B''C''D''E''$ .  
 $\text{pentagon } A'B'C'D'E' \cong \text{pentagon } A''B''C''D''E''$

c. Enrico translates figure  $Q$  to get figure  $Q'$ . Then he rotates figure  $Q'$  to get figure  $Q''$ . Explain why figure  $Q$  and figure  $Q''$  are congruent.

Figure  $Q =$  figure  $Q'$  and figure  $Q' =$  figure  $Q''$  because when a figure is translated, rotated, or reflected, the image and the original figure are congruent. Because figure  $Q =$  figure  $Q'$  and figure  $Q' =$  figure  $Q''$ , then figure  $Q =$  figure  $Q''$ .

**3 Reflect** How can you use a sequence of transformations to show that one figure is congruent to another?

Possible answer: You can find a sequence of translations, reflections, and rotations that maps one of the figures onto the other.

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**2 Look Ahead** Students should recognize that there is no limit to the number of transformations in a sequence of transformations. Each time a figure is translated, reflected, or rotated, its location and/or orientation may change, but the shape and the size of the figure do not change. The number of primes used to mark each vertex matches the number of transformations performed.

Ask a volunteer to rephrase the definitions of sequence of transformations and congruent. Support student understanding by having students use these new terms to describe their work on the Try It problem.

**CLOSE EXIT TICKET**

**3 Reflect** Look for understanding that when one figure maps onto another, it will exactly cover it. Therefore, corresponding side lengths will be equal and corresponding angle measures will also be equal.

**Common Misconception** If students state that a figure and its image after any sequence of rigid transformations are congruent, but believe they still need to measure sides and angles to confirm, then ask them to justify how a rigid transformation could have caused a change in a measurement.



**Facilitate Whole Class Discussion**

Call on students to share selected strategies. Remind speakers to project their voices and pause for questions as they explain their strategy. To engage all students, ask them to turn and talk about how they could decide whether the figures are the same size and shape.

Guide students to **Compare and Connect** the representations. To emphasize key mathematical ideas in students' explanations, ask others to repeat and rephrase the statements.

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**4 Monitoring**

Read the **ASK/LISTEN FOR** support for suggestions to help you authentically engage with students as they work and learn.

**5 Connecting**

Use the prompts to help students make connections with each other's strategies and give context to new ideas.

Contact your local representative  
to request a free sample!

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