Ready® Classroom

Grade 8 | Volume 1

Mathematics

Teacher's Guide

Sampler

Ready Classroom Mathematics

Grade 8 Teacher's Guide Sampler

In this sampler, you will see the *Ready Classroom Mathematics* Teacher's Guide pages for the Beginning and End of Unit 3 and two complete lessons.

Ready Classroom Mathematics
Grade 8 Table of Contents

Table of Contents (continued)

Beginning of Unit 3

Print Resources

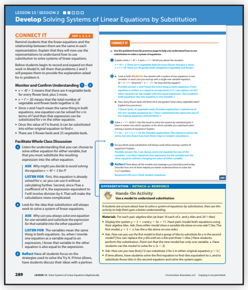
Reach all students using these print resources that support instruction, practice, assessment, and differentiation.

Resources can be found in the Student Worktext or Teacher's Guide or downloaded from the Teacher Toolbox section in the Teacher Digital Experience.

In-Class Instruction and Practice

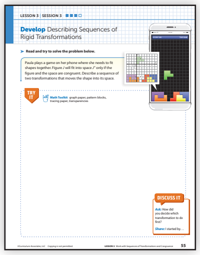
Teacher's Guide

- Learning Progression
- ELL Language Expectations
- Connect to Culture
- Discussion Prompts and Instructional Support



Student Worktext

Use the blue pages for in-class instruction and practice.

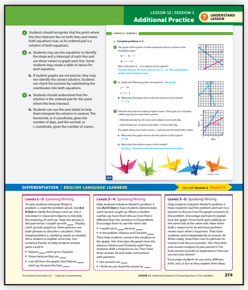


Instruction Page

Independent Practice for School or Home

Teacher's Guide

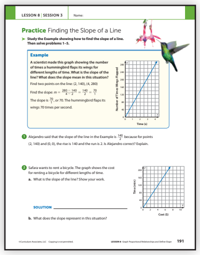
- Additional Practice
- Cumulative Practice



Student Worktext

Use the green pages for independent practice.

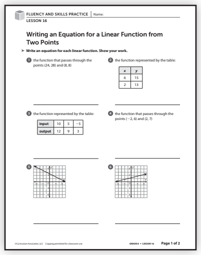
- Additional Practice
- Cumulative Practice



Additional Practice Page

Teacher Toolbox

- Fluency and Skills Practice
- Unit Game
- Cumulative Practice



Fluency and Skills Practice

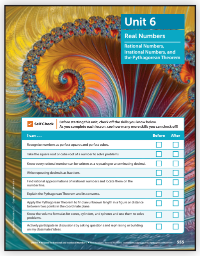
Assessments and Reports

Teacher's Guide

- Starts
- Support Whole Group/Partner Discussion
- Ask/Listen Fors
- Common Misconceptions
- Error Alerts
- Close: Exit Ticket

Student Worktext

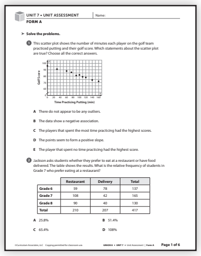
- Self Checks
- Apply It
- Reflect Questions
- Self Reflection
- Math Journal Questions
- Unit Review



Self Check

Teacher Toolbox

- Editable Lesson Quizzes
- Editable Unit Assessments

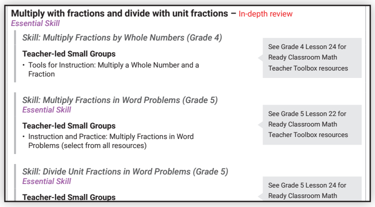


Unit Assessment

Differentiation

Before the Unit/Lesson: Prerequisites Report

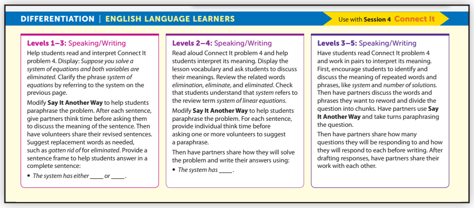
Recommended resources to support students' learning needs are highlighted in the Prerequisites report.



Prerequisites Report: Resources

During the Lesson: Teacher's Guide

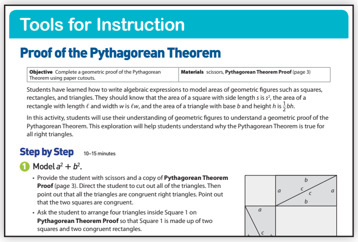
- Hands-On Activities or Visual Models
- Deepen Understanding
- ELL Differentiated Instruction
- Refine Sessions



ELL Differentiated Instruction

After the Lesson: Teacher Toolbox

- Reteach: Tools for Instruction
- Reinforce: Math Center Activities
- Extend: Enrichment Activities

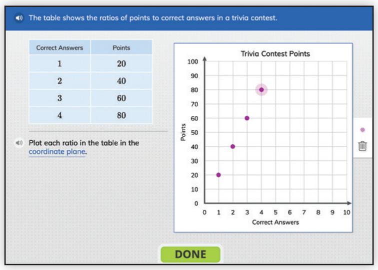


Reteach: Tools for Instruction

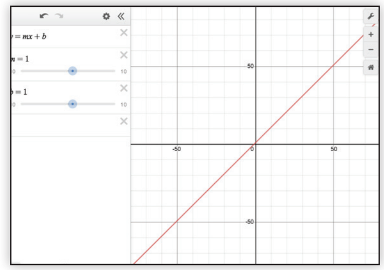
Digital Resources

Engage students with digital resources that provide interactive instruction, practice, assessment, and differentiation.

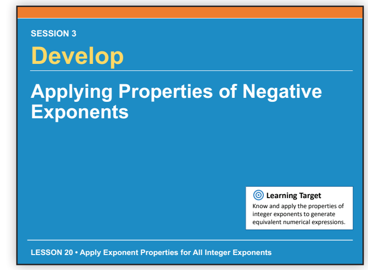
Resources can be found in the Student Digital Experience and the Teacher Digital Experience.



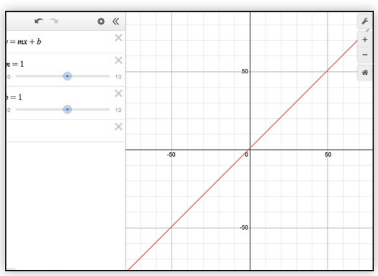
Interactive Tutorials



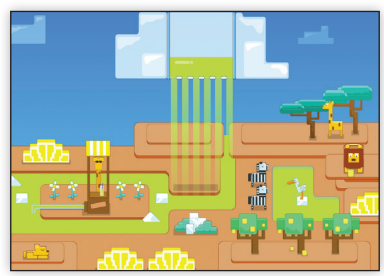
Digital Math Tools powered by Desmos



PowerPoint® Slides

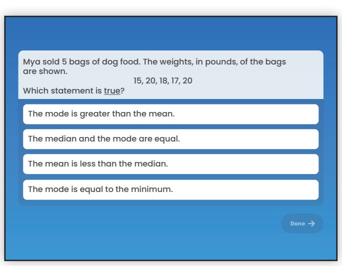


Digital Math Tools powered by Desmos

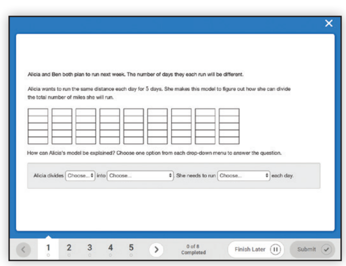


Learning Games

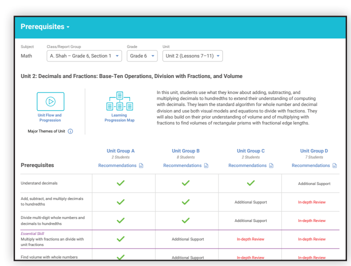
Digital Practice



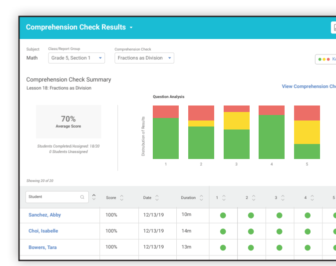
Diagnostic



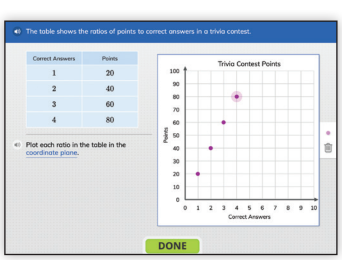
Lesson and Unit Comprehension Checks



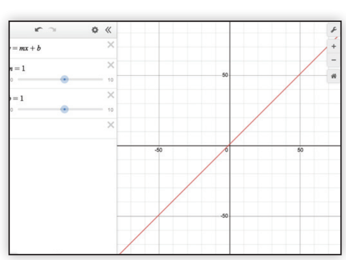
Prerequisites Report



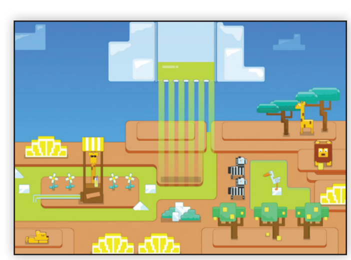
Comprehension Check Reports



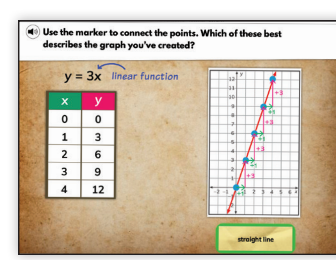
Interactive Tutorials



Digital Math Tools powered by Desmos



Learning Games



Optional Add-On: *i-Ready Personalized Instruction*

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UNIT 3

Linear Relationships

Slope, Linear Equations, and Systems

Before the Unit

- Self Check
- Prepare for Slope, Linear Equations, and Systems

After the Unit

- Math in Action
- Self Reflection
- Vocabulary Review
- Unit Review & Performance Task
- Unit Assessment, Forms A and B
- Digital Comprehension Checks

LESSON 8

Graph Proportional Relationships and Define Slope

PACING: 4 sessions

OBJECTIVES

- Understand that a proportional relationship is a linear relationship with an equation of the form $y = mx$.
- Interpret the unit rate of a proportional relationship as the slope of its graph.
- Understand that slope is the same between any two distinct points on a line.
- Find the slope of a line from two points by dividing the vertical change by the horizontal change or by using the slope formula.

STANDARDS

Focus: 8.EE.B.5, 8.EE.B.6 | **Developing:** none | **Applied:** 8.G.A.4, 8.G.A.5

VOCABULARY

NEW rate of change, slope

REVIEW congruent (≅), constant of proportionality, proportional relationship, right triangle, scale factor, similar (~), unit rate

ACADEMIC constant

LESSON-LEVEL DIFFERENTIATION

PREPARE **Ready Prerequisite Lessons**

Grade 6 Lesson 16 Use Unit Rates to Solve Problems
Grade 7 Lesson 4 Represent Proportional Relationships
Grade 8 Lesson 5 Perform and Describe Transformations Involving Dilations

RETEACH **Tools for Instruction**

Slope and Unit Rate

REINFORCE **Center Activity**

Find the Slope

EXTEND **Enrichment Activity**

Slope Rules

See the Lesson Overview for additional session-level differentiation.

LESSON 9

Derive and Graph Linear Equations of the Form $y = mx + b$

PACING: 5 sessions

OBJECTIVES

- Derive the equations $y = mx$ for a line through the origin and $y = mx + b$ for a line that intercepts the y-axis at b.
- Understand that when the equation of a line is given in slope-intercept form $y = mx + b$, m is the slope and b is the y-intercept.
- Understand that slope can be positive, negative, 0, or undefined.
- Graph linear equations in any form.

STANDARDS

Focus: 8.EE.B.6 | **Developing:** 8.F.A.3, 8.EE.C.7b | **Applied:** 8.EE.B.5

VOCABULARY

NEW linear equation, slope-intercept form, y-intercept

REVIEW slope

ACADEMIC define, derive, undefined

LESSON-LEVEL DIFFERENTIATION

PREPARE **Ready Prerequisite Lessons**

Grade 7 Lesson 3 Understand Proportional Relationships
Grade 7 Lesson 4 Represent Proportional Relationships
Grade 8 Lesson 8 Graph Proportional Relationships and Define Slope

RETEACH **Tools for Instruction**

The Equation of a Line

REINFORCE **Center Activity**

Use Slope-Intercept Vocabulary

EXTEND **Enrichment Activity**

Line Slide

See the Lesson Overview for additional session-level differentiation.

LESSON 10

Solve Linear Equations in One Variable

PACING: 4 sessions

OBJECTIVES

- Use one-variable linear equations with rational number coefficients to solve real-world and mathematical problems.
- Solve linear equations with the variable on both sides, including equations that require applying the distributive property and collecting like terms.

STANDARDS

Focus: 8.EE.C.7, 8.EE.C.7b | **Developing:** none | **Applied:** 8.G.A.5

NEW none

REVIEW coefficient, distributive property, like terms, perimeter, term, variable

ACADEMIC times as many

PREPARE **Ready Prerequisite Lessons**

Grade 6 Lesson 21 Write and Solve One-Variable Equations

Grade 7 Lesson 14 Use the Four Operations with Negative Numbers

Grade 7 Lesson 18 Write and Solve Multi-Step Equations

RETEACH **Tools for Instruction**

Solve Equations with the Variable on Both Sides

REINFORCE **Center Activity**

Match the Solution

EXTEND **Enrichment Activity**

Nesting Equations

See the Lesson Overview for additional session-level differentiation.

LESSON 11

Determine the Number of Solutions to One-Variable Equations

PACING: 4 sessions

OBJECTIVES

- Identify equations with infinitely many solutions or no solution.
- Write equations that have exactly one solution, infinitely many solutions, or no solution.
- Determine what constant term or variable term to use to complete an equation for a given number of solutions.

STANDARDS

Focus: 8.EE.C.7, 8.EE.C.7a | **Developing:** none | **Applied:** 8.EE.C.7b

NEW none

REVIEW distributive property, expression, like terms, linear equation, term, variable

ACADEMIC eliminate, in terms of, infinitely many

PREPARE **Ready Prerequisite Lessons**

Grade 6 Lesson 20 Understand Solutions of Equations

Grade 7 Lesson 15 Write Equivalent Expressions Involving Rational Numbers

Grade 8 Lesson 10 Solve Linear Equations in One Variable

RETEACH **Tools for Instruction**

Solutions of Linear Equations

REINFORCE **Center Activity**

Write an Equation

EXTEND **Enrichment Activity**

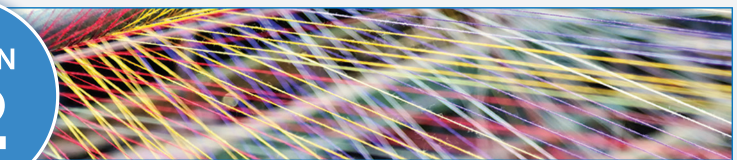
Deep Equations

. . . continued

UNIT 3

Linear Relationships

Slope, Linear Equations, and Systems



LESSON 12

Understand Systems of Linear Equations in Two Variables

PACING: 3 sessions

OBJECTIVES

- Understand that a system of linear equations is two or more related equations that are solved together to find a common solution. The solution is the set of ordered pairs that makes all equations in the system true.
- Use graphs and tables to identify the solutions to systems of two linear equations in two variables.
- Determine whether a system of two linear equations has one solution, infinitely many solutions, or no solution.

STANDARDS

Focus	Developing	Applied
8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b	8.F.B.4, 8.EE.C.8c	8.EE.C.7a, 8.EE.C.7b

VOCABULARY

NEW system of linear equations
REVIEW linear equation, slope
ACADEMIC common, context, intersection

LESSON-LEVEL DIFFERENTIATION

PREPARE **Ready Prerequisite Lessons**
Grade 6 Lesson 20 Understand Solutions of Equations
Grade 8 Lesson 9 Derive and Graph Linear Equations of the Form $y = mx + b$
Grade 8 Lesson 11 Determine the Number of Solutions to One-Variable Equations

RETEACH **Tools for Instruction**
Solutions of Systems of Linear Equations

REINFORCE **Center Activity**
Make Systems of Equations

EXTEND **Enrichment Activity**
System Solutions

See the Lesson Overview for additional session-level differentiation.

LESSON 13

Solve Systems of Linear Equations Algebraically

PACING: 5 sessions

OBJECTIVES

- Estimate the solution of a system of linear equations by graphing.
- Use substitution and elimination to solve systems of linear equations.
- Determine whether a system of linear equations has one solution, no solution, or infinitely many solutions.
- Identify efficient ways to solve a system of linear equations.

STANDARDS

Focus	Developing	Applied
8.EE.C.8, 8.EE.C.8b	8.EE.C.8c, 8.F.B.4	8.EE.C.8a, 8.EE.C.7a, 8.EE.C.7b

VOCABULARY

NEW none
REVIEW coefficient, system of linear equations
ACADEMIC algebraically, elimination, substitution

LESSON-LEVEL DIFFERENTIATION

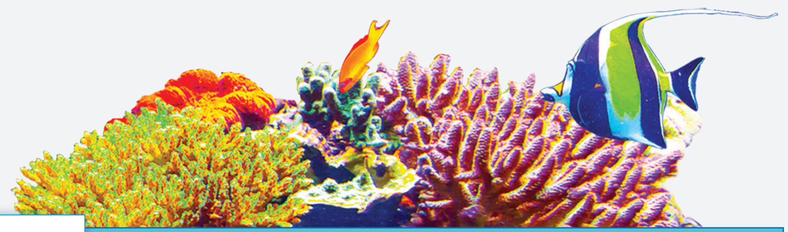
PREPARE **Ready Prerequisite Lessons**
Grade 7 Lesson 18 Write and Solve Multi-Step Equations
Grade 8 Lesson 10 Solve Linear Equations in One Variable
Grade 8 Lesson 12 Understand Systems of Linear Equations in Two Variables

RETEACH **Tools for Instruction**
Solve Systems of Equations by Substitution

REINFORCE **Center Activity**
Find Four Solutions

EXTEND **Enrichment Activity**
Salt Solutions

See the Lesson Overview for additional session-level differentiation.



LESSON 14

Represent and Solve Problems with Systems of Linear Equations

PACING: 4 sessions

OBJECTIVES

- Represent mathematical and real-world problems with two related linear equations in two variables.
- Graph systems of linear equations to estimate solutions.
- Solve systems of linear equations algebraically.
- Understand that a system of linear equations may have one solution, no solution, or infinitely many solutions.

STANDARDS

Focus: 8.EE.C.8, 8.EE.C.8c | **Developing:** 8.F.B.4 | **Applied:** none

NEW none

REVIEW expression, parallel lines, system of linear equations, *y*-intercept

ACADEMIC determine

PREPARE **Ready Prerequisite Lessons**
Grade 6 Lesson 22 Analyze Two-Variable Relationships
Grade 7 Lesson 18 Write and Solve Multi-Step Equations
Grade 8 Lesson 13 Solve Systems of Linear Equations Algebraically

RETEACH **Tools for Instruction**
Solve Real-World Systems of Equations

REINFORCE **Center Activity**
Match Scenarios and Systems

EXTEND **Enrichment Activity**
Space Challenge

See the Lesson Overview for additional session-level differentiation.

Math in Action

Math Lessons from the Real World

Coral Nursery | Analyzing Growth Data | Designing an Experiment

PACING: 2 sessions

OBJECTIVES

- Write and solve systems of linear equations.
- Write linear equations in slope-intercept form.
- Graph linear equations.
- Interpret slopes, *y*-intercepts, and points of intersection shown on a graph within the context of problems.
- Solve one-variable equations with variables on both sides.

STANDARDS

Focus: 8.EE.B.5, 8.EE.C.7a, 8.EE.C.7b, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c

Digital Resources

- Unit and Lesson Comprehension Checks
- Comprehension Check Reports
- Prerequisites Report
- Digital Tools powered by Desmos®
- Interactive Tutorials
- Learning Games (available 2020)
- Digital Practice (available 2021)

i-Ready

A personalized instruction path helps students fill prerequisite gaps and build up grade-level skills.

Linear Relationships

Slope, Linear Equations, and Systems

Which lessons are students building upon?

Grade 6, Lesson 16 Use Unit Rates to Solve Problems 6.RP.A.3b, 6.RP.A.3d	**Grade 7, Lesson 4** Represent Proportional Relationships 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2d	**Grade 8, Lesson 5** Perform and Describe Transformations Involving Dilations 8.G.A.3, 8.G.A.4
Grade 7, Lesson 3 Understand Proportional Relationships 7.RP.A.2b, 7.RP.A.2c	**Grade 7, Lesson 4** Represent Proportional Relationships 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2d	**Grade 8, Lesson 8** Graph Proportional Relationships and Define Slope 8.EE.B.5, 8.EE.B.6
Grade 6, Lesson 21 Write and Solve One-Variable Equations 6.EE.B.7	**Grade 7, Lesson 14** Use the Four Operations with Negative Numbers 7.NS.A.3, 7.EE.B.3	**Grade 7, Lesson 18** Write and Solve Multi-Step Equations 7.EE.B.4a
Grade 6, Lesson 20 Understand Solutions of Equations 6.EE.B.5	**Grade 7, Lesson 15** Write Equivalent Expressions Involving Rational Numbers 7.EE.A.1	**Grade 8, Lesson 10** Solve Linear Equations in One Variable 8.EE.C.7b
Grade 6, Lesson 20 Understand Solutions of Equations 6.EE.B.5	**Grade 8, Lesson 9** Derive and Graph Linear Equations of the Form $y = mx + b$ 8.EE.B.6	**Grade 8, Lesson 11** Determine the Number of Solutions to One-Variable Equations 8.EE.C.7a
Grade 7, Lesson 18 Write and Solve Multi-Step Equations 7.EE.B.4a	**Grade 8, Lesson 10** Solve Linear Equations in One Variable 8.EE.C.7b	**Grade 8, Lesson 12** Understand Systems of Linear Equations in Two Variables 8.EE.C.8a, 8.EE.C.8b
Grade 6, Lesson 22 Analyze Two-Variable Relationships 6.EE.C.9	**Grade 7, Lesson 18** Write and Solve Multi-Step Equations 7.EE.B.4a	**Grade 8, Lesson 13** Solve Systems of Linear Equations Algebraically 8.EE.C.8b

Lesson Progression

In this unit . . .

Which lessons are students preparing for?

In this unit . . .	Which lessons are students preparing for?	
Lesson 8 Graph Proportional Relationships and Define Slope 8.EE.B.5, 8.EE.B.6	**Grade 8, Lesson 9** Derive and Graph Linear Equations of the Form $y = mx + b$ 8.EE.B.6	**Grade 8, Lesson 16** Use Functions to Model Linear Relationships 8.F.B.4
Lesson 9 Derive and Graph Linear Equations of the Form $y = mx + b$ 8.EE.B.6	**Grade 8, Lesson 16** Use Functions to Model Linear Relationships 8.F.B.4	**Grade 8, Lesson 17** Compare Different Representations of Functions 8.F.A.2, 8.EE.B.5
Lesson 10 Solve Linear Equations in One Variable 8.EE.C.7b	**Grade 8, Lesson 13** Solve Systems of Linear Equations Algebraically 8.EE.C.8b	**Algebra** Reasoning with Equations and Inequalities HSA.REI.B.3
Lesson 11 Determine the Number of Solutions to One-Variable Equations 8.EE.C.7a	**Grade 8, Lesson 12** Understand Systems of Linear Equations in Two Variables 8.EE.C.8a, 8.EE.C.8b	**Algebra** Reasoning with Equations and Inequalities HSA.REI.A.1
Lesson 12 Understand Systems of Linear Equations in Two Variables 8.EE.C.8a, 8.EE.C.8b	**Grade 8, Lesson 13** Solve Systems of Linear Equations Algebraically 8.EE.C.8b	**Algebra** Reasoning with Equations and Inequalities HSA.REI.C.6
Lesson 13 Solve Systems of Linear Equations Algebraically 8.EE.C.8b	**Grade 8, Lesson 14** Represent and Solve Problems with Systems of Linear Equations 8.EE.C.8c	**Algebra** Reasoning with Equations and Inequalities HSA.REI.C.5
Lesson 14 Represent and Solve Problems with Systems of Linear Equations 8.EE.C.8c	**Algebra** Creating Equations HSA.CED.A.2	**Algebra** Reasoning with Equations and Inequalities HSA.REI.C.6

Professional Learning

Teacher Talk Moves That Engage Students in Discourse and Mathematical Thinking

Adapted from *Effective Teaching Strategies* by Grace Kelemanik and Amy Lucenta

Mathematical discourse is a powerful sense-making tool, but it doesn't just *happen*. These teacher talk moves are crucial supports for developing students' capacity to engage in productive mathematical discussion (Kazemi and Hintz, 2014; Chapin, O'Connor, and Anderson, 2009).

Individual Think Time

Think time provides students a short time—typically 10 seconds to 2 minutes—to think about a question or problem. It increases the quantity and quality of student talk because it gives students time to make sense of information and gather their thoughts.

Getting Started with Individual Think Time

- Consider when students will need to process ideas and how much time they may require.
- Craft a consistent prompt for individual think time. For example, *Take a moment to think by yourself* or *We'll start with individual think time*.
- Remind yourself to prompt think time by building an icon into your slide deck or putting a sticky by the document camera used to show student work.
- Pay attention to timing. What happens when think time is too short? Too long? Adjust accordingly.

Turn and Talk

Turn and Talk provides a safe space for students to rehearse ideas and language. It ensures all students, not just the fraction of them who speak in class, have opportunities to "talk math." Teachers often use this move to prepare students for a conversation or when they go silent during a class discussion (Kelemanik, Lucenta, and Janssen Creighton, 2016). Turn and Talk also provides English learners the option to use their home language.

Getting Started with Turn and Talk

- Give students a purpose, prompt, and product so they know what to talk about and why. *Will Olivia's strategy always work? Turn and talk with a partner to decide.*
- Specify a time frame. Start with 30 seconds to a minute to add some urgency to getting started.
- Provide sentence frames to support language development and act as the Turn and Talk product.

The Four Rs—Repeat, Rephrase, Reword, Record

The Four Rs strategy strings together one or more discrete talk moves to help students develop mathematical understanding and the language to communicate it. While helpful for all students, the Four Rs provides critical support for English learners, language minority students, and students with learning disabilities by giving them multiple passes at hearing the ideas and the language.

- **Repeat** A student restates an idea to give everyone a chance to hear it, or hear it again.
- **Rephrase** A student restates an idea in their own words—often building on the idea by adding details or new ideas to deepen everyone's understanding.
- **Reword** Students restate an idea using mathematical language, increasing the precision of the idea and the language with which it is conveyed.
- **Record** The teacher records an idea and/or specific language for students to reference.

Getting Started with the Four Rs

- Anticipate ideas students will share and list some math words or phrases that can help them communicate those ideas with mathematical precision. **Record** the terms if they come up in conversation or if you ask students to **reword** using them, or restate an idea using more precise language.
- Let students know that you will call on them to **rephrase** the ideas being shared. Remind them to look at and listen to the person who is talking.
- Start by prompting students to **repeat** and **rephrase**. After an idea is shared, ask one student to repeat it and then ask another student to rephrase. For example, *What did Morgan just say?* and *Who can share what they think they heard Sharice say?*

Grace Kelemanik
Grace Kelemanik is cofounder of Fostering Math Practices, an organization devoted to providing teachers and districts with practical strategies to improve student learning, and co-author of *Routines for Reasoning: Fostering the Mathematical Practices in All Students*. Central to her work is a focus on instructional routines that promote learning for all students.

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Differentiation for English Language Learners

Language Expectations

The chart below shows examples of what English learners at different levels of English language proficiency can do in connection with one of the Common Core State Standards (CCSS) addressed in this unit. As you plan for this unit, use these examples of language expectations to help you differentiate instruction to meet the needs of English learners.

Standard 8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

LANGUAGE DOMAINS	BEGINNING Level 1	INTERMEDIATE Level 2	Level 3	ADVANCED/ADVANCED HIGH Level 4	Level 5
LISTENING	Follow a series of one-step oral directions to write a linear equation using teacher modeling and illustrated examples.	Follow multi-step oral directions to write a linear equation using teacher modeling and illustrated examples.	Follow multi-step oral directions to write a linear equation using illustrated examples and peer support.	Follow detailed oral directions to write a linear equation with peer support.	Follow detailed oral directions to write a linear equation.
SPEAKING	Disagree with an equation written to represent a situation by pointing to the error, writing another equation, and matching each part of the second equation with corresponding parts of the problem.	Disagree with an equation written to represent a situation by identifying the error using a pattern sentence. Write another equation and use sentence frames to explain how each part matches the problem.	Disagree with an equation written to represent a situation by identifying the error and showing why it is incorrect. Write another equation and use sentence frames to explain how each part matches the problem.	Disagree with an equation written to represent a situation by identifying the error and showing why it is incorrect. Write another equation and, with partner support, explain how each part matches the problem.	Disagree with an equation written to represent a situation by identifying the error and showing why it is incorrect. Write another equation and explain how each part matches the problem.
READING	Make sense of word problems that are read aloud and demonstrated. Identify the y-intercept and slope from a graph and the problem using sentence frames and interactive support.	Follow along as word problems are read aloud. Make sense of the problem by identifying the y-intercept and slope from a graph and the problem using sentence frames and partner support.	Work with a partner to read and make sense of word problems. Identify the y-intercept and slope from a graph and the problem using sentence frames and partner support.	Work with a partner to read and paraphrase word problems. Identify the y-intercept and slope from a graph and the problem with partner support.	Read and paraphrase word problems. Confirm paraphrases with a partner. Prepare to write equations by identifying the y-intercept and slope from a graph and the problem and then comparing with a partner.
WRITING	With teacher support, complete sentence frames to explain what variables in an equation mean in the context of a problem.	Work with a partner to complete sentence frames to explain what variables in an equation mean in the context of a problem.	Work with a partner to write an explanation of the meaning of variables in the context of a problem.	Work with a partner to write an explanation of the meaning of variables in the context of a problem. Share explanations with another pair and revise as needed.	Write an explanation of the meaning of variables in the context of a problem. Share the explanation with a partner and revise as needed based on feedback.

Models, Progressions, and Teaching Tips

As you plan lessons, use this information to unpack the learning progressions and make connections between key concepts.

Unit Themes

The major themes of this unit are:

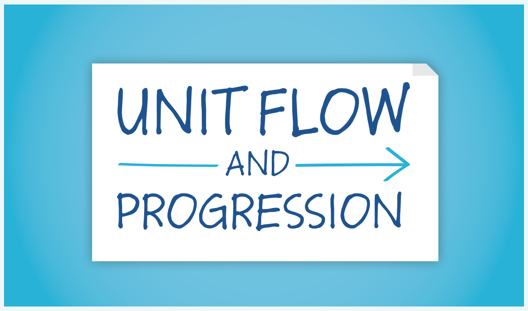
- A linear equation with two variables has a graph that is a straight line. Knowing about ratios and unit rate can help you make sense of the slope and *y*-intercept of a line.
- When you solve a linear equation in one variable, it will have solution, no solutions, or infinitely many solutions.
- A system of linear equations is a group of related linear equations where an ordered-pair solution makes all the equations true at the same time.

Prior Knowledge

Students will build on prior understandings of proportionality.

Students should:

- know that solving an equation means finding a value that makes the equation true.
- be able to solve equations of the form $px + q = r$ and $p(x + q) = r$.
- understand the idea of unit rate and constant of proportionality.
- be able to graph proportional relationships.



Watch the video
See the flow and progression of math concepts in this unit come to life with tips and insights on using models and making connections.

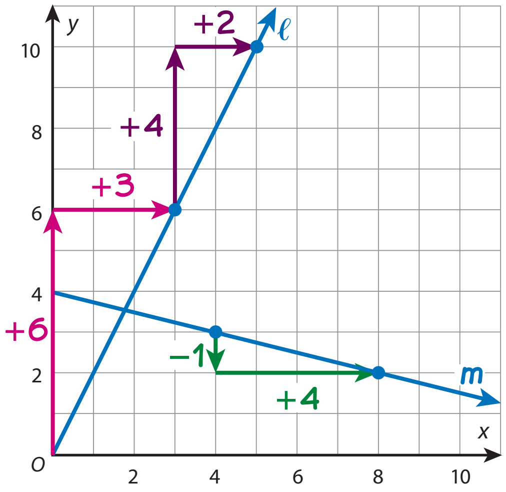
Slope of a Line

INSIGHTS ON . . .

The Concept of Slope

- ✔ What students know about unit rate helps them understand slope as a measure of the steepness of a line. The steeper the line, the faster the rate of change between the two related quantities.
- ✔ Students visualize slope by comparing vertical change (rise) to horizontal change (run) and find that the slope of a line is constant, regardless of which two points on the line were used to find it.
- ✔ Students see that the sign of the slope indicates direction. Lines with positive slopes point up to the right because both quantities are increasing. Lines with negative slopes point down to the right because as one quantity increases, the other decreases. If the vertical quantity changes more quickly than the horizontal quantity, the slope will be greater than 1.

Students **understand slope** as a measure of the **steepness** and **direction** of a line.



slope of $\ell = \frac{6}{3}$

$= \frac{4}{2}$

slope of $m = -\frac{1}{4}$

Slope of a Line *(continued)*

INSIGHTS ON . . .

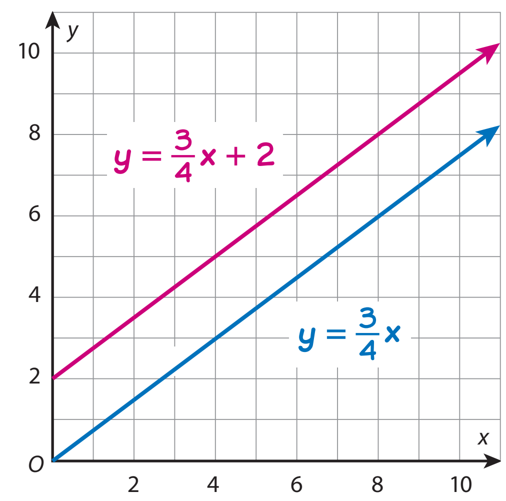
Slope-Intercept Form of an Equation

- ✔ Students explore linear relationships and make sense of the similarities and differences between those that are proportional and those that are not.
- ✔ In addition to slope, the other main feature of the graph of a linear relationship to examine is the *y*-intercept, which is the *y*-coordinate of the point where the graph intersects the *y*-axis.
- ✔ Any linear relationship can be modeled by the equation $y = mx + b$, where m is the slope and b is the *y*-intercept.
- ✔ Students learn that the graph of any proportional relationship has a *y*-intercept of 0, so the relationship can be modeled by $y = mx + 0$, or $y = mx$. The slope is equal to the unit rate.

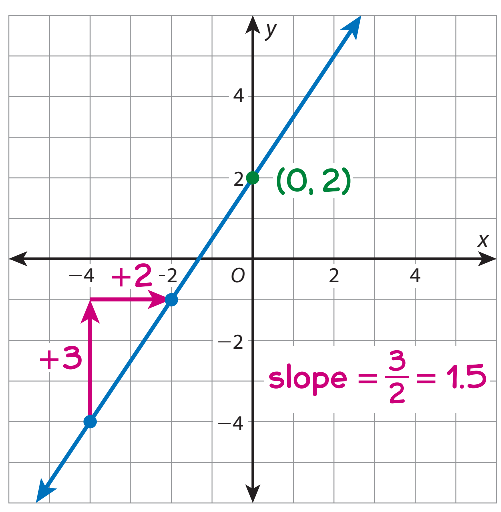
  EXAMPLE If $\frac{3}{4}$-inch cubes are stacked, the height of the stack is modeled by $y = \frac{3}{4}x$, where x = number of cubes. The *y*-intercept is 0 because if no cubes are stacked, the height is 0 inches.
- ✔ Students learn that the graph of any nonproportional linear relationship does not pass through the origin, and therefore has a nonzero *y*-intercept, *b*. The slope is a constant rate of change, but it does not represent a unit rate.

  EXAMPLE If $\frac{3}{4}$-inch cubes are stacked on top of a 2-inch block, the height is modeled by $y = \frac{3}{4}x + 2$. Even if no cubes are stacked, the height is 2 inches. In both situations, the height increases by $\frac{3}{4}$ in. with each additional cube.

Students compare graphs of **proportional relationships** to linear **nonproportional relationships** . . .



. . . to understand the **slope-intercept form of an equation**.



$$y = 1.5x + 2$$

slope   *y*-intercept

One-Variable Linear Equations

INSIGHTS ON . . .

Solving Linear Equations

✔ Students solved linear equations with one variable in Grades 6 and 7. In Grade 8, they solve more complex equations that may have the variable on both sides of the equal sign, and may also require using the distributive property to expand or factor expressions.

✔ Continue to support students' understanding of the equal sign to mean *the same as* and to see each step in solving an equation as maintaining balance.

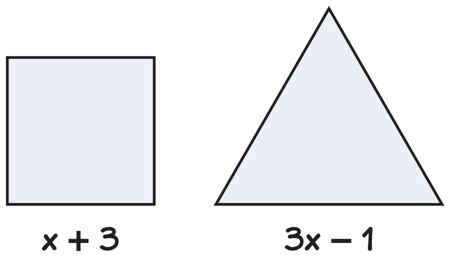
✔ Encourage students to inspect and reason about equations before solving to make sense of the relationship.

EXAMPLE Students should recognize that the solution of $4x + 8 = 3$ will not be a positive number, because when the quantity $4x$ is being increased by 8, the result is less than 8.

✔ **Common Error** Students may need extra support with distributing negatives in equations, such as $4 - (x - 5) = 6x$. They can benefit from thinking of the negative sign outside of parentheses as a factor of -1, which needs to be distributed to every term inside the parentheses just as a positive number would be.

Students **write equations** to model real-world situations . . .

The figures have the same perimeter. Solve for x.



	Square	Equilateral Triangle
Side Length	$x + 3$	$3x - 1$
Perimeter	$4(x + 3)$	$3(3x - 1)$

↓ ↓

$$4(x + 3) = 3(3x - 1)$$

. . . and **solve equations** using properties of operations.

$$4(x + 3) = 3(3x - 1)$$

$$4x + 12 = 9x - 3$$

$$4x - 4x + 12 = 9x - 4x - 3$$

$$12 + 3 = 5x - 3 + 3$$

$$15 = 5x$$

$$3 = x$$

One-Variable Linear Equations *(continued)*

INSIGHTS ON . . .

Linear Equations with No Solution or Infinitely Many Solutions

- ✔ In earlier grades, students learned that the solution of an equation is the value for a variable that makes the equation true. They now expand on this understanding by working with equations that have no solution or infinitely many solutions.
- ✔ Some students may find the idea of no solution or many solutions unsettling because it conflicts with their prior assumptions. Take advantage of this to engage them in conversations about what real-world situations might produce these types of equations, and what the solution means.

  EXAMPLE Suppose you have bank accounts with \$100 and \$200. If you deposit \$50 into each account every month, they will never have the same balance. This means if you solve $100 + 50x = 200 + 50x$, there will be no solution.
- ✔ Looking ahead, students will use these ideas to solve systems of equations with two variables that might have one solution, no solutions, or infinitely many solutions.

Students **solve linear equations** and find that there can be **one solution,** . . .

$$4x + 8 = 3x + 13$$

$$4x - 3x + 8 - 8 = 3x - 3x + 13 - 8$$

$$x = 5 \leftarrow \text{only true for one value}$$

. . . **no solution** . . .

$$5x + 2 = 5x + 5$$

$$5x - 5x + 2 = 5x - 5x + 5$$

$$2 = 5 \leftarrow \text{never true}$$

. . . or **infinitely many solutions**.

$$1 + 4x = 2x + 1 + 2x$$

$$1 + 4x = 4x + 1$$

$$1 = 1 \leftarrow \text{always true}$$

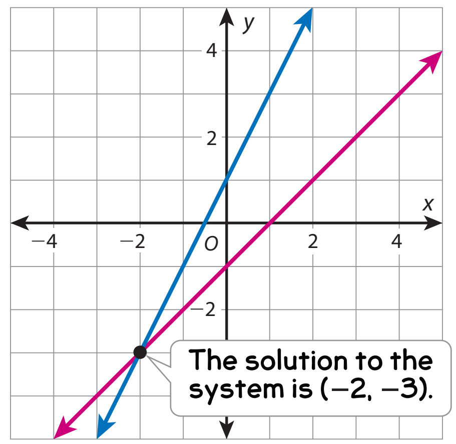
Systems of Linear Equations

INSIGHTS ON . . .

Understanding Systems of Equations

- ✔ Students extend their understanding of solving equations by asking: *How can I find x- and y- values that will make both of two different equations true?*
- ✔ Students learn that they can solve a system of equations to find the conditions that make two statements about a real-world situation true.
- ✔ Graphing a system of linear equations allows students to visualize the solution as any points that the lines have in common, as well as to estimate the solution.

Students **graph two lines** to visualize the *x*- and *y*-values that make two different equations true.



INSIGHTS ON . . .

Solving Systems of Linear Equations

- ✔ Students will explore both the substitution and elimination methods for solving systems.
- ✔ Students use bar models to help them visualize how an expression for one variable can be substituted into the other equation to focus on only one variable at a time.
- ✔ With experience, students begin to see that they are making connections between the two equations to write one equation with one variable.
- ✔ Students will make use of structure and recognize that some systems are easier to solve with substitution, while others are easier to solve with elimination. Be sure students recognize that either method will give the same solution.
- ✔ **Error Alert** Students might find the value of only one of the variables when solving a system. Remind students that a solution to a system is an ordered pair, so they must provide both coordinates. Ask: *Each equation represents a line. At what point do the lines intersect?*

Students find solutions of systems of equations by **substituting** from one equation into the other . . .

Solve: $x = 2y + 5$ and $x + y = 14$

$x = 2y + 5$

x		
y	y	5

$x + y = 14$

14	
x	y

↓

$2y + 5 + y = 14$

14			
y	y	5	y

. . . and by **eliminating** one of the variables.

Solve: $3m + 6n = 33$ and $3m - 3n = 6$

$$\begin{aligned} 3m + 6n &= 33 \\ 2(3m - 3n &= 6) \end{aligned} \quad \rightarrow \quad \begin{array}{r} 3m + 6n = 33 \\ +\ 6m - 6n = 12 \\ \hline 9m + 0n = 45 \end{array}$$

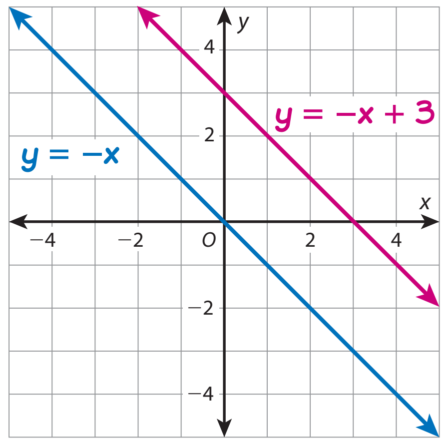
Systems of Linear Equations *(continued)*

INSIGHTS ON . . .

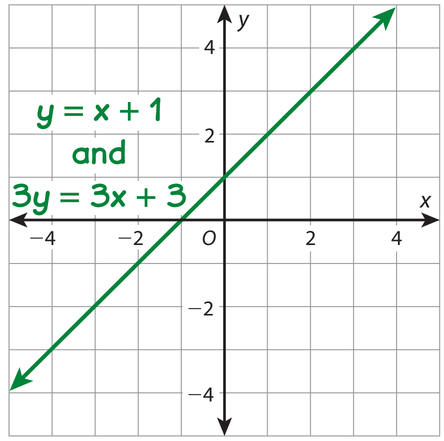
Systems with No Solution or Infinitely Many Solutions

- ✔ Just as with linear equations in one variable, students see that systems of linear equations in two variables have one, zero, or infinitely many solutions.
- ✔ Give special attention to what it means when a system has no solution or infinitely many solutions by referring back to the problem context.
- ✔ Students relate graphs of the equations and the structure of the equations to conclude that:
  - When the lines have different slopes, there is exactly one solution given by the point where they cross.
  - When the lines have the same slope and the same *y*-intercept, they are the same line and every point on that line is a solution.
  - When the lines have the same slope and different *y*-intercepts, they are parallel lines that never cross so there is no solution.
- ✔ **Common Misconception** Students may think that a system has infinitely many solutions any time the equations in the system have the same slope. If so, be sure to provide many examples of systems that represent parallel lines. Help them make the connection that if slopes are equal, then lines are parallel and will never intersect.

Students find that systems of equations can also have **no solution** . . .



. . . or **infinitely many solutions**.



Future Learning

Students will move on to using functions to model relationships.

Students will:

- interpret and write linear and nonlinear functions.
- explore and compare increasing and decreasing functions.
- compare functions written in different forms.
- determine the average rate of change of a function along a specific interval.

Unit Big Ideas

The major themes of this unit are:

- A linear equation with two variables has a graph that is a straight line. Knowing about ratios and unit rate can help you make sense of the slope and y-intercept of a line.
- When you solve a linear equation in one variable, there might be one solution, no solutions, or infinitely many solutions.
- A system of linear equations is a group of related linear equations where a solution makes all the equations true at the same time. You can use what you know about solving equations to solve systems of equations.

This unit introduces students to the concept of slope, and solving linear equations and systems of equations. Students preview the skills they will be learning in this unit and assess what they know and do not know about them. Students record their progress after completing each lesson and reflect on their learning at the end of the unit.

Self Check

- Take a few minutes to have each student independently read through the list of skills. Ask students to consider each skill and check the box if it is a skill they think they already have.
- Remind students that these skills are likely to all be new to them and that over time, they will be able to check off more and more skills.

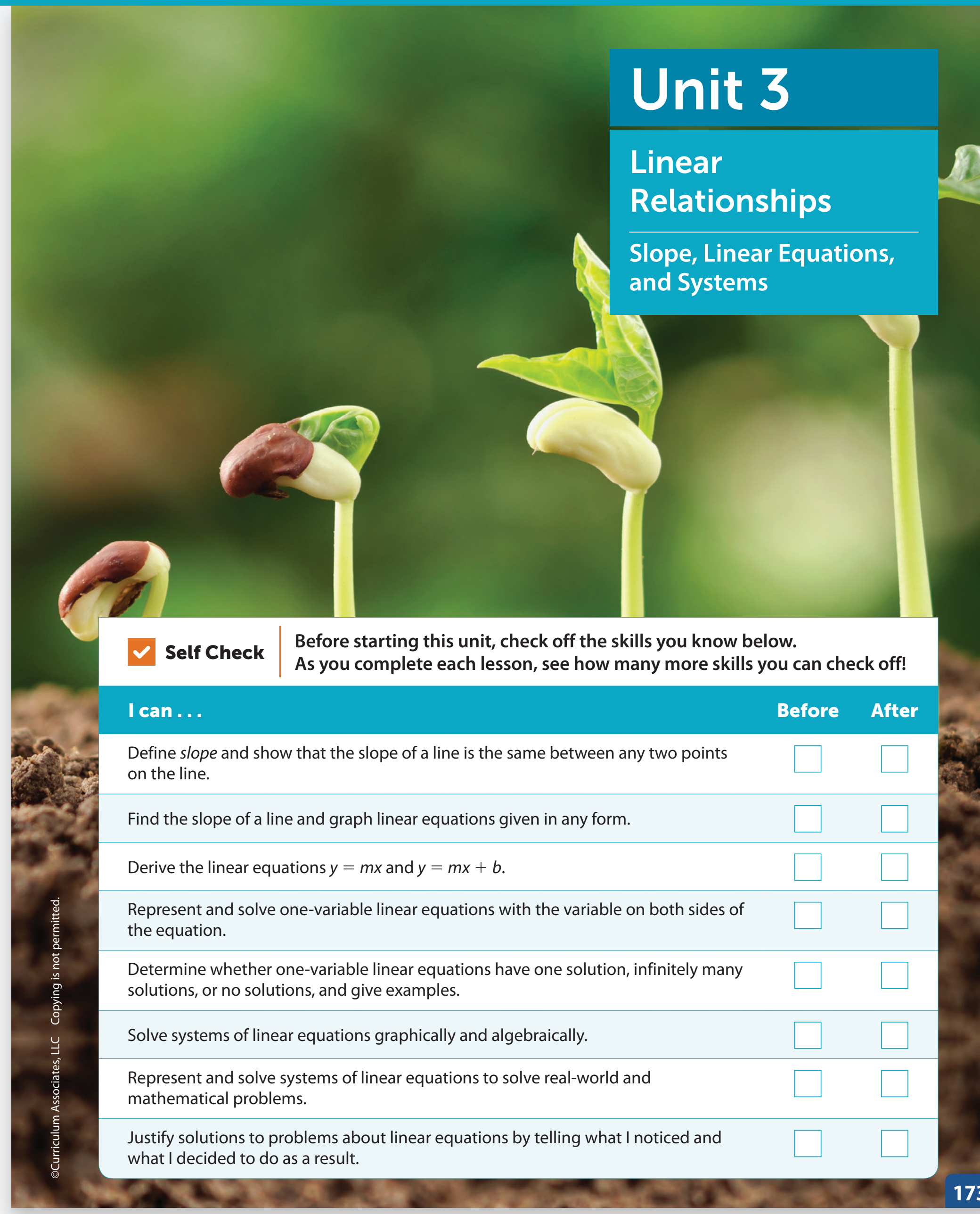
Support Whole Class Discussion

Engage students in a discussion about the skills with questions such as:

- *Which skills seem related to something you already know?*
- *Which skills do you think you would use in your everyday life? Why?*

Support Growth Mindset

At the end of the unit have students review the skills on the **Student Book Self Reflection** page and work in pairs to respond to the prompts. Encourage students to revisit the work they did in each lesson.



Unit 3

Linear Relationships

Slope, Linear Equations, and Systems

Self Check

Before starting this unit, check off the skills you know below.
As you complete each lesson, see how many more skills you can check off!

I can . . .	Before	After
Define *slope* and show that the slope of a line is the same between any two points on the line.		
Find the slope of a line and graph linear equations given in any form.		
Derive the linear equations $y = mx$ and $y = mx + b$.		
Represent and solve one-variable linear equations with the variable on both sides of the equation.		
Determine whether one-variable linear equations have one solution, infinitely many solutions, or no solutions, and give examples.		
Solve systems of linear equations graphically and algebraically.		
Represent and solve systems of linear equations to solve real-world and mathematical problems.		
Justify solutions to problems about linear equations by telling what I noticed and what I decided to do as a result.		

Unit Skills

Skill	Lesson(s)
Define *slope* and show that the slope of a line is the same between any two points on the line.	8
Find the slope of a line and graph linear equations given in any form.	8, 9
Derive the linear equations $y = mx$ and $y = mx + b$.	9
Represent and solve one-variable linear equations with the variable on both sides of the equation.	10
Determine whether one-variable linear equations have one solution, infinitely many solutions, or no solutions, and give examples.	11
Solve systems of linear equations graphically and algebraically.	12, 13
Represent and solve systems of linear equations to solve real-world and mathematical problems.	14
Justify solutions to problems about linear equations by telling what you noticed and what you decided to do as a result.	8–14

Prepare For Unit 3

- Read the directions and the headings in the boxes or have a student do so. Call on volunteers to explain the task in their own words.
- Have students write what they know about proportional relationships in the graphic organizer.
- Have students work with a partner to share their ideas. Give students time to revise and add onto what they have written in their graphic organizers. Circulate and validate responses or clarify any misconceptions.
- After most students have finished, debrief with a whole class discussion.

Build Academic Vocabulary

- Display academic terms used throughout this unit: *constant, determine,* and *system*. Students will likely have some prior knowledge of the terms from math learning in previous grade levels or other content areas. Use the **Academic Vocabulary** routine described in Unit 1 Professional Learning to provide explicit instruction and active engagement.
- Academic vocabulary for each lesson is listed in the Lesson Overview. The chart below includes the Spanish cognates for academic vocabulary introduced in the unit and in each lesson. To support students whose primary language is Spanish, use the **Cognate Support** routine described in Unit 1 Professional Learning.
- Support students as they move from informal language to formal academic language by using the **Collect and Display** routine. Have students refer to the chart during discussion and writing.

UNIT 3 Prepare for Slope, Linear Equations, and Systems

Think about what you know about rates, proportional relationships, and plotting points in a coordinate plane. Write what you know about proportional relationships in the boxes. Share your ideas with a partner and add any new information to the organizer.

Possible answers:

Proportional Relationships

Equations
$y = kx$
$y = 5x$
$\frac{2}{3}x = y$

Descriptions
The quantities in a proportional relationship all vary by the same factor.
The graphs are always straight lines that go through the origin.

Related Terms
- unit rate
- constant rate of change
- constant of proportionality

Examples
- A robot traveling at a constant speed is a proportional relationship between distance and time.
- The total cost for a number of items that each have the same price is a proportional relationship between number of items and their price.

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Cognates for Academic Vocabulary in Unit 4

ACADEMIC WORD	SPANISH COGNATE	ACADEMIC WORD	SPANISH COGNATE
algebraically	*algebraicamente*	eliminate	*eliminar*
common	*común*	elimination	*eliminación*
constant	*constante*	intersection	*intersección*
context	*contexto*	substitution	*sustitución*
define	*definir*	system	*sistema*
derive	*derivar*	undefined	*indefinido(a)*
determine	*determinar*		

Lesson 12

Understand Systems of Linear Equations in Two Variables

Overview | Understand Systems of Linear Equations in Two Variables

UNDERSTAND: What does it mean to solve a system of linear equations?

MATH FOCUS

This Understand lesson extends students' understanding of linear equations and their graphs to solving systems of two linear equations in two variables. Foundational understanding established in this lesson supports students in developing strategies to solve systems of linear equations algebraically.

Focus Standards

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

See Unit 3 Pacing Guide for developing and applied standards.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 2, 3, and 7 are integrated into the Understand lesson structure.*

This lesson provides additional support for:

4 Model with mathematics.

* See page XX to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Understand that a system of linear equations is two or more related equations that are solved together to find a common solution. The solution is the set of ordered pairs that makes all equations in the system true.
- Use graphs and tables to identify the solutions to systems of two linear equations in two variables.
- Determine whether a system of two linear equations has one solution, infinitely many solutions, or no solution by graphing or by analyzing equations.

Language Objectives

- Discuss advantages and disadvantages of using graphs and tables to solve systems of equations in partner and whole-class discussions.
- Use lesson vocabulary and graphs to describe the solution to a system of equations with one solution, infinitely many solutions, or no solution.
- Explain in writing and speaking what a solution of a system means in terms of the problem context.
- Describe a way to test a strategy or value to see if it is true.

Prior Knowledge

- Understand that linear equations in one variable have exactly one solution, infinitely many solutions, or no solution.
- Identify slopes and y-intercepts of lines from linear equations and graphs.
- Write linear equations in two variables in slope-intercept form.
- Graph linear equations.

Vocabulary

Math Vocabulary

system of linear equations a group of related linear equations in which a solution makes all the equations true at the same time. A system of equations can have zero, one, or infinitely many solutions.

Review the following key terms.

linear equation an equation whose graph is a straight line.

slope for any two points on a line, the $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$. It is a measure of the steepness of a line.

Academic Vocabulary

common shared by two or more things.

context the situation in which something happens.

intersection the place or point where two or more things meet or cross.

Learning Progression

In Grade 7, students used variables to represent quantities and wrote expressions and equations to model real-world situations.

Earlier in Grade 8, students learned to graph linear equations and that the graph represents all solutions of the equation. They worked with slope-intercept form and slope.

In this lesson, students consider systems of two linear equations. They learn that an ordered pair is a solution of a system if it satisfies both equations and describes a point that lies on both lines. Students find the solution to systems of two linear equations graphically and classify systems of linear equations as having one solution, infinitely many solutions, or no solution.

In the next lesson, students solve systems of two linear equations in two variables algebraically.

In subsequent lessons, they will model and solve real-world problems using systems of linear equations.

In high school, students will solve systems that include both linear and nonlinear equations.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Systems of Linear Equations in Two Variables (35–50 min)		
• **Start** (5 min) • **Model It** (5 min) • **Discuss It** (5–10 min) • **Model It** (5–10 min) • **Discuss It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 273–274)	Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model **Materials** For display: a large four-quadrant coordinate plane
SESSION 2 **Develop** Understanding of the Number of Solutions to a System of Linear Equations (45–60 min)		
• **Start** (5 min) • **Model It: No Solution** (5 min) • **Discuss It** (5–10 min) • **Model It: Infinitely Many Solutions** (5 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 277–278)	Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: transparency markers, transparency of Activity Sheet *Coordinate Plane: Four Quadrants* **REINFORCE** Fluency & Skills Practice
SESSION 3 **Refine** Ideas About Systems of Linear Equations in Two Variables (45–60 min)		
• **Start** (5 min) • **Apply It** (35–50 min) • **Close: Exit Ticket** (5 min)	Presentation Slides	
Lesson 12 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 ■ □ □

Model It Schools and libraries often have reading contests to encourage students to improve their reading skills as they explore different types of books. In these contests, students record how many pages or how many books they have read. Invite students to talk about a time that they tracked the number of pages or books they read over a certain time period. If students have participated in a reading contest, ask them to share the best thing they discovered about reading.

SESSION 2 ■ ■ □

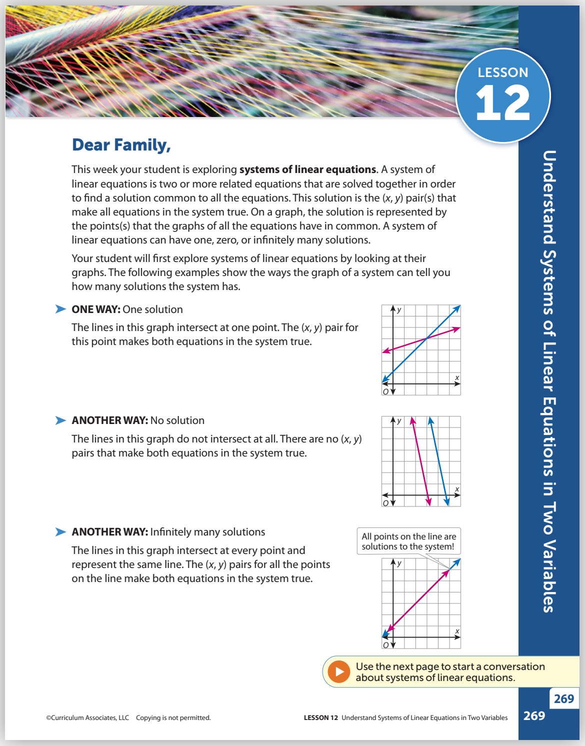
Model It Ask if any of your students have hiked all or part of the Appalachian Trail or know someone who has. Invite them to share their experiences with the class. The Appalachian National Scenic Trail is a marked hiking trail that extends from Maine to Georgia. The full trail passes through 14 states and is approximately 2,200 miles long, which is about 5,000,000 steps. People who hike the entire trail within a calendar year are called thru-hikers, and most take about 6 months to complete their journey. Ask students to estimate the farthest distance they have hiked in a single day. Make a dot plot of the class' data.

SESSION 3 ■ ■ ■

Apply It **Problem 4** If time allows or at a later time, ask students to share their experiences watching or participating in a cross-country race or another running event. Unlike racing events that take place in lanes marked on tracks with smooth surfaces, cross-country races take place outside on surfaces that are not always paved. Runners may encounter muddy conditions or uneven surfaces. Sometimes the path is marked only with flags that show runners where to turn. International cross-country competitions were first held in 1973.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**


LESSON 12

Dear Family,

This week your student is exploring **systems of linear equations**. A system of linear equations is two or more related equations that are solved together in order to find a solution common to all the equations. This solution is the (x, y) pair(s) that make all equations in the system true. On a graph, the solution is represented by the points(s) that the graphs of all the equations have in common. A system of linear equations can have one, zero, or infinitely many solutions.

Your student will first explore systems of linear equations by looking at their graphs. The following examples show the ways the graph of a system can tell you how many solutions the system has.

➤ **ONE WAY:** One solution

The lines in this graph intersect at one point. The (x, y) pair for this point makes both equations in the system true.

➤ **ANOTHER WAY:** No solution

The lines in this graph do not intersect at all. There are no (x, y) pairs that make both equations in the system true.

➤ **ANOTHER WAY:** Infinitely many solutions

The lines in this graph intersect at every point and represent the same line. The (x, y) pairs for all the points on the line make both equations in the system true.

All points on the line are solutions to the system!

Use the next page to start a conversation about systems of linear equations.

Understand Systems of Linear Equations in Two Variables

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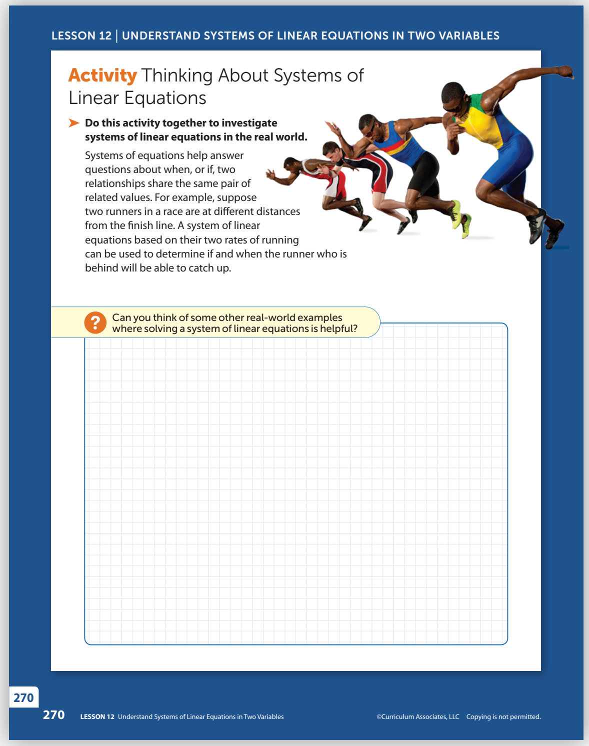
LESSON 12 | UNDERSTAND SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES

Activity Thinking About Systems of Linear Equations

➤ **Do this activity together to investigate systems of linear equations in the real world.**

Systems of equations help answer questions about when, or if, two relationships share the same pair of related values. For example, suppose two runners in a race are at different distances from the finish line. A system of linear equations based on their two rates of running can be used to determine if and when the runner who is behind will be able to catch up.

Can you think of some other real-world examples where solving a system of linear equations is helpful?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Model It**

MATH TERMS

To *intersect* means to "cross."

An *ordered pair* is a pair of numbers, (x, y), that describes the location of a point in the coordinate plane.

ACADEMIC VOCABULARY

Exact means "accurate and precise."

Levels 1–3: Reading/Speaking

Read Model It problem 2 aloud and have students follow along. Prepare students to respond to the questions by using a **Co-Constructed Word Bank**. Add *common*, *solution*, *intersect*, and *intersection* if students do not suggest them. After students graph the equations, check for understanding by having them touch the part of their graph that matches the words as you say: *common point*, *intersection*, *solution*, *point with coordinates* $(-1, 1)$. Observe that they point to the point of intersection for each. Then have them verify the solution.

Levels 2–4: Reading/Speaking

Read Model It problem 2 with students. Use **Co-Constructed Word Bank** to support students in responding to the questions. Have them use the terms *intersect* and *intersection* using sentence starters:

- *The lines cross or _____.*
- *The point where they cross is the _____.*

Have students graph the equations and locate the intersection. Then ask students to turn to a partner and discuss how they can verify their solutions. Have them complete:

- *If the point I identified is the solution, then _____.*

Levels 3–5: Reading/Speaking

Have students turn to a partner to read Model It problem 2 and review vocabulary terms in the problem. Call on volunteers to explain how the word *intersect* relates to the word *intersection*. Have students use the two terms as they discuss the problem. Have partners graph and discuss the equations and name the ordered pair for the intersection. Ask them to take turns telling how they can verify that the point of intersection is the solution. Then have students verify the solution and explain the steps using the word *substitute*.

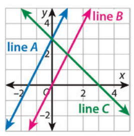
Explore Systems of Linear Equations in Two Variables

Purpose

- **Explore** the idea of using a graph or a table to find the solution(s) common to two linear equations.
- **Understand** that the solution of a system of equations is represented by the point(s) where the graphs of the equations intersect.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different



Possible Solutions

Lines *A* and *B* have the same slope but different *y*-intercepts.

Lines *A* and *C* have different slopes but the same *y*-intercept.

Lines *B* and *C* have different slopes and different *y*-intercepts.

WHY? Prepare students to explore graphs of systems of equations by identifying slopes and *y*-intercepts.

MODEL IT SMP 2, 4

Read the *Understand* question at the top of the page. Remind students that they know how to graph a linear equation that is in slope-intercept form.

1 – 2 See ***Connect to Culture*** to support student engagement. Tell students that they are going to use what they know about graphs of linear equations to investigate the solutions to systems of linear equations. Read the problems and call on students to rephrase them to confirm understanding. Have students turn and talk about how the equations of the lines and their graphs are related.

Error Alert If students get the wrong point of intersection, then they may have graphed the equations incorrectly or imprecisely. Review how to identify the slope and *y*-intercept from a linear equation and how to use these values to graph the line. Encourage use of a straightedge for better precision. Remind students that the process is the same when variables other than *x* and *y* are used.

LESSON 12 | SESSION 1

UNDERSTAND: What does it mean to solve a system of linear equations?

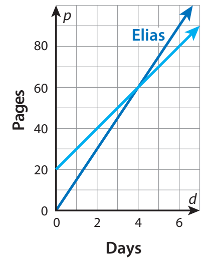
Explore Systems of Linear Equations in Two Variables

Model It

Complete the problems about graphing related linear equations.

1 Elias reads 15 pages of his book each day. The graph of $p = 15d$ shows how many pages Elias reads in *d* days.

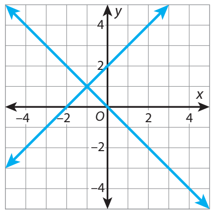
Before Elias starts reading, Jaime has already read 20 pages of the same book. Jaime then reads 10 pages each day. The equation $p = 10d + 20$ tells how many pages Jaime has read since Elias started reading.



a. Graph Jaime's equation in the same coordinate plane as Elias's graph. What point is on both lines? See graph. (4, 60)

b. On what day have both Elias and Jaime read the same number of pages? How many pages have they read on that day? Use your answer to problem 1a to explain.
Day 4; 60 pages; (4, 60) is on both lines. This means $d = 4$ and $p = 60$ for both equations, and so apply to both boys.

2 The related equations in problem 1 for which you found a common solution form a **system of linear equations**. You can graph the equations in a system to see if the lines intersect. Any point that is common to both lines represents a solution to the system of equations. This means that the ordered pair for this point makes both equations in the system true.



a. Graph the equations $y = x + 2$ and $y = -x$. Where do the lines intersect? See graph. (−1, 1)

b. You cannot always know the exact coordinates of a point of intersection from a graph. How can you be sure that the point you identified in problem 2a is a solution to the system?
Possible answer: If the ordered pair is a solution, then it will make both equations true. So, I can substitute −1 for *x* and 1 for *y* in both equations to make sure I get true statements.

DISCUSS IT

Ask: What can the graph of a system of equations tell you?

Share: Seeing the graphs of the equations is helpful because . . .

Learning Targets SMP 2, SMP 3, SMP 4, SMP 7
Analyze and solve pairs of simultaneous linear equations.
- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

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DISCUSS IT SMP 3, 7

Support Partner Discussion

After students complete problems 1 and 2, have them respond to Discuss It with a partner. Encourage them to think about what the graphs of the equations represent. Listen for understanding that:

- the line for each equation represents all the ordered pairs—(*d*, *p*) for problem 1 and (*x*, *y*) for problem 2—that make the equation true.
- the point of intersection is on both lines, so its ordered pair makes both equations true.

Facilitate Whole Class Discussion

Have students look back at the graph for problem 1 and think about what information the graph reveals about the situation.

ASK *For which days is Jaime's line is above Elias' line? What does this mean?*

LISTEN FOR Jaime's line is above Elias' line for days 0, 1, 2, and 3. This means that Jaime has read more pages than Elias on those days.

MODEL IT

SMP 2

3 Tell students that they will now think about using a table to find the solution to a system of linear equations. Prompt students to discuss how finding the solution using a table is similar to and different from using a graph.

Error Alert If students complete the table and think the solution of the system is $(-2, -2)$, then ask them to read the labels for each row. Remind students that the second and third row each name y-coordinates of points that have the same x-coordinate, which is named in the first row.

DISCUSS IT

SMP 3, 7

Support Partner Discussion

After students complete problem 3a, have them respond to Discuss It with a partner. Encourage them to compare graphing and using a table.

Listen for understanding that:

- when using a table to find a solution, you look for an x-value that has the same y-value for both equations.
- a table shows only some of the ordered pairs that are solutions of each equation, while a graph represents all solutions.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Find solutions to systems of equations using tables and graphs.

If students are unsure about using tables and graphs to identify solutions to systems of linear equations, then use this activity to provide an additional example.

Materials For display: a large four-quadrant coordinate plane

- Display the equation $y = x + 2$. Set up a table, with columns for x and y. List integer values from -3 to 3 in the x column.
- For each x-value, have a student enter the corresponding y-value and plot the point.
- When all the points have been plotted, draw a line through them.
- Follow this process to make a table and graph for $y = 2x - 1$, using the same coordinate plane.
- Ask students to identify the point of intersection. Circle the coordinates of the point in both tables.
- Emphasize that the point of intersection is a solution of both equations by having students substitute the coordinates into both equations to check.

LESSON 12 | SESSION 1

Model It

➤ **Complete the problems about solving systems of linear equations.**

3 Another way to find a solution to a system of equations is to list ordered pairs that make each equation in the system true. An ordered pair that makes *both* equations true is a solution to the system.

Look at the system of equations below.

$y = 2x$

$y = x - 1$

a. Complete the table. For each equation, find the y-value for each x-value. What is the solution to the system of equations? How do you know?

x	-4	-3	-2	-1	0	1	2	3	4
$y = 2x$	-8	-6	-4	-2	0	2	4	6	8
$y = x - 1$	-5	-4	-3	-2	-1	0	1	2	3

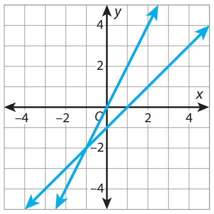
$(-1, -2)$; For both equations, $y = -2$ when $x = -1$.

DISCUSS IT

Ask: How does using a table help you find the solution to a system of equations?

Share: Using a table is different from using a graph because . . .

b. Graph the system of equations to check that your answer to problem 3a is reasonable.



4 **Reflect** How is the solution to the system of equations in problem 3 related to the solutions of the individual equations $y = 2x$ and $y = x - 1$?

Possible answer: The equations $y = 2x$ and $y = x - 1$ each have infinitely many solutions. The solution to the system is the only ordered pair that is a solution to both of the equations.

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Facilitate Whole Class Discussion

Prompt students to talk about using graphs and tables to identify solutions to systems of equations.

ASK *What do you think are the advantages and disadvantages of using a table to find the solution to a system of equations? What do you think are the advantages and disadvantages of using a graph?*

LISTEN FOR A table shows exact values, but the system's solution may not be listed. The graph represents all solutions for each equation, but the solution of the system is an estimation, and you need to check that it makes both equations true.

CLOSE EXIT TICKET

4 **Reflect** Look for understanding that the solution of a system is a solution of each of the individual equations.

Common Misconception If students think they can always identify an exact solution by graphing, then explain that sometimes the solution point will be between grid lines and must be estimated. Encourage students to always check a solution they find by graphing by substituting it into both equations.

Prepare for Systems of Linear Equations in Two Variables

Support Vocabulary Development

Assign **Prepare for Systems of Linear Equations in Two Variables** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *linear equation*. Encourage students to consider both words in the term as they construct their responses. Students should be able to see that the word *linear* is related to the word *line* and should acknowledge that they are familiar with the term *equation*.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the words, illustrations, graphs, and examples given.

Have students look at the equation in problem 2 and discuss with a partner different strategies for sketching the graph of the equation.

Problem Notes

1. Students should understand that a linear equation is an equation whose graph is a straight line. Student responses might include different forms of linear equations. Students may recognize that most linear equations can be written in slope-intercept form and that this form is useful for graphing the equation.

2. Students should recognize that the graph of the equation $y = x - 2$ has slope 1 and y-intercept -2, so the line goes through the point $(0, -2)$. Student responses might include a table of values to help identify points on the graph.

LESSON 12 | SESSION 1 Name:

Prepare for Systems of Linear Equations in Two Variables

1. Think about what you know about linear equations. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

What Is It?

A linear equation is an equation whose graph is a line.

What I Know About It

The linear equation of a nonvertical line can be written in the form $y = mx + b$, where m is the slope, or rate of change, for the line, and b is the y-intercept.

A vertical line has an equation of the form $x = c$. Its slope is undefined.

linear equation

Examples

$y = 7x - 16$

$2x - 3y = 12$

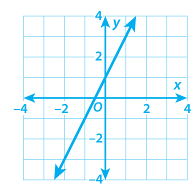
$y = 4x$

$y = 5$

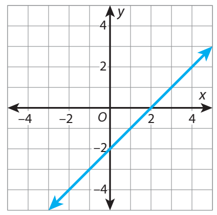
$x = -9$

Examples

The graph of $y = 2x + 1$:



2. Graph the linear equation $y = x - 2$.



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REAL-WORLD CONNECTION

Paleontologists have discovered fossilized remains of dinosaur footprints, called *trackways*, preserved in rock. By using the lengths of the footprints and the distance between them, scientists can approximate how fast a dinosaur was moving when the tracks were made. By writing and solving systems of equation for two different sets of prints, scientists can hypothesize about whether one dinosaur would have been able to catch up to another. Ask students to think of other real-world examples in which systems of linear equations might be useful.

3 Students should recognize that the point where the lines intersect lies on both lines and makes both equations true, so its ordered pair is a solution of both equations.

4 **a.** Students may use the equations to identify the slope and *y*-intercept of each line and use these values to graph each line. Some students may create a table of values for each equation.

b. If student graphs are not precise, they may not identify the correct solution. Students can check the solution by substituting the coordinates into both equations.

5 **a.** Students should understand that the solution is the ordered pair for the point where the lines intersect.

b. Students can use the axes labels to help them interpret the solution in context. The horizontal, or *d*-coordinate, gives the number of days, and the vertical, or *c*-coordinate, gives the number of cranes.

➤ **Complete problems 3–5.**

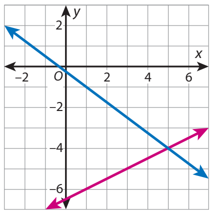
3 The graph of the system of linear equations below is shown in the coordinate plane.

$4y = -3x - 1$

$2y = x - 13$

Why is the point $(5, -4)$ a solution to the system?

Possible answer: The lines intersect at $(5, -4)$. This ordered pair makes both equations true.



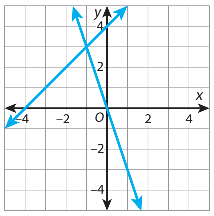
4 **a.** Graph the following system of equations. See graph.

$y = -3x$

$y = x + 4$

b. What does the graph show to be the solution of the system?

$(-1, 3)$



5 DeAndre and Leah are making origami cranes. Their goal is to complete 1,000 cranes by the end of the summer.

- DeAndre already has 30 cranes and makes 5 more each day.
- Leah already has 10 cranes and makes 15 more each day.

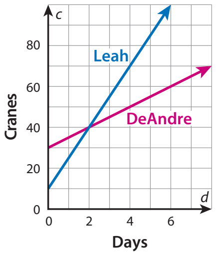
The graph shows how many cranes, *c*, each person has made after *d* days.

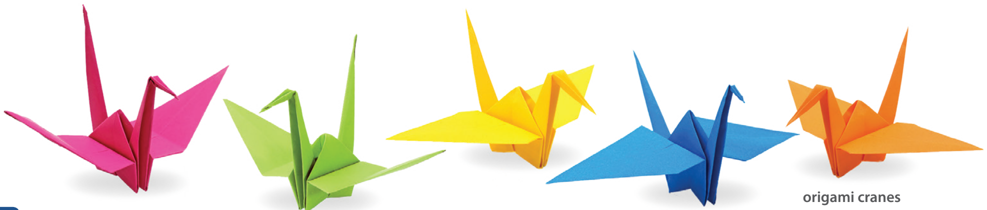
a. What does the graph show to be the solution of the system?

(2, 40)

b. What does the solution mean in this context?

On day 2, DeAndre and Leah have each made 40 cranes.




origami cranes

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Model It**

Levels 1–3: Speaking/Writing

To help students interpret Model It problem 2, read the problem aloud. Use **Act It Out** to clarify the phrase *catch up*. Use a volunteer or classroom objects to role play the meaning of *catch up*. State the phrase in the past tense: *I caught up with ____*. Display *catch up* and *caught up*. Have partners use both phrases to describe a situation. Then reread problem 2, clarifying words as needed. Ask a student to explain *same rate*. Use sentence frames to help students answer parts a and b:

- *Paloma ____ catch up to Charlotte.*
- *I know because they are ____.*
- *I can tell from the graphs that Paloma ____ catch up, because the lines ____.*

Levels 2–4: Speaking/Writing

Help students interpret Model It problem 2. Use **Act It Out** to have students demonstrate *catch up* and *caught up*. When a student catches up, have them discuss how that is different than the situation in the problem. Encourage them to use the word *rate*:

- *I caught up to ____ because ____.*
- *In the problem, Paloma and Charlotte ____.*

Then help students connect the situation to the graph. Ask: *How does the graph show the distance Paloma and Charlotte walk?* Have students draft a response to 2a. Then have them answer 2b and make connections with partners:

- *Our answers are ____.*
- *I think we can check the answer by ____.*

Levels 3–5: Speaking/Writing

Help students interpret Model It problem 2. Have students read the problem and turn to a partner to discuss how the graph connects to the problem. Encourage partners to explain how the graph shows both girls walking on the same trail at the same rate. Have them draft a response to 2a and have partners review each other's responses. Then have students work independently to answer 2b. When ready, have them turn to partners to connect and discuss answers. Ask: *How does your answer compare to your partner's? Do both answers include an explanation? How can you test your answer?*

Encourage students to use *same, different, both, and,* or *but* as they explain their ideas.

Develop Understanding of the Number of Solutions to a System of Linear Equations

Purpose

- **Develop** the idea that a system of linear equations can have exactly one solution, no solution, or infinitely many solutions.
- **Understand** that the number of solutions a system has can be determined by examining the graph or the equations of the system.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A $y = 3x - 4$	B $y = 2x - 4$
C $3x - y = 5$	D $2 + 0.5y = 1.5x$

Possible Solutions

B is the only equation whose graph has a slope that is not 3.

C is the only equation whose graph does not cross the *y*-axis at (0, −4).

D the only equation with decimal coefficients.

WHY? Reinforce the connection between a linear equation and key features of its graph.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they build onto their responses to the Discuss It question.

HOW? Encourage students to listen for ideas they agree with during whole-class discussion. Have students share their ideas about what is the same about the equations. Explain that one way to add to, or build on, someone's explanation is to give another example that shows that the idea makes sense. Use: *I also think ____ and I can add onto this idea by ____.*

MODEL IT

SMP 2

1 – 2 See ***Connect to Culture*** to support student engagement. As students complete the problems, have them identify that a system of linear equations has no solution if the lines do not intersect.

Common Misconception If students think the lines could intersect eventually, ask them to identify the slopes. Have them explain what it means when two lines have the same slope and different *y*-intercepts. [the lines are parallel and do not intersect]

LESSON 12 | SESSION 2

UNDERSTAND: What does it mean to solve a system of linear equations?

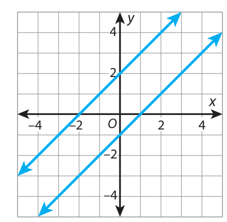
Develop Understanding of the Number of Solutions to a System of Linear Equations

Model It: No Solution

➤ **Try these two problems involving systems of linear equations with no solution.**

1 You have seen that a solution to a system of equations is represented on its graph as a point of intersection. The ordered pair for this point makes both equations true.

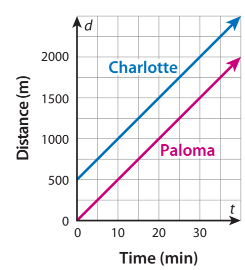
a. Graph the system $y = x + 2$ and $y = x - 1$. **See graph.**



b. Does this system have a solution? How do you know?

No; There is no point of intersection, so there is no ordered pair that makes both equations true.

2 Charlotte and Paloma both walk the same trail at the same rate. Charlotte starts to walk first. The graph shows each girl's distance along the trail for the first 40 minutes of Paloma's hike.



a. Does Paloma catch up to Charlotte? How do you know?

No; Possible explanation: The girls are hiking at the same rate, so Charlotte is always 500 m ahead of Paloma.

b. Look at the system of equations. How can you tell by looking at the equations that the system has no solution?

Paloma: $d = 50t$

Charlotte: $d = 50t + 500$

Possible answer: There is no solution because *d* cannot equal both 50*t* and 50*t* + 500 at the same time.

hiking trail

DISCUSS IT

Ask: How are the equations of the system in problem 1 like the equations of the system in problem 2?

Share: I can tell there is no solution if . . .

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DISCUSS IT

SMP 3, 7

Support Partner Discussion

After students complete problems 1 and 2, have them respond to Discuss It with a partner. Support as needed with questions such as:

- *The equations in each system are in the form $y = mx + b$. How do the values of m compare? How do the values of b compare?*
- *What do the values of m and b tell you about how the lines are related?*

Facilitate Whole Class Discussion

For each problem, have students talk about how the graph and equations of the system indicates that the system has no solution.

ASK *How does the graph show that the system of equations has no solution?*

LISTEN FOR The lines never intersect, so there is no point that lies on both lines.

ASK *How could you use the equations to predict that the system has no solution?*

LISTEN FOR The equations are in slope-intercept form, so I can identify the slope and *y*-intercept. The lines have the same slope and different *y*-intercepts, so they are different lines, and they are parallel. This means the system has no solution.

MODEL IT

SMP 2

3 As students complete the problem, have them compare the equations as well as the graphs. Students can make a table of values or rewrite the equation $y - 1 = 2x$ in slope-intercept form to confirm that both equations represent the same line.

DISCUSS IT

SMP 3, 7

Support Partner Discussion

After students complete problem 3, have them respond to Discuss It with a partner. Support as needed with questions such as:

- *How would the equations change to model the new situation? How would the graphs change?*

Facilitate Whole Class Discussion

Have students discuss strategies for comparing the equations in a system. Encourage them to add reasons or examples to ideas they agree with during discussion.

ASK *How could you predict that a system will have infinitely many solutions without graphing?*

LISTEN FOR I can write the equations in slope-intercept form. If both equations have the same slope and the same y-intercept, then they represent the same line. The system will have an infinite number of solutions.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Compare equations to identify the solutions of a system.

If students are unsure about how to identify the number of solutions a system of linear equations has, then use this activity to spark discussion.

Materials For each student: transparency markers, transparency of Activity Sheet *Coordinate Plane: Four Quadrants*

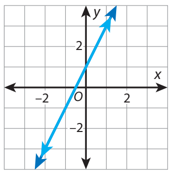
- Tell each student to write an equation in the form $y = mx + b$ on their transparency, selecting m and b from the set of numbers: 1, 2, and 3. Then have students graph their equations.
- Have students circulate and compare equations and overlay their graphs matching up the axes. As they compare equations, ask students to discuss the number of solutions the system has.
- After students have finished their comparisons, ask them to share what they have learned. Have students connect the number of solutions to the values of m and b in the equations for each system.

LESSON 12 | SESSION 2

Model It: Infinitely Many Solutions

➤ **Try this problem about a system of linear equations with infinitely many solutions.**

3 The graph of the equation $y - 1 = 2x$ is shown.



a. Graph the equation $y = 2x + 1$ in the same coordinate plane to represent a system. See graph.

b. At which point(s) do the two lines intersect?
They intersect at every point.

c. How many ordered pairs are solutions of the system? Explain.
infinitely many; Possible explanation: An infinite number of ordered pairs make both equations true.

DISCUSS IT

Ask: Suppose Charlotte and Paloma start at the same time in problem 2. Why would the system representing this context have infinitely many solutions?

Share: I can tell there are infinitely many solutions if . . .

CONNECT IT

➤ **Complete the problems below.**

4 Look at problems 1–3. In each system of equations, both lines have the same slope. Can two lines with the same slope ever intersect at exactly one point? Explain.
No; Possible explanation: If two lines have the same slope, they are either parallel and do not intersect at all, or they are the same line and share infinitely many points.

5 What values of m and b will result in a system with no solution?

$y = 4x + 5$

$y = mx + b$

$m = 4$; Possible answer: $b = 10$ (any number except 5)

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CONNECT IT

SMP 2, 3, 7

4 Student responses should show understanding that lines with the same slope are either different parallel lines or the same line.

CLOSE EXIT TICKET

5 Look for understanding that for the system to have no solution, the lines must have the same slope but different y-intercepts.

Error Alert If students choose m and b so that the two equations have the same y-intercept and different slopes, have them sketch the graphs of the equations of the system they have created. They should observe that the lines intersect at the y-intercept, so the system has one solution. Ask students what m and b indicate about the line and what values of m and b would guarantee that the two lines do not intersect.

Practice Determining the Number of Solutions to a System of Linear Equations

Problem Notes

Assign **Practice Determining the Number of Solutions to a System of Linear Equations** as extra practice in class or as homework.

1. *Basic*
2. Students should recognize that a system of linear equations has exactly one solution when the lines have different slopes. ***Medium***
3. Students should recognize that a system of linear equations for lines with the same slope but different *y*-intercepts will never intersect. Any *b* value except 1 creates such a system. ***Medium***
4. **a.** Students should draw a line parallel to the given line. ***Basic***

   b. Students should draw a line that intersects the given line at exactly one point. ***Basic***

   c. Students should draw a line that is the same as the given line. ***Basic***

LESSON 12 | SESSION 2 Name:

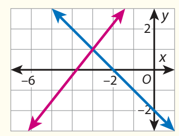
Practice Determining the Number of Solutions to a System of Linear Equations

➤ **Study the Example showing how to determine the number of solutions to a system of linear equations. Then solve problems 1–7.**

Example

How many solutions does each system of equations have?

a.



b. $y = 7x + 3$

$y = 7x$

a. The lines intersect at one point. There is exactly one solution.

b. y cannot equal both $7x + 3$ and $7x$ at the same time. There is no solution.

1. What does the graph show to be the solution of the system in part a of the Example? $(-3, 1)$

2. Find a value for m that will give you a system of equations with exactly one solution.

   $y = 6x + 1$
   $y = mx + 1$

   Possible answer: 2 (any number except 6)

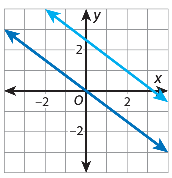
3. Find a value for b that will give you a system of equations with no solution.

   $y = 6x + 1$
   $y = 6x + b$

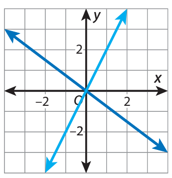
   Possible answer: 2 (any number except 1)

4. Draw a line in each coordinate plane so that the lines represent a system of equations with the given number of solutions. Possible answers shown.

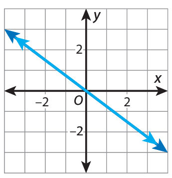
   a. no solution

   b. exactly one solution

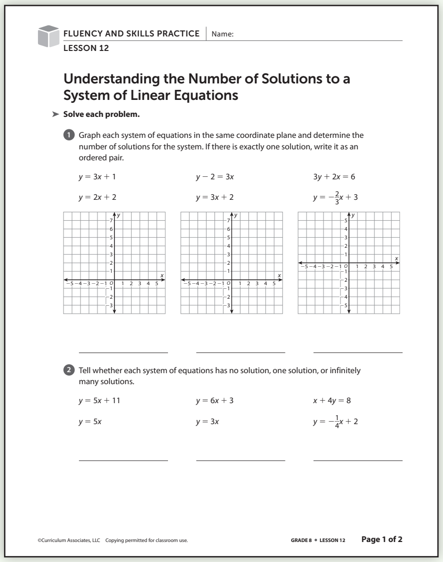
   c. infinitely many solutions

Fluency & Skills Practice

Understanding the Number of Solutions to a System of Linear Equations

In this activity, students determine the number of solutions to a system of linear equations by identifying whether each system of equations has no solution, one solution, or infinitely many solutions. Students also practice graphing systems of linear equations.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 12

Understanding the Number of Solutions to a System of Linear Equations

➤ Solve each problem.

1 Graph each system of equations in the same coordinate plane and determine the number of solutions for the system. If there is exactly one solution, write it as an ordered pair.

$y = 3x + 1$, $y = 2x + 2$ — $y - 2 = 3x$, $y = 3x + 2$ — $3y + 2x = 6$, $y = -\frac{2}{3}x + 3$

2 Tell whether each system of equations has no solution, one solution, or infinitely many solutions.

$y = 5x + 11$, $y = 5x$ — $y = 6x + 3$, $y = 3x$ — $x + 4y = 8$, $y = -\frac{1}{4}x + 2$

Page 1 of 2

Additional Practice

UNDERSTAND LESSON

5 **a.** Students should select equations for two lines with different slopes. ***Medium***

b. Students should select equations for two lines with the same slope and different y-intercepts. ***Medium***

6 **a.** Students can multiply both sides of the second equation by -1 to see that the equations represent the same line. ***Medium***

b. Students may reason that y cannot equal both $3x$ and $3x - 10$ at the same time. ***Medium***

c. Students may recognize the lines have different slopes, so they must intersect. ***Medium***

7 **a.** Students should recognize that each graph represents the number of stairs climbed in a particular number of minutes after the tower opens. Jayden starts after Elizabeth but climbs at a faster rate, so they will meet at one point. ***Medium***

b. Students should recognize that the slope represents climbing speed and that Robert and Mel have the same climbing speed. However, because they started climbing at different times, they will never meet one another. ***Medium***

LESSON 12 | SESSION 2

5 Use the equations below.

$y = 4x + 2$ $\quad$ $y = 9x + 2$ $\quad$ $y = 9x + 5$

a. Use two of the equations to write a system of equations with exactly one solution.

$y = 4x + 2$ and $y = 9x + 2$ OR $y = 4x + 2$ and $y = 9x + 5$

b. Use two of the equations to write a system of equations with no solution.

$y = 9x + 2$ and $y = 9x + 5$

6 Tell whether each system of equations has *no solution, one solution,* or *infinitely many solutions.*

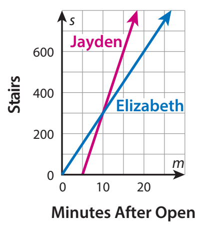
a. $y = x$
$-y = -x$
infinitely many solutions

b. $y = 3x$
$y = 3x - 10$
no solution

c. $y = x$
$y = 2x$
one solution

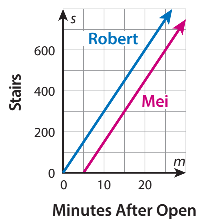
7 Four friends plan to meet at the top of the Eiffel Tower. Each arrives about the time the tower opens and starts climbing the stairs to the top. The graphs show the number of stairs each has climbed, s, in the m minutes since the tower opened. Tell how many solutions each system has. What does each solution mean in this context?

a.



One solution; Possible answer: Elizabeth and Jayden are both on the 300th stair 10 min after the tower opens.

b.



No solution; Possible answer: Robert and Mei are never on the same stair at the same time.

Eiffel Tower, Paris, France

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Reading/Speaking

Help students prepare to answer Apply It problem 3. Read the problem aloud as students follow along. Prepare students to respond by using a **Co-Constructed Word Bank.** Add *extend, intersect,* and *parallel* to the bank if students do not suggest them.

Then remind students to build on ideas they agree with or explain why they disagree with an idea. Show students how to use the words in the question, *Do you agree or disagree with Rachel?* to begin their response. Suggest these sentence frames:

- *I _____ with Rachel.*
- *If you _____, the lines will _____.*

Levels 2–4: Reading/Speaking

Help students prepare to answer Apply It problem 3. Read the problem with students and prepare them to respond by using a **Co-Constructed Word Bank.** Add *extend, intersect,* and *parallel* to the bank if students do not suggest them. Ask students to think of words that express agreement or disagreement and add them to the bank. Have students refer to the bank to answer the question in the problem. Allow think time and have students turn to a partner to discuss. Have students discuss what the answer means in the context of the problem. Provide a sentence starter:

- *The answer means Rachel _____.*

Levels 3–5: Reading/Speaking

Have students discuss Apply It problem 3 by creating a **Co-Constructed Word Bank.**

When students finish adding words and definitions or examples to the bank, ask them to turn to a partner and compare. Then have students refer to the bank as they respond to the problem.

Allow time for students to meet with partners again and compare answers. Have them share the words they used to express agreement or disagreement. Then have them explain what the answers mean in the context of the problem. Ask: *What does your answer mean? Why?*

Refine Ideas About Systems of Linear Equations in Two Variables

Purpose

- **Refine** understanding of systems of linear equations and their solutions by reasoning about the graphical representations of the equations.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

A A system of two linear equations with the same slope has infinitely many solutions.

B A system of two linear equations with different slopes has no solution.

C A system of two linear equations with different slopes has exactly one solution.

Solutions

A is sometimes true.

B is never true.

C is always true.

WHY? Reinforce understanding of the characteristics of the equations in a system that determine whether the system has exactly one solution, no solution, or infinitely many solutions.

APPLY IT

SMP 2, 3, 4, 7

Have students work independently or with a partner for problems 1–3.

1 **Generalize** Look for understanding that the lines in a system of linear equations with exactly one solution will have different slopes. If two lines have the same slope, they will either not intersect at all, so the system has no solution, or they will intersect at every point, so the system has infinitely many solutions.

UNDERSTAND: What does it mean to solve a system of linear equations?

Refine Ideas About Systems of Linear Equations in Two Variables

Apply It

➤ **Complete problems 1–5.**

1 **Generalize** A system of linear equations has exactly one solution. What can you say about the slopes of the lines when the equations are graphed? How do you know?

The slopes of the two lines are different. Possible explanation: If the slopes of the lines were the same, either they would be parallel lines and the system would have no solution, or they would be the same line and the system would have infinitely many solutions.

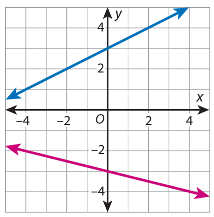
2 **Analyze** Can you identify the solution of this system without graphing the equations? Explain.

$x = 4$

$y = 6$

Yes; Possible explanation: All of the points on the graph of $x = 4$ have an x-coordinate of 4, and all of the points on the graph of $y = 6$ have a y-coordinate of 6. So, the point that is common to both lines must be (4, 6).

3 **Examine** Rachel graphs this system of equations. She says there is no solution to the system because the lines do not intersect. Do you agree or disagree with Rachel? Explain.



Disagree; Possible explanation: The lines are not parallel, so they must intersect at some point. If the graph is extended far enough to the left, the intersection point will be visible.

2 **Analyze** Look for understanding that the equations represent a horizontal line and a vertical line. Remind students as they discuss that clear explanations use complete sentences and precise vocabulary. Prompt the discussion with questions such as:

- *What kind of line is $y = 6$? What do all the points on the line have in common?*
- *What kind of line is $x = 4$? What do all the points on the line have in common?*
- *How can you use this information to identify the point of intersection?*

Common Misconception If students do not recognize that the equations represent a system with one horizontal line and one vertical line, ask them to list and plot three points that satisfy each equation. Encourage students to brainstorm ways to remember that an equation in the form $x = a$ represents a vertical line and an equation of the form $y = b$ represents a horizontal line.

3 **Examine** Look for understanding that each line extends beyond the visible portion of the graph. Have students form an opinion about Rachel's claim, then extend the sketches of the lines to see if they are correct.

4 See ***Connect to Culture*** to support student engagement. Before students begin, read the first part of the problem aloud and ask which numbers in the problem describe the slopes of the lines and which describe the y-intercepts. Then have students read the directions for parts A, B, and C and have them rephrase to confirm that they understand each part of the task.

As students work on their own, walk around to ensure that they are graphing and interpreting the system correctly.

Have students share their description from part C with a partner and explain their reasoning.

CLOSE EXIT TICKET

5 **Math Journal** Look for understanding that a solution to a system of linear equations is an ordered pair that makes both equations true and that a system may have one solution, no solution, or infinitely many solutions. Students may sketch graphs that illustrate each possible outcome or write equations and analyze slopes and y-intercepts.

Error Alert If students forget to mention all three possibilities for the number of solutions, suggest that they try to visualize all the possible ways two lines can be related. Students should recognize that the lines may intersect at one point, may never intersect, or may be the same line. Each of these possibilities corresponds to a possible number of solutions.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they think about what it means when someone says: *We have a lot in common.* Have them relate this to the idea of two lines having a point in common.

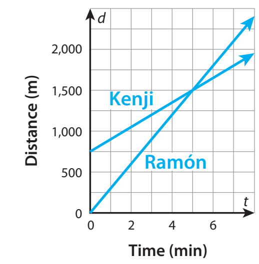
LESSON 12 | SESSION 3

4 Kenji and Ramón are running cross country. Kenji runs at a rate of 150 meters per minute. Kenji has already run 750 meters before Ramón starts running. Ramón runs at a rate of 300 meters per minute.

PART A The system of equations represents the distance, d, from the starting point of each runner t minutes after Ramón starts running. Graph the system and label each line with the runner it represents.

$d = 150t + 750$

$d = 300t$



PART B What does the graph show to be the solution of the system in Part A? What does the solution mean in the context of the problem?

(5, 1500); After Ramón has run 5 minutes, Ramón and Kenji are both 1,500 meters from the starting point.

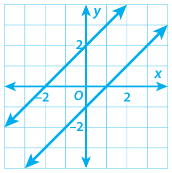
PART C Describe a situation in which Kenji and Ramón are running cross country but are never the same distance from the starting point at the same time. Write a system of equations or draw a graph to model the situation. How many solutions does the system have?

Possible answer: Kenji starts first and runs 200 meters per minute. After Kenji runs 500 meters, Ramón starts running at 200 meters per minute; $d = 200t + 500$, $d = 200t$; There is no solution.

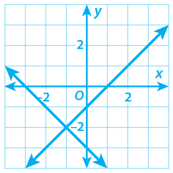
5 **Math Journal** What does it mean to solve a system of linear equations? Use models to show the possible numbers of solutions a system can have.

Possible answer: Solving a system of linear equations means finding any ordered pairs that make both equations true at the same time.

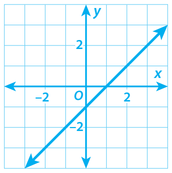
no solution



one solution



infinitely many solutions



End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Write a new entry for *common*. What does it mean when a point is *common* to two lines?

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Short Response Scoring Rubric (2 points) — **Problem 4**

Points	Expectations
PART A	
2	The sketches of both graphs are accurate **AND** labeled.
1	The graphs are inaccurate **OR** labeled incorrectly.
PART B	
2	The solution is identified **AND** its meaning is explained in the context of the problem.
1	The solution is identified **OR** its meaning is explained in the context of the problem.
PART C	
2	An appropriate situation is described. It is represented accurately with a graph or system of equations. The correct number of solutions is identified.
1	Two of the three elements of the problem are correct.

Lesson Quiz

TESTED SKILLS

Assesses 8.EE.C.8a, 8.EE.C.8b

Problems on this assessment require students to be able to determine the number of solutions to a system of equations. Students need to be able to examine a graphed system of equations to determine if the system of equations has no solution, one solution, or infinitely many solutions. They will also need to be able to describe the slope and *y*-intercept for systems with no solution, one solution, and infinitely many solutions. Students will select equations that create a system of equations and explain why the system of equations has no solution. Students will graph a system of linear equations given a real-world situation. Students will also need to be familiar with graphing linear equations, determining the number of solutions to one-variable linear equations, and determining unit rates.

Alternately, teachers may assign the **Digital Comprehension Check** online to assess student understanding of this material.

Error Alert Errors may result if students:

- confuse *x*- and *y*-coordinates when graphing equations.
- misinterpret situations in which there are infinitely many solutions as having no solutions.
- misrepresent given information about rate and initial value when writing equations to represent real-world scenarios.

Problem Notes

1 **B is correct.** Students could solve the problem by noticing that the lines intersect at exactly one point, (4, 4).

A is not correct. This answer represents two lines that do not intersect.

C is not correct. This answer represents misinterpreting two lines as having two solutions; lines can either intersect at zero, one, or infinitely many points, having no solution, one solution, or infinitely many solutions.

D is not correct. This answer represents two lines that intersect at all points.

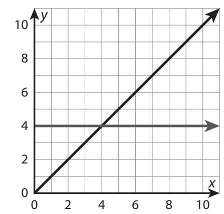
(1 point)

DOK 1 | **8.EE.C.8a**

LESSON 12 • QUIZ   Name:

➤ **Solve the problems.**

1 The graph of a system of two linear equations is shown. How many solutions does the system of equations have? *(1 point)*



A No solution

(B) One solution

C Two solutions

D Infinitely many solutions

2 A system of linear equations has infinitely many solutions. What is known about the slopes and *y*-intercepts of the graphed equations? Explain your reasoning. *(2 points)*

SOLUTION The slopes and *y*-intercepts of the two lines are the same; Possible explanation: If the slopes of the two lines were different, the system of equations would have exactly one solution. If the slopes were the same but with different *y*-intercepts, the graphs of the equations would be parallel lines, and the system of equations would have no solution.

3 Use two of the equations below to write a system of equations with no solution.

$y = 3x + 1$ $y = 3x - 1$ $-y = -3x + 1$

Explain your reasoning. *(2 points)*

SOLUTION $y = 3x + 1$ and $y = 3x - 1$ or $y = 3x + 1$ and $-y = -3x + 1$; Possible explanation: If the equations were graphed in the coordinate plane, they would both have a slope of 3 but would have different *y*-intercepts of -1 and 1. Therefore, the system of equations has no solution.

Short Response Scoring Rubric

Points	Expectations
2	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of concepts and/or procedures.
1	Response contains mostly correct solution(s) and shows partial understanding of concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of concepts and/or procedures.

Graphing Coordinate Grid Scoring Rubric

Points	Expectations
2	Response has both lines correctly graphed with the intersection shown.
1	Response has only one line correctly graphed.
0	Response has incorrect lines graphed.

2 Students should recognize that a system of linear equations has infinitely many solutions if the equations are represented by the same line.

(2 points)

DOK 2 | 8.EE.C.8a

3 Students could recognize that all three equations represent lines with a slope of 3. The first line has a y-intercept of 1, and the others have a y-intercept of -1. Students should select the first equation and either of the remaining two equations.

(2 points)

DOK 2 | 8.EE.C.8b

4 **Part A**

Students could find the y-intercept for each graph. Then graph the lines using the rate for each driver as the slope.

(2 points)

DOK 2 | 8.EE.C.8b

Part B

Students could recognize that if Kylie and Matt leave at different times and drive at the same rate, they will remain the same distance apart and therefore never be at the same distance from the starting point. In order to drive the same rate, the slope needs to be the same.

(2 points)

DOK 3 | 8.EE.C.8b

LESSON 12 • QUIZ Name:

4 Kylie and Matt are driving out of town, leaving from the same house in different cars. Matt drives at a rate of 40 miles per hour. Matt drives 50 miles before Kylie begins traveling. Kylie drives at a rate of 50 miles per hour.

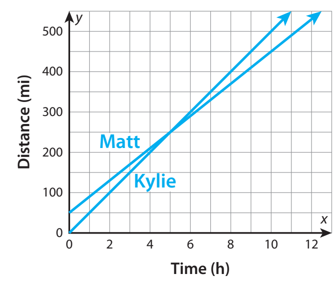
PART A

The system of equations represents the distance, y, from the starting point of each driver x minutes after Kylie starts driving.

$y = 40x + 50$

$y = 50x$

Graph the system in the coordinate plane and label each line with the driver it represents. *(2 points)*



PART B

Describe a situation in which Kylie and Matt are traveling but are never the same distance from the starting point at the same time. Write a system of equations to model the situation. How many solutions does the system have? Explain your reasoning. *(2 points)*

SOLUTION **Possible answer: $y = 50x$, $y = 50x + 50$; There is no solution.**
Possible explanation: Kylie and Matt could leave the house at different times but drive at the same rate. Since they are never the same distance from the starting point at the same time, there is no solution.

DIFFERENTIATION

RETEACH Tools for Instruction

Students who require additional support for prerequisite or on-level skills will benefit from activities that provide targeted skills instruction.

GRADE 8

- **Lesson 12** TFI title TK

REINFORCE Math Center Activity

Students who require practice to reinforce concepts and skills and deepen understanding will benefit from small group collaborative games and activities (available in on-level, below-level, and above-level versions).

GRADE 8

- **Lesson 12** Center Activity title TK

EXTEND Enrichment Activity

Students who have achieved proficiency with concepts and skills and are ready for additional challenges will benefit from group collaborative games and activities that extend understanding.

GRADE 8

- **Lesson 12** Enrichment title TK

Lesson 13

Solve Systems of Linear Equations Algebraically

Overview | Solve Systems of Linear Equations Algebraically

MATH FOCUS

Focus Standards

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

See Unit 3 Pacing Guide for developing and applied standards.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

7 Look for and make use of structure.

* See page XX to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Estimate the solution of a system of linear equations by graphing.
- Use substitution and elimination to solve systems of linear equations.
- Determine whether a system of linear equations has one solution, no solution, or infinitely many solutions.
- Identify efficient ways to solve a system of linear equations.

Language Objectives

- Justify solutions for systems of linear equations by referring to graphs and checking if solutions are reasonable.
- Respond to clarifying questions about the processes of substitution and elimination during partner and class discussions.
- Explain why systems of linear equations have one, infinitely many, or no solutions using precise mathematical language.
- Describe and evaluate methods of solving a system of equations using lesson vocabulary when speaking and writing.
- Listen for understanding by asking clarifying questions or requesting more information during partner and class discussions.

Prior Knowledge

- Understand the definition of a system of linear equations.
- Graph a system of linear equations to determine its solution.
- Solve an equation for one variable in terms of another.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

coefficient a number that is multiplied by a variable.

system of linear equations a group of related equations in which a solution makes all the equations true at the same time. A system of equations can have zero, one, or infinitely many solutions.

Academic Vocabulary

algebraically in a way that involves variables and the rules of algebra.

elimination the method or process used to remove something.

substitution the process of replacing one thing with another that is equivalent.

Learning Progression

Earlier in Grade 8, students graphed linear equations and rewrote them in slope-intercept form.

In the previous lesson, students learned about systems of linear equations. They graphed both equations of a system and identified any points of intersection. They determined whether a system had zero, one, or infinitely many solutions by graphing and by considering the values of m and b in the slope-intercept form of the equations.

In this lesson, students will use the substitution and elimination methods to solve systems of linear equations. They will understand how the graph of a system can help them estimate the solution before finding the exact solution algebraically. They will recognize that when an algebraic solution leads to an equation that is never true, the system has no solution, and when it leads to an equation that is always true, the system has infinitely many solutions.

Later in Grade 8, students will learn how to write systems of equations to model both mathematical and real-world problems. Then they will apply the skills learned in this lesson to solve the systems.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 Explore Solving Systems of Linear Equations Algebraically (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 285–286)	Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model
SESSION 2 Develop Solving Systems of Linear Equations by Substitution (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 291–292)	**Math Toolkit** graph paper, straightedges Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: algebra tiles (at least 10 each of *x*- and *y*-tiles and 20 1-tiles) **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 Develop Solving Systems of Linear Equations by Elimination (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 297–298)	**Math Toolkit** graph paper, straightedges Presentation Slides	**RETEACH or REINFORCE** Visual Model **Materials** For display: a large four-quadrant coordinate plane **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 Develop Determining When a System Has Zero or Infinitely Many Solutions (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 303–304)	**Math Toolkit** graph paper, straightedges Presentation Slides	**RETEACH or REINFORCE** Visual Model **Materials** For display: a large four-quadrant coordinate plane **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 5 Refine Solving Systems of Linear Equations Algebraically (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each pair: algebra tiles (at least 10 each of *x*- and *y*-tiles and 20 1-tiles) **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 13 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1

Try It A yard sale, or garage sale, is a sale of used household goods or personal items, typically held at a private property. The self-proclaimed World's Largest Yard Sale spans 690 miles along a highway across Georgia, Alabama, Tennessee, Kentucky, Ohio, and Michigan. The sale takes place the first Thursday through Sunday in August. During this event, individual homeowners, as well as storeowners and vendors, set up booths to try to sell their belongings and merchandise. The event is said to involve hundreds of thousands of shoppers and vendors. Ask students to share their experiences either shopping at a yard sale or hosting a yard sale of their own.

SESSION 2

Try It Ask students to raise their hands if they have ever seen or worked in a garden on the roof of a building. Encourage these students to share their experiences. Rooftop gardens are becoming more popular because they add beauty to the city and can provide fresh food. Rooftop gardens also have significant environmental benefits. They cool the building in summer and help to insulate it in winter, leading to less energy use. They make use of rainwater that might otherwise end up in storm sewers or cause flooding. Rooftop gardens also help to reduce noise in the building and provide habitats for birds and helpful insects. Ask students to suggest other possible benefits of rooftop gardens that they can add to the list.

SESSION 3

Try It Many museums and other tourist attractions have gift shops in which souvenirs are sold. The U.S. souvenir store industry is estimated to include around 23,000 stores. Total sales from these stores is estimated to be $16 billion. Ask students to describe gift shops they have visited or souvenirs they have collected.

SESSION 4

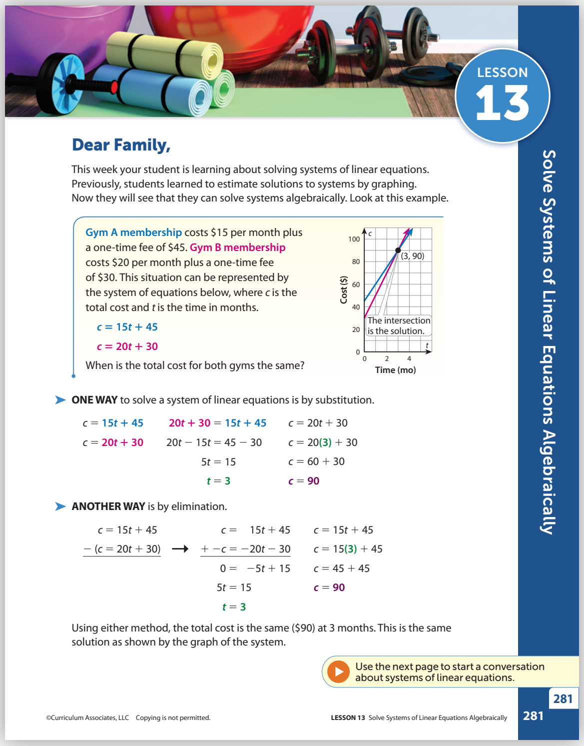
Apply It **Problem 8** Making a picture frame can be a simple project. However, the detail in a more professional-looking frame may require special equipment. In a rectangular picture frame, each of the four pieces of wood that form the sides of the frame has a 45°-angle at each end. These can be cut by hand, but cutting with a miter saw produces a more accurate angle and cleaner cut. Also, many frames have a piece of glass or clear plastic in front of the picture. This is held in place by a special groove called a rabbet. To cut a groove like this, woodworkers use a router. Ask students to describe woodworking projects they have done or power tools they have used.

SESSION 5

Apply It **Problem 7** Communities that want to encourage students to ride their bikes to school can get results by providing bike racks, adding crossing guards at critical intersections, and providing families with information about bike routes and safety. Write different methods for getting to school on the board and have students place sticky notes by the method they use. Discuss the results.

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



LESSON 13

Solve Systems of Linear Equations Algebraically

Dear Family,

This week your student is learning about solving systems of linear equations. Previously, students learned to estimate solutions to systems by graphing. Now they will see that they can solve systems algebraically. Look at this example.

Gym A membership costs $15 per month plus a one-time fee of $45. Gym B membership costs $20 per month plus a one-time fee of $30. This situation can be represented by the system of equations below, where *c* is the total cost and *t* is the time in months.

$c = 15t + 45$

$c = 20t + 30$

When is the total cost for both gyms the same?

ONE WAY to solve a system of linear equations is by substitution.

$c = 15t + 45$	$20t + 30 = 15t + 45$	$c = 20t + 30$
$c = 20t + 30$	$20t - 15t = 45 - 30$	$c = 20(3) + 30$
	$5t = 15$	$c = 60 + 30$
	$t = 3$	$c = 90$

ANOTHER WAY is by elimination.

$c = 15t + 45$		$c = 15t + 45$	$c = 15t + 45$
$-(c = 20t + 30)$	$\rightarrow$	$+ -c = -20t - 30$	$c = 15(3) + 45$
		$0 = -5t + 15$	$c = 45 + 45$
		$5t = 15$	$c = 90$
		$t = 3$	

Using either method, the total cost is the same ($90) at 3 months. This is the same solution as shown by the graph of the system.

Use the next page to start a conversation about systems of linear equations.

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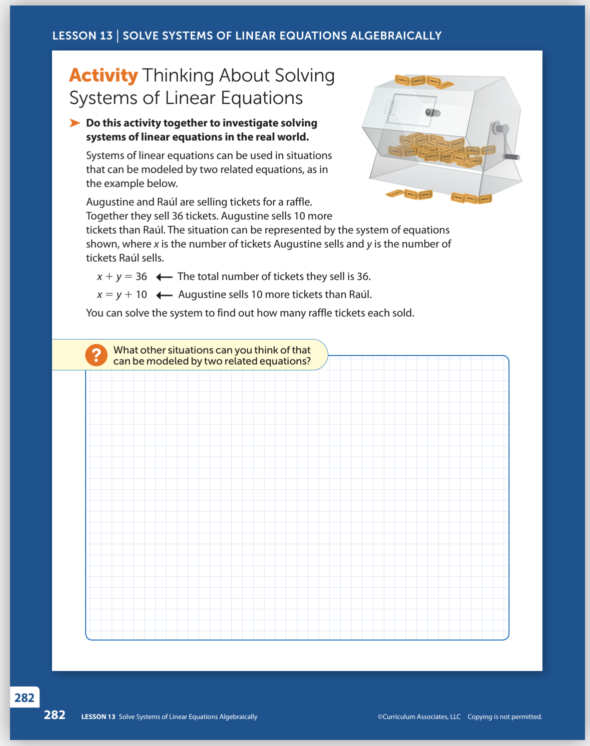

LESSON 13 | SOLVE SYSTEMS OF LINEAR EQUATIONS ALGEBRAICALLY

Activity Thinking About Solving Systems of Linear Equations

Do this activity together to investigate solving systems of linear equations in the real world.

Systems of linear equations can be used in situations that can be modeled by two related equations, as in the example below.

Augustine and Raúl are selling tickets for a raffle. Together they sell 36 tickets. Augustine sells 10 more tickets than Raúl. The situation can be represented by the system of equations shown, where *x* is the number of tickets Augustine sells and *y* is the number of tickets Raúl sells.

$x + y = 36$ ⟵ The total number of tickets they sell is 36.

$x = y + 10$ ⟵ Augustine sells 10 more tickets than Raúl.

You can solve the system to find out how many raffle tickets each sold.

What other situations can you think of that can be modeled by two related equations?

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Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Reading/Writing

Help students prepare to answer Connect It problem 2a. Read the problem aloud as students follow along. To help them understand, ask what words in the first sentence mean *equal to*. Have students underline the words. Then have them **Say It Another Way** by using *equal to* instead of *the same as*.

Next, break the second sentence into parts. After reading the first part, explain that a fact is something that is true. Ask: *What is the fact?* Then read the second part. Ask: *How many equations are you being asked to write?* Have partners work together to write one equation, share their equation, and tell what variable they used. Ask: *Is there more than one equation you can write?*

Levels 2–4: Reading/Writing

Help students prepare to answer Connect It problem 2a. Have them read the problem with a partner. Refer students back to the Try It section to review the facts of the problem. Ask students to discuss the value of a book compared to the value of a puzzle.

Encourage students to identify the words in the problem that mean *equal to*. Ask for other synonyms, like *interchangeable* or *same value*. Have students use **Say It Another Way** to confirm understanding. Then have them tell what they are being asked to do in problem 2a.

Next, have students work together to write the equation using only one variable. If they struggle, remind them to think of what it means for two items to be equal in value.

Levels 3–5: Reading/Writing

Have students prepare to answer Connect It problem 2a. Have them state what they are being asked to do in their own words. Then have students work independently to write one equation using only one variable.

Have students share their equations and explain their reasoning. Encourage students to listen to their partners and ask clarifying questions. If the partners used different variables, have them discuss why both of their equations are correct. Encourage students to use precise mathematical language to clarify how they used the variables.

Explore Solving Systems of Linear Equations Algebraically

Purpose

- **Explore** and reinforce the idea that graphing should be used only to estimate or help confirm the solution of a system of equations.
- **Understand** that you cannot rely only on graphing to find the exact solution of a system of equations.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: $y = 6x + 1$	B: $y = 6x - 1$
C: $y = 12x + 1$	D: $2y = 12x + 2$

Possible Solutions

D is the only equation not in slope-intercept form.

C is the only equation whose line does not have a slope of 6.

B is the only equation whose line has a negative y-intercept.

WHY? Support students' facility in comparing slopes, intercepts, and equations of lines.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask students what the problem is about. After the second read, ask students what they are trying to determine. After the third read, ask them what the important quantities and relationships are.

DISCUSS IT SMP 2, 3, 6

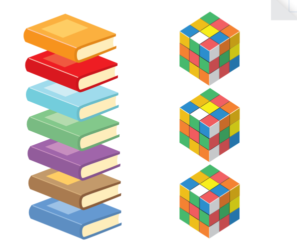
Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the solution of the system is the point of intersection on the graph, but it can be hard to tell what its exact coordinates are.
- reasoning, guess-and-check, or an algebraic method can be used to find the exact solution.

LESSON 13 | SESSION 1 ■ □ □ □ □

Explore Solving Systems of Linear Equations Algebraically



Previously, you learned how to estimate or check solutions to systems of linear equations by graphing. In this lesson, you will learn how to solve systems of linear equations algebraically.

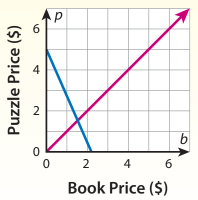
➤ **Use what you know to try to solve the problem below.**

Neena buys books and puzzles at a yard sale. The price of a book is the same as the price of a puzzle. She spends \$15. The situation can be represented by the graph and system of equations shown, where b is the price of each book and p is the price of each puzzle.

$7b + 3p = 15$ ⟵ Neena spends \$15 buying 7 books and 3 puzzles.

$b = p$ ⟵ A book is the same price as a puzzle.

What is the price of each book? What is the price of each puzzle?



TRY IT

Possible work:

SAMPLE A

The point of intersection looks to be about (1.5, 1.5).

$b = 1.5$ and $p = 1.5$: $7(1.5) + 3(1.5) = 15$

The price of each book and each puzzle is \$1.50.

SAMPLE B

She bought 10 items, all at the same price, for \$15. I can just divide 15 by 10 to find the price of 1 item.

$\frac{15}{10} = 1.5$

The price of each item is \$1.50.

DISCUSS IT

Ask: What strategy did you use to find the exact prices of the books and puzzles?

Share: To find the exact prices, I …

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

Analyze and solve pairs of simultaneous linear equations.

- Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

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Error Alert If students get different prices for a book and a puzzle, then have them reread the problem. Have them explain what the equation $b = p$ means in the context of the problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- estimation of the solution from the given graph
- guess-and-check method
- logical reasoning
- equation in one variable written and solved

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that good listeners use engaged body language, such as looking at the speaker and nodding to show understanding.

Guide students to **Compare and Connect** the representations. To prompt students to use precise academic language, call on volunteers to reword vague or unclear statements.

ASK *What are some of the advantages and disadvantages of each of these strategies?*

LISTEN FOR Using a graph does not require calculations, but you often need to estimate where the lines intersect. Guess-and-check is simple, but it may take several guesses. Writing one equation gives an exact solution, but can take more work than some of the other methods.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that the graph can be used to estimate the solution of the system. The exact value can be found using guess-and-check (starting with the estimate from the graph), reasoning, or an algebraic method.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use a table to solve a system of linear equations.

If students are unsure of how to solve the system of linear equations, then use this activity to help them understand the guess-and-check method by using a table.

- Ask: *How many books does Neena buy? How many puzzles? How much does she spend?* [7; 3; $15]
- Display the graph and a table with three columns. Label the columns: *price of a book, price of a puzzle, cost for 7 books and 3 puzzles*.
- Start with $1 as the price of a book. Have volunteers complete the other columns in the row. [$1; $10] Ask students to explain the entries.
- Ask: *Is this the correct amount spent? What should be changed?* [No. It is too little. The price must be greater.]
- Have a student locate the point (1, 1) on the graph of the system. Note that it is on the line for $b = p$, but below the line for $7b + 3p = 15$.
- Repeat this for a book price of $2. After students complete the row, have them suggest the next price, using the total cost to decide if it should be less or greater. Repeat to find the correct price.

LESSON 13 | SESSION 1

CONNECT IT

1 **Look Back** What is the price of each book? What is the price of each puzzle? How did you find your answers?

$1.50; $1.50; Possible answer: From the graph, I estimated the point of intersection to be about (1.5, 1.5). I substituted these *x*- and *y*-values into the equations and got two true statements.

2 **Look Ahead** In earlier lessons, you learned strategies for solving a one-variable equation. You can also use those strategies to solve a system of equations. You can do this by finding a way to combine the two equations with two variables into one equation with one variable.

a. In **Try It**, the price of each book is the same as the price of each puzzle. How can you use this fact to write a one-variable equation for the money Neena spends? What is the equation?

Possible answer: Since $b = p$, I can let b also be the price of a puzzle; The new equation is $7b + 3b = 15$.

b. Solve the equation you wrote in problem 2a. Do you get the same prices for the books and puzzles as in problem 1?

$b = 1.50$; Yes, the price of each book and each puzzle is $1.50.

c. Why might someone prefer using the strategy in problems 2a and 2b over solving a system by graphing? How can a graph be helpful when solving a system?

Possible answer: Using equations can be quicker than graphing, and you can get an exact answer without guessing. A graph can help you estimate the solution, or check to see if your solution is reasonable.

3 **Reflect** Explain how you used both equations in the system to write the one-variable equation in problem 2a.

Possible answer: The equation I wrote, $7b + 3b = 15$, is the same as the equation $7b + 3p = 15$ from the Try It problem except that I used b for both unknowns. I knew I could use one variable for both unknowns because the other equation, $b = p$, tells me that the unknowns have the same value.

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2 **Look Ahead** Point out that a system of equations can be solved by combining the two equations, which each have two variables, into one equation with one variable. In this case, the second equation $b = p$ means that the values of b and p are always the same. So, b can be substituted for p in the first equation $7b + 3p = 15$. This gives the equation $7b + 3b = 15$, which has only one variable. After solving, the value of b can be used to find the value of p.

CLOSE **EXIT TICKET**

3 **Reflect** Look for understanding of how to generate a single equation in one variable when given two equations in two variables.

Common Misconception If students do not understand why the system with two variables can be combined to get one equation with one variable, ask them to look at $b = p$. Ask questions like: *If b is [number], what is p?* Students should come to see that the equation means that the two variables always have the same value. Guide students to see that either p can replace b or b can replace p in the other equation. To avoid students concluding they can always “swap” variables, have them consider if $b = p + 1$ instead of $b = p$.

Prepare for Solving Systems of Linear Equations Algebraically

Support Vocabulary Development

Assign **Prepare for Solving Systems of Linear Equations Algebraically** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *system of linear equations*. Have them discuss what it means to be part of a *system* as well as what constitutes a *linear equation*.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of the definitions, known facts, and examples given.

Have students look at the equation in problem 2 and discuss with a partner what has to be true about the second linear equation in the system to result in the desired number of solutions for each part of the problem.

Problem Notes

1. Students should understand that a system of linear equations is a group of related linear equations in which a solution makes all the equations true at the same time. Student responses might include an example shown as a graph of two lines and may mention that the solution is the ordered pair for the point where the lines intersect. Students should recognize that the equations in a system use the same set of variables.

2. **a.** Any equation for a line with a slope of -2 and a y-intercept other than 0.5 is correct.

   b. Any equation for a line with a slope other than -2 is correct.

   c. Any form of the equation for a line with slope of -2 and a y-intercept of 0.5 is correct.

LESSON 13 | SESSION 1 Name:

Prepare for Solving Systems of Linear Equations Algebraically

1. Think about what you know about systems of linear equations. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

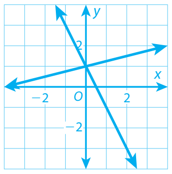
system of linear equations

Definition
a groups of related linear equations where a solution makes all the equations true at the same time

What I Know About It
You can estimate or check the solution of a system of equations by graphing.
A solution of a system of equations is an ordered pair.

Examples
$3x + 7y = -5$
$x + y = 1$

Examples
$y = -2x + 1$
$y = -2x - 1$

Examples


2. Start with the linear equation $y = -2x + 0.5$. Write an equation that results in a system of equations with the given number of solutions.

   a. No solution Possible answer: $y = -2x - 0.5$

   b. One solution Possible answer: $y = 2x + 0.5$

   c. Infinitely many solutions Possible answer: $2y = -4x + 1$

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REAL-WORLD CONNECTION

A common problem that can be solved by using a system of linear equations is determining when the costs for two different repair technicians is the same. For example, suppose one technician charges an hourly rate to fix an appliance. Another has an upfront service charge to assess the job but then charges a lower hourly rate than the first technician. A system of equations can be used to determine the number of hours a job would have to take for the costs to be the same. This might help determine which technician to use. Ask students to think of other real-world examples where solving a system of equations might be useful.

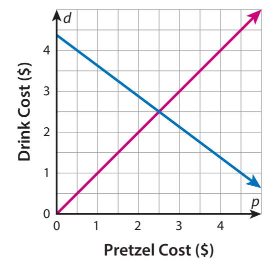
Additional Practice

3 Problem 3 provides another look at using a system of equations to solve a problem. This problem is similar to the problem about the prices of books and puzzles at the yard sale. In both problems, there are two unknowns and two linear equations, and a graph of both equations in the same coordinate plane is provided. This problem asks about the costs of pretzels and drinks at the movies.

Suggest that students use **Three Reads**, asking themselves one of the following questions after each read:

- *What is this problem about?*
- *What am I trying to find out?*
- *What are the important quantities and relationships in the problem?*

3 Riley and her friends buy snacks at the movies. They buy 3 pretzels and 4 drinks for \$17.50. The cost of a pretzel is the same as the cost of a drink. The situation can be represented by the graph and the system of equations shown, where p is the cost of a pretzel and d is the cost of a drink.



$3p + 4d = 17.50$ ⟵ They spend \$17.50 on 3 pretzels and 4 drinks.

$p = d$ ⟵ A pretzel costs the same as a drink.

a. What is the cost of each pretzel? What is the cost of each drink? Show your work.

Possible work:

The point of intersection seems to be (2.5, 2.5).

Check \$2.50:

$3(2.50) + 4(2.50) \stackrel{?}{=} 17.50$

$7.50 + 10.00 = 17.50$ TRUE

SOLUTION Each pretzel is \$2.50 and each drink is \$2.50.

b. Check your answer to problem 3a. Show your work.

Possible work:

All 7 items have the same cost, so I can divide 17.50 by 7 to find the cost for 1 item.

$\frac{17.50}{7} = 2.50$

The cost of each item is \$2.50.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Apply It**

Levels 1–3: Reading/Listening

Help students interpret Apply It problem 7. Use **Three Reads.** Focus on the first paragraph in the first read. Explain the difference between *scores* and *earns points.* Have students state the number of points earned for a touchdown and a field goal. Then have volunteers use **Act It Out** with one person tossing a coin 7 times while other students record points and the final score. Let heads be a touchdown and tails be a field goal. Reread the second paragraph and have students relate their coin toss game to the equations in the problem. For the last read, read the problem. Ask: *What is the system of equations? What do the variables represent?* Have students solve the problem.

Levels 2–4: Reading/Listening

Use **Three Reads** to help students interpret Apply It problem 7. After the first read, have partners turn and talk about the difference between scoring multiple times and earning points. Point to the score board and ask: *What does the number on the scoreboard represent?* Have students share what they know about earning points in sports. After the second read, have students use a coin toss to **Act It Out** to show the way teams earn points in a football game. Let heads be a touchdown and tails be a field goal.

After the last read, display these questions:

- *What do the variables represent?*
- *What information is in each equation?*

Levels 3–5: Reading/Listening

Modify **Three Reads** to help students interpret Apply It problem 7. After the first read, have partners discuss the meanings of *scores* and *earns*. Have them talk about different ways to score and earn points in sports.

After the next read, have partners discuss the quantities and relationships in the problem. Display these questions to support discussions:

- *What do the variables represent?*
- *What information is in each equation?*

Reinforce with students that clear explanations use complete sentences and precise vocabulary.

Develop Solving Systems of Linear Equations by Substitution

Purpose

- **Develop** strategies for solving a system of linear equations by substituting for one of the variables.
- **Recognize** how to use substitution efficiently to solve systems of linear equations.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: $y = 5$	B: $x = 8$
C: $y = 4x + 2$	D: $2x + y = -1$

Possible Solutions

A is the only equation for a horizontal line.

B is the only equation for a vertical line.

C is the only equation written in $y = mx + b$ form where m is not 0.

D is the only equation for a line with a negative slope.

WHY? Support students' facility with connecting linear equations and their graphs.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of *solved for* in mathematical language.

HOW? During Model It, display: *You can rewrite the second equation so both equations are solved for v.* Underline *solved for* and ask students what this phrase is asking them to do. Explain that *solving for* a variable means that you find the value of the variable. Have students share what other variable you could solve for in Model It.

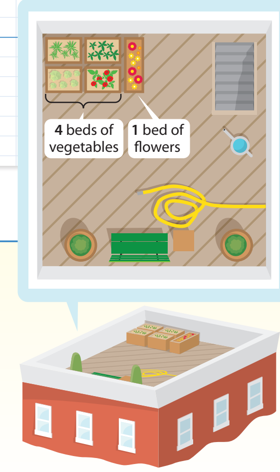
TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. After the problem is read aloud, have students respond to: *What is the situation about?* Ask students to suggest questions that might be asked about the situation. Display students' questions. If time allows at the end of the session, choose one or two of the questions to answer.

LESSON 13 | SESSION 2

Develop Solving Systems of Linear Equations by Substitution



Read and try to solve the problem below.

The residents of a downtown apartment building are starting a rooftop garden. They will plant 4 beds of vegetables for every 1 bed of flowers, plus have 5 extra beds of vegetables. They plan to plant a total of 30 garden beds. In the system of equations shown, v is the number of vegetable beds and f is the number of flower beds.

$v = 4f + 5$

$v + f = 30$

How many vegetable beds and how many flower beds will be in the rooftop garden?

TRY IT **Math Toolkit** graph paper, straightedges

Possible work:

SAMPLE A

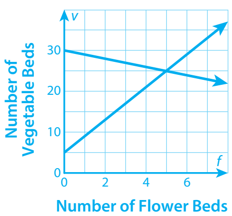
The lines seem to intersect at (5, 25).

Check both equations.

$25 = 4(5) + 5$ TRUE

$25 + 5 = 30$ TRUE

There will be 25 vegetable beds and 5 flower beds in the rooftop garden.



SAMPLE B

For every flower bed, there are 4 vegetable beds, plus there are an extra 5 vegetable beds.

1 flower bed: 4 veg. beds + 5 veg. beds ⟶ 10 garden beds

3 flower beds: 12 veg. beds + 5 veg. beds ⟶ 20 garden beds

5 flower beds: 20 veg. beds + 5 veg. beds ⟶ 30 garden beds

Check $v = 25$ and $f = 5$ make other equation true: $25 = 4(5) + 5$ TRUE

There will be 25 vegetable beds and 5 flower beds in the rooftop garden.

DISCUSS IT

Ask: How did you use the equations given in the problem?

Share: I began by . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Did you graph the equations? How did this help you?*
- *What strategy did you use to find v and f values that worked in both equations?*

Error Alert If students solve the problem by graphing and get the wrong solution, then their graphs may be incorrect or inaccurate. You might suggest that students write both equations in slope-intercept form and use the values of m and b to make their graphs. Remind students that when they find a solution to a system using a graph, they should always check their solution in both original equations. Graphs can provide an estimate or a check on a solution but should not be the only method used.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- bar models used to solve
- graph used to estimate the solution and guess-and-check used to refine
- logical reasoning and guess-and-check used to find solution
- substitution used to solve system

Facilitate Whole Class Discussion

Call on students to share selected strategies. Encourage students who share to explain how their approach is the same as a previous one, and how it is different.

Guide students to **Compare and Connect** the representations. If an explanation is unclear, ask students to rephrase what was said and confirm with the speaker.

> **ASK** *How did the strategies ensure that the values of v and f are solutions to* both *equations?*
>
> **LISTEN FOR** For the graph, (f, v) was on both lines. For guess-and-check, the values of v and f were chosen to satisfy the first equation, then they were checked to see if they satisfied the second. For the equations, one equation was used to find an expression to substitute into the other.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are the solutions in the Model Its similar?*
>
> **LISTEN FOR** In both, one equation is used to find an expression equal to v and that expression is substituted for v in the other equation to get an equation that only has f.

For the bar model method, prompt students to connect the model to the algebraic steps.

- *How do the first two bar models show the equations in the system?*
- *How does the last model show the substitution?*

For the algebraic method, prompt students to discuss how substitution was used.

- *Why can the expressions be set equal to each other?*

➤ **Explore different ways to solve a system of equations by substitution.**

The residents of a downtown apartment building are starting a rooftop garden. They will plant 4 beds of vegetables for every 1 bed of flowers, plus have 5 extra beds of vegetables. They plan to plant a total of 30 garden beds. In the system of equations shown, v is the number of vegetable beds and f is the number of flower beds.

$v = 4f + 5$

$v + f = 30$

How many vegetable beds and how many flower beds will be in the rooftop garden?

Model It

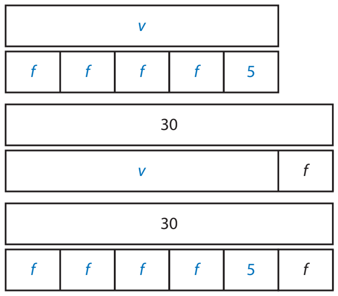
You can substitute an expression from one equation into the other equation.

Since $v = 4f + 5$, you can substitute $4f + 5$ for v in the other equation. You can also show this using bar models.

$v = 4f + 5$

$v + f = 30$

$v + f = 30 \longrightarrow 4f + 5 + f = 30$



Model It

You can rewrite the second equation so both equations are solved for v.

$v + f = 30 \longrightarrow v = -f + 30$

The system is now: $v = 4f + 5$

$v = -f + 30$

Set the two expressions for v equal to each other.

$4f + 5 = -f + 30$

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DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding
Justifying Substitution Choices

Point out that the second Model It solves the *second* equation for v and then substitutes the resulting expression for v into the *first* equation.

> **ASK** *What do you get if you substitute the expression for v back into the second equation instead? Why?*
>
> **LISTEN FOR** You get $-f + 30 + f = 30$, which simplifies to $30 = 30$. This is a true statement, which just confirms that you wrote the equation correctly. If you substitute a true value of v in the original equation, you would expect a true statement.
>
> **ASK** *Does this substitution help you find the solution to the system? Why?*
>
> **LISTEN FOR** It does not help you find a solution because you are working with just one equation. You are not taking both equations of the system into account.

Develop Solving Systems of Linear Equations by Substitution

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the linear equations and the relationship between them are the same in each representation. Explain that they will now use the representations to understand how to use substitution to solve systems of linear equations.

Before students begin to record and expand on their work in Model It, tell them that problems 2 and 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- $v = 4f + 5$ means that there are 4 vegetable beds for every flower bed, plus 5 more.
- $v + f = 30$ means that the total number of vegetable and flower beds together is 30.
- Since v and f each mean the same thing in both equations, one equation can be solved for v in terms of f and then that expression can be substituted for v in the other equation.
- Once the value of f is found, it can be substituted into either original equation to find v.
- There are 5 flower beds and 25 vegetable beds.

Facilitate Whole Class Discussion

3 Listen for understanding that you can choose to solve either equation for either variable, but that you must substitute the resulting expression into the other equation.

> **ASK** *Why might you decide to avoid solving the equation $v = 4f + 5$ for f?*
>
> **LISTEN FOR** First, this equation is already solved for v, so you can use it without calculating further. Second, since f has a coefficient of 4, the expression equivalent to f will involve division by 4. That will make the calculations more complicated.

4 Look for the idea that substitution will always work to solve a system of linear equations.

> **ASK** *Why can you always solve one equation for one variable and substitute the expression for that variable into the other equation?*
>
> **LISTEN FOR** The variables mean the same thing in both equations. So, when I rewrite one equation as a variable equal to an expression, I know that variable in the other equation is also equal to the expression.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use substitution to solve a system of equations.**

1 Explain what $v = 4f + 5$ and $v + f = 30$ tell you about the situation.

$v = 4f + 5$: There are 4 vegetable beds for every flower bed plus 5 more.
$v + f = 30$: There are 30 garden beds altogether (vegetable and flower).

2 **a.** Look at both **Model Its**. You started with a system of two equations in two variables. In each case you end up with a single one-variable equation, $4f + 5 + f = 30$ and $4f + 5 = -f + 30$. How did this happen?

Possible answer: v and f mean the same thing in both equations. If one equation is written as v equal to an expression in f, I can replace v in the other equation (or bar model) with this equal expression. This combines both equations into one equation in one variable, f.

b. How many flower beds will there be in the garden? How many vegetable beds? Explain how you know.

5 flower beds; 25 vegetable beds; Possible explanation: I solved one of the one-variable equations for f. Then I substituted this value into one of the original equations and solved for v.

3 Solve $v + f = 30$ for f. Use this result to solve the system by substituting for f. Does it matter into which equation or for which variable you substitute when solving a system of equations? Explain.

$f = 30 - v$; $f = 5$, $v = 25$; No; Possible explanation: The solutions will be the same, but one choice may have fewer steps or simpler calculations.

4 Do you think using substitution will always work when solving a system of equations? Explain.

Possible answer: Yes. I can always solve one equation for one of the variables. I can then substitute that expression for the variable into the other equation without changing the value of either variable.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a model to understand substitution.

If students are unsure about how to solve a system of equations by substitution, then use this activity to help them gain a better understanding.

Materials For each pair: algebra tiles (at least 10 each of x- and y-tiles and 20 1-tiles)

- Display the system $y = 3 + x$ and $y + 3x = 11$. Have pairs model both equations using their algebra tiles. Ask: *Does either model show a variable tile alone on one side?* [Yes; The first model, $y = 3 + x$, has the y-tile alone on one side.]
- Ask: *How can you use the first model to find a group of tiles to substitute for y in the second model?* [You can replace the y-tile with an x-tile and three 1-tiles.] Have students perform the substitution. Point out that the new model has only one variable, x. Have students use the model to solve for x. [$x = 2$]
- Ask: *How can you now find y?* [I can substitute 2 for x in either original equation; $y = 5$.]
- If time allows, have students solve the first equation to find tiles equivalent to x, and to substitute those tiles in the second equation and solve the system again.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, when graphing a system of linear equations in the coordinate plane, the lines likely will not be perfectly accurate. Remind students to check any solutions obtained by graphing by either substituting the solutions into both equations or by using another solution method.

6. You might ask students to suggest another possible way to start solving the system. Some students may want to complete the solution process. The solution is $\left(2\frac{3}{10}, -\frac{17}{20}\right)$.

7. Students may solve for f in terms of t and write $f = -t + 7$. They would then use substitution to obtain $6t + 3(-t + 7) = 27$. Here is a sample solution of this equation:

$$6t + 3(-t + 7) = 27$$
$$6t - 3t + 21 = 27$$
$$3t + 21 = 27$$
$$3t = 6$$
$$t = 2$$

The value of $t = 2$ can be substituted into either original equation to find $f = 5$.

Apply It

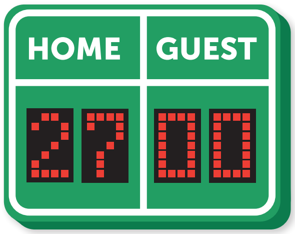
➤ **Use what you learned to solve these problems.**

6. Tara begins solving the system below. What do you think her next step will be?

$6x + 8y = 7$
$-2x + 4y = -8 \longrightarrow -2x = -4y - 8$
$x = 2y + 4$

Possible answer: Substitute $2y + 4$ for x in $6x + 8y = 7$ and solve for y.

7. A football team scores 7 times and earns 27 points. All their points come from 6-point touchdowns and 3-point field goals.



Let t be the number of touchdowns and f be the number of field goals. The equation $t + f = 7$ represents the number of times the team scores. The equation $6t + 3f = 27$ represents the total number of points scored. How many touchdowns and how many field goals does the team make? Show your work.

Possible work:

$t + f = 7$
$t = -f + 7$

$6(-f + 7) + 3f = 27$
$-6f + 42 + 3f = 27$
$-3f = -15$
$f = 5$

$t + f = 7$
$t + 5 = 7$
$t = 2$

SOLUTION The team makes 2 touchdowns and 5 field goals.

8. Solve this system of equations. Show your work.

$3x = 6y - 21$
$6x - 9y = -30$

Possible work:

$3x = 6y - 21 \longrightarrow x = 2y - 7$

$6(2y - 7) - 9y = -30$
$12y - 42 - 9y = -30$
$3y = 12$
$y = 4$

$6x - 9y = -30$
$6x - 9(4) = -30$
$6x - 36 = -30$
$6x = 6$
$x = 1$

SOLUTION (1, 4)

CLOSE EXIT TICKET

8. Students' solutions should show an understanding of:
   - how to use substitution to solve a system of linear equations.
   - the idea that once the value of one variable is found, it can be substituted into either original equation to find the value of the other variable.

Common Misconception If students only find the value of one of the variables, ask them to tell you what their answer means. Help students conclude that one value is not the complete answer; the solution to a system of equations is an ordered pair. Then ask them how they can find the value of the other variable.

Practice Solving Systems of Linear Equations by Substitution

Problem Notes

Assign **Practice Solving Systems of Linear Equations by Substitution** as extra practice in class or as homework.

1. Students may also describe solving $2y = -2 - 5x$ for y since the term with the variable y is isolated. If they do so, they will get $y = -1 - \frac{5}{2}x$. They can then substitute $-1 - \frac{5}{2}x$ for y into the first equation to get $x + \left(-1 - \frac{5}{2}x\right) = 1$ and solve for x. ***Challenge***

2. Because the first equation has s isolated on one side, it is efficient to substitute $t + 5$ for s in the second equation. However, students may use another method. For example, they may use the first equation to write $t = s - 5$, and then substitute $s - 5$ for t in the second equation. ***Medium***

LESSON 13 | SESSION 2 Name:

Practice Solving Systems of Linear Equations by Substitution

➤ **Study the Example showing how to solve a system of equations by substitution. Then solve problems 1–3.**

Example

What is the solution to the system of equations?

$\mathbf{x + y = 1}$

$\mathbf{2y = -2 - 5x}$

Solve the first equation for x. Then substitute the expression into the second equation.

$$x + y = 1$$
$$x = -y + 1$$

$$2y = -2 - 5(-y + 1)$$
$$2y = -2 + 5y - 5$$
$$7 = 3y$$
$$\frac{7}{3} = y$$

$$x + y = 1$$
$$x + \frac{7}{3} = 1$$
$$x = -\frac{4}{3}$$

1. Describe a different way to use substitution to solve the problem in the Example.

   Possible answer: Solve the first equation for y to get $y = -x + 1$. Then substitute $-x + 1$ into the second equation to get $-2x + 2 = -2 - 5x$ and solve for x.

2. Antonio is a set designer. He is gluing ribbon on 4 square and 2 triangular posters to be used as stage props in an upcoming play. Use the system of equations to find s, the amount of ribbon needed for a square poster, and t, the amount of ribbon needed for a triangular poster. Show your work.

   $s = t + 5$

   $4s + 2t = 110$

   Possible work:

   $$4(t + 5) + 2t = 110$$
   $$4t + 20 + 2t = 110$$
   $$6t = 90$$
   $$t = 15$$

   $$s = t + 5$$
   $$s = 15 + 5$$
   $$s = 20$$

   SOLUTION 20 in. for a square poster and 15 in. for a triangular poster

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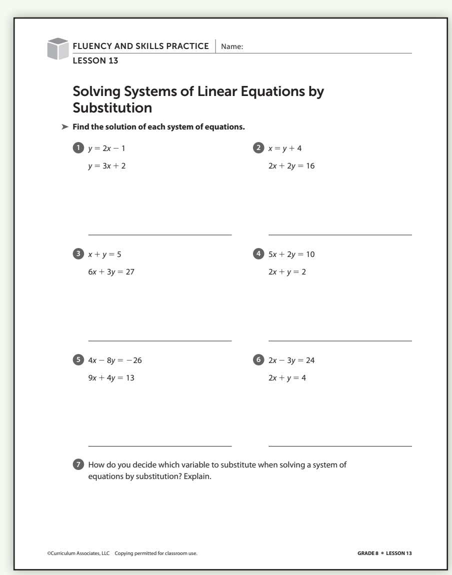
Fluency & Skills Practice

Solving Systems of Linear Equations by Substitution

In this activity, students solve systems of linear equations using substitution.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 13

Solving Systems of Linear Equations by Substitution

➤ Find the solution of each system of equations.

1. $y = 2x - 1$; $y = 3x + 2$
2. $x = y + 4$; $2x + 2y = 16$
3. $x + y = 5$; $6x + 3y = 27$
4. $5x + 2y = 10$; $2x + y = 2$
5. $4x - 8y = -26$; $9x + 4y = 13$
6. $2x - 3y = 24$; $2x + y = 4$
7. How do you decide which variable to substitute when solving a system of equations by substitution? Explain.

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Additional Practice

3 **a.** The solution on the student page is efficient, but there are other possibilities as well. For example, students may solve the second equation for x and substitute the resulting expression into the first equation. ***Medium***

b. Students might see that they can isolate the expression $3x$ on one side of each equation, to get $3x = 11 + 4y$ and $3x = 2 - 2y$. They can then set the right sides of these two equations equal, writing $11 + 4y = 2 - 2y$. ***Challenge***

c. Students should note the second equation gives x in terms of y. They can immediately use substitution to solve. ***Basic***

LESSON 13 | SESSION 2

3 What is the solution of each system of equations? Show your work.

a. $-6x - 5y = 6$

$4x + y = 3$

Possible work:

$$4x + y = 3 \qquad y = -4x + 3$$

$$-6x - 5y = 6$$
$$-6x - 5(-4x + 3) = 6$$
$$-6x + 20x - 15 = 6$$
$$14x = 21$$
$$x = \frac{3}{2}$$

$$4x + y = 3$$
$$4\left(\frac{3}{2}\right) + y = 3$$
$$6 + y = 3$$
$$y = -3$$

SOLUTION $\left(1\frac{1}{2}, -3\right)$

b. $3x - 4y = 11$

$3x + 2y = 2$

Possible work:

$$3x + 2y = 2$$
$$2y = -3x + 2$$
$$y = -\frac{3}{2}x + 1$$

$$3x - 4\left(-\frac{3}{2}x + 1\right) = 11$$
$$3x + 6x - 4 = 11$$
$$9x - 4 = 11$$
$$9x = 15$$
$$x = \frac{5}{3}$$

$$3\left(\frac{5}{3}\right) + 2y = 2$$
$$5 + 2y = 2$$
$$2y = -3$$
$$y = -\frac{3}{2}$$

SOLUTION $\left(\frac{5}{3}, -\frac{3}{2}\right)$

c. $8x + 9y = 20$

$x = -3y$

Possible work:

$$8x + 9y = 20$$
$$8(-3y) + 9y = 20$$
$$-24y + 9y = 20$$
$$-15y = 20$$
$$y = -\frac{4}{3}$$

$$x = -3y$$
$$x = -3\left(-\frac{4}{3}\right)$$
$$x = 4$$

SOLUTION $\left(4, -\frac{4}{3}\right)$

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Model It**

MATH TERMS

A *variable* is a letter that stands for an unknown number.

Like terms are terms that have the same variable or no variable.

Opposite numbers are the same distance from 0 on the number line but in opposite directions.

Levels 1–3: Reading/Speaking

Help students understand Model It by reading it aloud. Explain that both Model Its use a process called *elimination* and that elimination is a way to remove a variable term. Clarify by erasing something. Reread the first Model It and display *Elimination gives you a ____ for ____* and have students complete it. Then reread the second Model It and do the same. Review the Math Terms. Have pairs read the two solutions together and point out examples of the Math Terms. Then have pairs describe a solution using *eliminate* and *elimination.*

Levels 2–4: Speaking/Listening

Help students understand Model It by having partners take turns reading aloud each problem. Modify **Say It Another Way** by having students discuss words and phrases before rephrasing. Display the Math Terms and ask students for examples. Help them connect *eliminate* with *elimination*. Guide students to explain that the word *opposites* refers to terms with opposite numbers as the coefficient.

Guide students to underline words that signal a sequence (*first, then*) or a result (*so*). Have partners share the words they underlined with each other.

Levels 3–5: Speaking/Listening

Help students understand Model It by having them discuss how the lesson vocabulary and Math Terms relate to the problem. Ask students to explain the mathematical meaning of *opposites.*

Point out that sentences often include signal words to help readers identify the cause, sequence, or results of something, for example, *because, first, next, then, finally, so,* and *as a result.* Ask students to discuss the meaning of signal words in Model It.

Have partners take turns rephrasing each Model It, being careful to express the sequence or results with these signal words or others.

Develop Solving Systems of Linear Equations by Elimination

Purpose

- **Develop** strategies for solving a system of linear equations by eliminating one of the variables.
- **Recognize** when elimination is an efficient method for solving systems of linear equations.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $5x$ and $15x$	B: $4y$ and $10y$
C: $-2y$ and $-6y$	D: $3x$ and $4x$

Possible Solutions

All pairs are like terms and all coefficients are integers.

The first term in A and C can be multiplied by 3 to get the second term in the pair.

The pairs in A, B, and C have a common factor besides 1.

The pair in D has only a common factor of 1.

WHY? Support students' facility with identifying common factors to facilitate solving systems of linear equations by elimination.

DEVELOP ACADEMIC LANGUAGE

WHY? Use word parts to support understanding of *elimination* and *substitution*.

HOW? Have students read Connect It problem 4 and identify the two words that name processes. Guide them to share and explain other related words. [*eliminated, eliminating, substituted,* and *substituting*] Encourage students to tell the related word that names what they did in Model It. [*elimination*] Then have students discuss what they do when they *substitute* a variable. [*replace*]

TRY IT

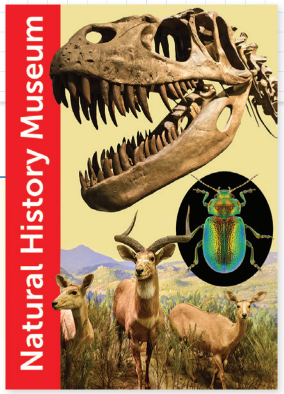
SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding of what the variables in the system of equations represent.

LESSON 13 | SESSION 3 ■■■□□

Develop Solving Systems of Linear Equations by Elimination



➤ **Read and try to solve the problem below.**

Amare is buying museum souvenirs. He buys 3 animal figures and 6 bookmarks for \$33. Later he goes back to buy 3 more figures and return 3 of the bookmarks, spending another \$6. In the system, f is the price of a figure and b is the price of a bookmark.

$3f + 6b = 33$

$3f - 3b = 6$

How much does each figure cost? How much does each bookmark cost?

TRY IT **Math Toolkit** graph paper, straightedges

Possible work:

SAMPLE A

$3f + 6b = 33$	$3f - 3b = 6$	$3f + 6b = 33$
$3f = -6b + 33$	$3(-2b + 11) - 3b = 6$	$3f + 6(3) = 33$
$f = -2b + 11$	$-6b + 33 - 3b = 6$	$3f + 18 = 33$
	$-9b = -27$	$3f = 15$
	$b = 3$	$f = 5$

Each figure costs \$5 and each bookmark costs \$3.

SAMPLE B

$3f - 3b = 6$	$3f + 6b = 33$	$3f - 3b = 6$
$-3b = -3f + 6$	$3f + 6(f - 2) = 33$	$3(5) - 3b = 6$
$b = f - 2$	$3f + 6f - 12 = 33$	$15 - 3b = 6$
	$9f = 45$	$-3b = -9$
	$f = 5$	$b = 3$

Figures cost \$3 each and bookmarks cost \$5 each.

DISCUSS IT

Ask: What strategy did you use?

Share: I began by …

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Did you graph the system of equations? Were you able to make a good estimate of the solution?*
- *Did you use substitution? How did you decide which equation to use to solve for a variable?*
- *Where there any challenges when you substituted?*

Common Misconception Listen for students who think that substitution cannot be used to solve this system of equations because neither given equation has a variable isolated on one side. As students share their strategies, help them see that any equation with two variables can be solved for one variable in terms of the other.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- graph used to estimate solution
- guess-and-check
- (misconception) solution assumes substitution cannot be used because of forms of the equations
- substitution, starting with solving the first or second equation for m
- substitution, starting with solving the first or second equation for b

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that a good explanation describes what you did and why you decided to do it.

Guide students to **Compare and Connect** the representations. If an explanation is unclear, ask a student to reword it to confirm their understanding.

ASK *What makes some strategies more efficient than others?*

LISTEN FOR Some strategies require fewer algebraic steps or allow you to work with integer constants and coefficients. Strategies that give you an exact solution, rather than an estimate that you need to check, are more efficient.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How do the Model It strategies differ?*

LISTEN FOR One adds the equations to eliminate b, and the other adds them to eliminate f.

For the algebraic solution eliminating *b*, prompt students to see why it is useful to multiply the second equation by 2.

- *After multiplying the second equation by 2, what do you notice about the coefficients of b in the two equations?*
- *What happens when the equations are added?*

For the algebraic solution eliminating *f*, prompt students to relate this method to subtraction.

- *Why is multiplying the second equation by −1 and adding it to the first the same as subtracting the second equation from the first?*

LESSON 13 | SESSION 3

➤ Explore different ways to solve systems of equations by elimination.

Amare is buying museum souvenirs. He buys 3 animal figures and 6 bookmarks for \$33. Later he goes back to buy 3 more figures and return 3 of the bookmarks, spending another \$6. In the system, f is the price of a figure and b is the price of a bookmark.

$3f + 6b = 33$

$3f - 3b = 6$

How much does each figure cost? How much does each bookmark cost?

Model It

You can first eliminate the variable b.

Multiply the second equation by 2 so the b terms are opposites. Then add the like terms in the two equations. This gives you a one-variable equation for f.

$$\begin{array}{r} 3f + 6b = 33 \\ 2(3f - 3b = 6) \end{array} \longrightarrow \begin{array}{r} 3f + 6b = 33 \\ \underline{+\ 6f - 6b = 12} \\ 9f + 0b = 45 \end{array}$$

Model It

You can first eliminate the variable f.

Multiply the second equation by -1 so the f terms are opposites. Then add the like terms in the two equations. This gives you a one-variable equation for b.

$$\begin{array}{r} 3f + 6b = 33 \\ \underline{-(3f - 3b = 6)} \end{array} \longrightarrow \begin{array}{r} 3f + 6b = 33 \\ \underline{+\ -3f + 3b = -6} \\ 0f + 9b = 27 \end{array}$$

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DIFFERENTIATION | EXTEND

SMP 4

Deepen Understanding

Using a Model to Visualize a System of Equations Solved by Elimination

Prompt students to consider how the graph of a system of equations relates the graphs of the sums of the equations found during the elimination method.

ASK *Suppose you graphed the equations in the system, with f on the horizontal axis and b on the vertical axis. At what point would the lines intersect? How do you know?*

LISTEN FOR They would intersect at (5, 3), because that is the solution of the system.

ASK *Now suppose you graphed the sum of the equations, $9f + 0b = 45$, from the first Model It. What would the graph look like?*

LISTEN FOR It would be the vertical line with equation $f = 5$.

ASK *Suppose you graphed the sum of the equations from the second Model It, $0f + 9b = 27$. What would the graph look like?*

LISTEN FOR It would be the horizontal line with equation $b = 3$.

ASK *How do these graphs confirm the solution to the system of equations?*

LISTEN FOR The graphs intersect at the point (5, 3), which is the solution of the system.

Develop Solving Systems of Linear Equations by Elimination

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the system of equations is the same in each representation. Explain that they will now use the representations to reason about using elimination to solve a system of equations.

Before students begin to record and expand on their work in Model It, tell them that problems 2 and 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 3

- $3f + 6b = 33$ means that 3 figures and 6 bookmarks cost \$33; $3f - 3b = 6$ means buying 3 figures and returning 3 bookmarks costs \$6.
- Multiplying both sides of an equation by the same number does not change the solutions. You can verify this by writing the original and new equations in slope-intercept form.
- Adding one equation to another is the same as adding the same quantity to both sides of either equation, so the result is a true equation.
- The final solution to the system is the same no matter which variable is eliminated.

Facilitate Whole Class Discussion

4 To engage all students, ask them to turn and talk to their partner before sharing their answers.

> **ASK** *Can you use either substitution or elimination to solve any system of linear equations? If so, why do you think it is useful to understand both methods?*
>
> **LISTEN FOR** Any system can be solved by either substitution or elimination. However, depending upon the equations, one method may require fewer steps or have simpler calculations than the other.

> **ASK** *When might you choose to use elimination to solve a system?*
>
> **LISTEN FOR** Elimination makes sense when like terms are opposites or multiples of each other.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use elimination to solve a system of equations.**

1 Explain what $3f + 6b = 33$ and $3f - 3b = 6$ tell you about the situation.

$3f + 6b = 33$: Buying 3 figures and 6 bookmarks costs \$33.
$3f - 3b = 6$: Buying 3 figures and returning 3 bookmarks costs \$6.

2 **a.** Look at the first **Model It**. Why does the equation $6f - 6b = 12$ have the same solutions as $3f - 3b = 6$?

Possible explanation: When I write both equations in slope-intercept form they are identical, so they have the same solutions.

b. How does knowing that $6f - 6b = 12$ allow you to add $6f - 6b$ to one side and 12 to the other side of the equation $3f + 6b = 33$?

If $6f - 6b$ is equal to 12, then the same quantity is being added to each side of the equation $3f + 6b = 33$. So, the resulting equation is also true.

c. How much does each figure cost? How much does each bookmark cost?

Each figure costs \$5 and each bookmark cost \$3.

3 Solve the equation $0f + 9b = 27$ in the second **Model It** for b. Do you get the same answer as you did in problem 1c? **$b = 3$; Yes.**

4 How could you use substitution to find how much each figure and bookmark cost? When do you think you might choose to use substitution to solve a system instead of elimination?

Possible answer: I could solve either equation for f or for b. Then substitute this expression for the variable in the other equation. I would use substitution if one of the equations is already solved for a variable or could easily be solved for a variable.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use a model to understand the elimination method.

If students are unsure about using elimination to solve systems of linear equations, then use this activity to help them understand the process.

Materials For display: a large four-quadrant coordinate plane

- Write and display the system: $4x + 2y = 32$ and $2x - 5y = -8$. Have students take turns finding and plotting points to graph the system.
- Ask: *Where do the lines intersect?* [(6, 4)]
- Have students write the equation that results from multiplying both sides of the second equation by -2. Then have them take turns finding and plotting points to graph this new equation on the same coordinate plane.
- Ask: *What do you notice about this line?* [It is the same as the second line.]
- Have students add this new equation to the first equation. Ask: *What is the sum of the equations?* [$12y = 48$] Have students solve for y. Ask: *How does this value of y compare to the y-coordinate of the intersection point of the two lines?* [It is the same, 4.]
- Have students use this value to solve for x and confirm their solution, (6, 4).

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision if they choose to graph the system of equations. For the purpose of this session, graphs should serve as a way for students to reinforce their understanding of the problems and solutions.

6 Because this problem specifically instructs students to use elimination, an efficient method is to multiply one equation by -1 and add the equations to eliminate y (or to subtract one equation from the other). However, some students may choose to eliminate x by multiplying the first equation by 5, or -5, and the second by -3, or 3, and adding the resulting equations.

7 **B, C, and D are correct.** Any of these strategies will eliminate one of the variables.

A is not correct. Multiplying the first equation by 2 makes the coefficients of y the same, rather than opposites, so y will not be eliminated when the equations are added.

E is not correct. Multiplying the second equation by $\frac{1}{2}$ makes the coefficients of y the same, rather than opposites, so y will not be eliminated when the equations are added.

Apply It

➤ **Use what you learned to solve these problems.**

6 Adela has 24 bracelets. She makes 3 more each day. Isaiah has 10 bracelets. He makes 5 more each day. The equations represent the number of bracelets y each person has after x days.

Adela: $y = 3x + 24$

Isaiah: $y = 5x + 10$

Use elimination to solve the system of equations. What does the solution mean in the situation?

(7, 45); After 7 days, they will each have 45 bracelets.

7 Which of these strategies will eliminate a variable to help you solve the system of equations to the right? Select all that apply.

$12x + 5y = 7$
$-4x + 10y = -7$

A Multiply the first equation by 2 and add it to the second equation.

B Multiply the first equation by -2 and add it to the second equation.

C Multiply the first equation by $\frac{1}{3}$ and add it to the second equation.

D Multiply the second equation by 3 and add it to the first equation.

E Multiply the second equation by $\frac{1}{2}$ and add it to the first equation.

8 Solve the system of equations. Show your work.

$-6x + 4y = -28$
$-9x + 5y = -33$

Possible work:

$$\begin{aligned} 3(-6x + 4y &= -28) \\ -2(-9x + 5y &= -33) \end{aligned} \rightarrow \begin{aligned} -18x + 12y &= -84 \\ +\quad 18x - 10y &= 66 \\ \hline 2y &= -18 \\ y &= -9 \end{aligned} \qquad \begin{aligned} -6x + 4(-9) &= -28 \\ -6x - 36 &= -28 \\ -6x &= 8 \\ x &= -\tfrac{4}{3} \end{aligned}$$

SOLUTION $\left(-\frac{4}{3}, -9\right)$

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CLOSE EXIT TICKET

8 Students' solutions should show an understanding that:

- the elimination method requires multiplying one or both equations by a number to create variable terms with opposite coefficients and then adding the resulting equations.
- once the value of one variable is found, it can be substituted into either original equation to find the value of the other variable.

Error Alert If students multiply the equations by numbers to create opposite terms, but still get the wrong answer, then encourage them to go back and check their solution steps for computational errors. It can help some students to write notes about what they are doing as part of their work, such as, *Multiply equation 1 by 3.*

Practice Solving Systems of Linear Equations by Elimination

Problem Notes

Assign **Practice Solving Systems of Linear Equations by Elimination** as extra practice in class or as homework.

1. Students may also solve by multiplying the second equation by -1 and adding the equations (or by subtracting the equations) to eliminate y. ***Basic***

2. **a.** Students should understand that regardless of the forms of the equations, all systems of linear equations may be solved by either method: elimination or substitution. ***Medium***

   b. Students may recognize that elimination is an efficient choice because the coefficients of the x-terms are opposites. ***Medium***

LESSON 13 | SESSION 3 Name:

Practice Solving Systems of Linear Equations by Elimination

➤ **Study the Example showing how to use elimination to solve systems of equations. Then solve problems 1–4.**

Example

What is the solution of the system of equations?

$$-5x + 2y = 22$$
$$10x + 2y = -8$$

$$2(-5x + 2y = 22) \longrightarrow \begin{array}{r} -10x + 4y = 44 \\ \underline{+\ \ 10x + 2y = -8} \\ 6y = 36 \\ y = 6 \end{array} \qquad \begin{array}{r} 10x + 2y = -8 \\ 10x + 2(6) = -8 \\ 10x + 12 = -8 \\ 10x = -20 \\ x = -2 \end{array}$$

The solution is $(-2, 6)$.

1. Show a different way to use elimination to solve the system in the Example.

   Possible work:

$$\begin{array}{r} 10x + 2y = -8 \\ \underline{-\ (-5x + 2y = 22)} \\ 15x = -30 \\ x = -2 \end{array} \qquad \begin{array}{r} -5(-2) + 2y = 22 \\ 10 + 2y = 22 \\ 2y = 12 \\ y = 6 \end{array}$$

2. Cheryl says that the system of equations at the right MUST be solved by elimination rather than by substitution.

$$-7x + 12y = 13$$
$$7x - 11y = -9$$

   a. Explain why Cheryl is not correct.

   Possible answer: Any system of linear equations can be solved by substitution.

   b. Why might Cheryl think this is true?

   Possible answer: It makes sense to use elimination. The equations can be added and a variable eliminated without any additional steps.

Vocabulary

system of linear equations
a group of related linear equations in which a solution makes all the equations true at the same time.

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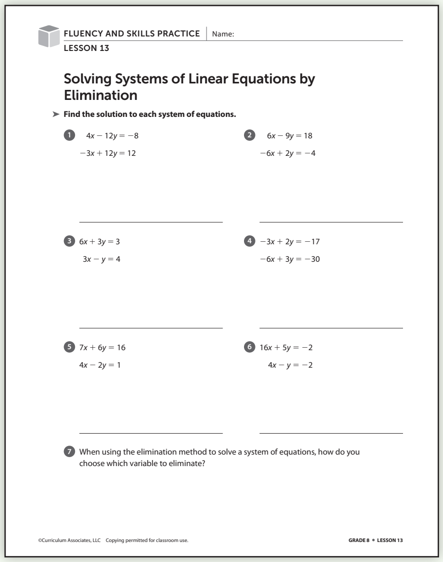
Fluency & Skills Practice

Solving Systems of Linear Equations by Elimination

In this activity, students solve systems of linear equations using elimination.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 13

Solving Systems of Linear Equations by Elimination

➤ Find the solution to each system of equations.

1. $4x - 12y = -8$; $-3x + 12y = 12$
2. $6x - 9y = 18$; $-6x + 2y = -4$
3. $6x + 3y = 3$; $3x - y = 4$
4. $-3x + 2y = -17$; $-6x + 3y = -30$
5. $7x + 6y = 16$; $4x - 2y = 1$
6. $16x + 5y = -2$; $4x - y = -2$
7. When using the elimination method to solve a system of equations, how do you choose which variable to eliminate?

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Additional Practice

3 **a.** Some students may also suggest multiplying the second equation by $\frac{1}{3}$. Although the calculations would involve fractions, this method would work. ***Basic***

b. Students may also suggest multiplying the second equation by $-\frac{1}{2}$. ***Basic***

c. The solution steps will depend on which variable students choose to eliminate, but the solution will be the same for either choice. ***Medium***

4 **a.** There are many ways to solve this system, but since the *x*-terms and *y*-terms are multiples of each other, elimination may be a good choice. ***Medium***

b. Students who chose elimination may solve by multiplying the second equation by −2 and then adding it to the first equation to eliminate *y*. They may also choose to multiply the first equation by −3 and add it to the second equation to eliminate *x*. ***Challenge***

LESSON 13 | SESSION 3

3 Use the system of equations shown. $-2x - 4y = 24$
$6x - 8y = 28$

a. How could you change one of the equations so that you could add it to the other equation and eliminate the *x* terms?

Possible answer: Multiply $-2x - 4y = 24$ by 3.

b. How could you change one of the equations so that you could add it to the other equation and eliminate the *y* terms?

Possible answer: Multiply $-2x - 4y = 24$ by -2.

c. What is the solution of the system? Show your work.

Possible work:

$$-2(-2x - 4y = 24) \longrightarrow \begin{array}{r} 4x + 8y = -48 \\ +\ 6x - 8y = 28 \\ \hline 10x = -20 \\ x = -2 \end{array} \qquad \begin{array}{r} -2(-2) - 4y = 24 \\ 4 - 4y = 24 \\ -4y = 20 \\ y = -5 \end{array}$$

SOLUTION $(-2, -5)$

4 Use the system of equations shown. $3x + 4y = -9$
$9x + 2y = 3$

a. Would you choose substitution or elimination to solve this system? Explain.

Possible answer: Elimination; If I use substitution, I will need to work with a fraction if I solve for either variable. With elimination, I could multiply either equation by an integer and add the equations together to eliminate a variable.

b. Solve the system using the method you chose in problem 4a. Show your work.

Possible work:

$$-3(3x + 4y = -9) \longrightarrow \begin{array}{r} -9x - 12y = 27 \\ +\ \ 9x + \ 2y = 3 \\ \hline -10y = 30 \\ y = -3 \end{array} \qquad \begin{array}{r} 9x + 2(-3) = 3 \\ 9x - 6 = 3 \\ 9x = 9 \\ x = 1 \end{array}$$

SOLUTION $(1, -3)$

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Connect It**

Levels 1–3: Speaking/Writing

Help students read and interpret Connect It problem 4. Display: *Suppose you solve a system of equations and both variables are eliminated.* Clarify the phrase *system of equations* by referring to the system on the previous page.

Modify **Say It Another Way** to help students paraphrase the problem. After each sentence, give partners think time before asking them to discuss the meaning of the sentence. Then have volunteers share their revised sentences. Suggest replacement words as needed, such as *gotten rid of* for *eliminated*. Provide a sentence frame to help students answer in a complete sentence:

- *The system has either* ____ *or* ____.

Levels 2–4: Speaking/Writing

Read aloud Connect It problem 4 and help students interpret its meaning. Display the lesson vocabulary and ask students to discuss their meanings. Review the related words *elimination*, *eliminate*, and *eliminated*. Check that students understand that *system* refers to the review term *system of linear equations*.

Modify **Say It Another Way** to help students paraphrase the problem. For each sentence, provide individual think time before asking one or more volunteers to suggest a paraphrase.

Then have partners share how they will solve the problem and write their answers using:

- *The system has* ____.

Levels 3–5: Speaking/Writing

Have students read Connect It problem 4 and work in pairs to interpret its meaning. First, encourage students to identify and discuss the meaning of repeated words and phrases, like *system* and *number of solutions*. Then have partners discuss the words and phrases they want to reword and divide the question into chunks. Have partners use **Say It Another Way** and take turns paraphrasing the question.

Then have partners share how many questions they will be responding to and how they will respond to each before writing. After drafting responses, have partners share their work with each other.

Develop Determining When a System Has Zero or Infinitely Many Solutions

Purpose

- **Develop** strategies for determining whether a system of linear equations has no solution, one solution, or infinitely many solutions.
- **Recognize** how slopes, y-intercepts, and coefficients can be used to determine the number of solutions to a system of equations.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $2(x + 3) = x$	B: $16x + 20 = 4(4x + 5)$
C: $3(x + 8) = 3x + 7$	D: $5(x + 1) = 10x + 3$

Possible Solutions

All are one-variable linear equations.

All have parentheses.

A and D have one solution.

B has infinitely many solutions.

C has no solution.

WHY? Support students' facility with identifying the number of solutions of one-variable equations.

DEVELOP ACADEMIC LANGUAGE

WHY? Guide students to listen carefully to explanations and ask clarifying questions.

HOW? Have students practice good listening skills as they read their explanations to Connect It problem 4. Explain that they can take notes and then paraphrase their partner's explanation. Remind them to always ask a question like, *Did I understand what you said correctly?* and listen carefully to their partner's response. If needed, have the partner clarify the ideas.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Record students' observations and questions.

LESSON 13 | SESSION 4 ■ ■ ■ ■ □

Develop Determining When a System Has Zero or Infinitely Many Solutions

➤ **Read and try to solve the problem below.**

How many solutions does the system of equations have?

$$-\frac{1}{2}x + y = 3$$
$$x - 2y = 4$$

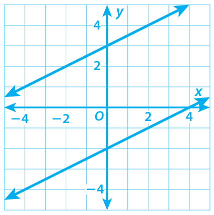
TRY IT Math Toolkit graph paper, straightedges

Possible work:

SAMPLE A

$-\frac{1}{2}x + y = 3$	
x	y
0	3
−6	0

$x - 2y = 4$	
x	y
0	−2
4	0



The lines are parallel. The system has no solution.

SAMPLE B

$$-\frac{1}{2}x + y = 3$$

$$y = \frac{1}{2}x + 3 \longrightarrow x - 2\left(\frac{1}{2}x + 3\right) = 4$$

$$x - x - 6 = 4$$

$$-6 = 4$$

When I solved one-variable equations and got a false statement like $-6 = 4$, it meant there was no solution.

DISCUSS IT

Ask: How did you find how many solutions the system has?

Share: I noticed that . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What are the possibilities for the number of solutions a system of linear equations can have?*
- *Do you have to solve the system to figure out how many solutions it has?*

Common Misconception Listen for students who think that because the equations represent lines with the same slope, the system has infinitely many solutions. As students share their strategies, ask them what the slope-intercept form of the equations tells them about the graphs. If no one presents a graph, work with the class to make one. Discuss the fact that because the lines have the same slope, but different y-intercepts, they are different, parallel lines. They never intersect, so the system has no solution.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- graph of the equations
- (misconception) conclusion that the same slope means infinitely many solutions
- equations written in slope-intercept form
- substitution method
- elimination method

Facilitate Whole Class Discussion

Call on students to share selected strategies. Reinforce that good listeners ask questions to clarify ideas or ask for more information during math discussions.

Guide students to **Compare and Connect** the representations. If students' ideas are unclear, you might rephrase them so that others understand. Confirm that your rewording is accurate.

ASK *How does each model show that the system has no solution?*

LISTEN FOR In the graph, the two lines do not intersect. When the equations are written in slope-intercept form, the slopes are the same but the *y*-intercepts are different. An algebraic solution results in a false statement.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How are the Model Its different?*

LISTEN FOR In the first Model It, the system is solved algebraically, resulting in false statements. In the second, the equations are written in slope-intercept form and the slopes and *y*-intercepts are compared.

For the algebraic solutions, prompt students to consider the work shown.

- *Using substitution, why do you think the second equation was solved for x, rather than the first?*
- *Using elimination, why was the first equation multiplied by 2?*

For the slope-intercept method, prompt students to think about why rewriting the equations is helpful.

- *What do the slopes and y-intercepts tell you about the system?*

➤ **Explore different ways to determine the number of solutions a system of equations has.**

How many solutions does the system of equations have?

$$-\frac{1}{2}x + y = 3$$
$$x - 2y = 4$$

Model It

You can solve the system.

Solve by substitution.

$$x - 2y = 4$$
$$x = 2y + 4 \longrightarrow -\frac{1}{2}(2y + 4) + y = 3$$
$$-y - 2 + y = 3$$
$$-2 = 3$$

Solve by elimination.

Multiply the first equation by 2.

$$2(-\frac{1}{2}x + y = 3) \longrightarrow -x + 2y = 6$$
$$x - 2y = 4 \qquad + \quad x - 2y = 4$$
$$0 = 10$$

Model It

You can write the equations in slope-intercept form and compare.

$$-\frac{1}{2}x + y = 3 \qquad\qquad x - 2y = 4$$
$$y = \frac{1}{2}x + 3 \qquad\qquad -2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$

The slopes are the same and the *y*-intercepts are different.

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding
Identifying Systems Where Both Variables May Be Eliminated

Prompt students to examine the coefficients of the variables in the system of equations.

ASK *How does the x-coefficient in the second equation compare to the x-coefficient in the first equation? How do the y-coefficients in the two equations compare?*

LISTEN FOR The *x*-coefficient in the second equation is −2 times the *x*-coefficient in the first equation. The same is true for the corresponding *y*-coefficients.

ASK *When you multiply the first equation by 2 to make the x-coefficients opposites, what happens to the y-coefficients? What happens when you add the equations?*

LISTEN FOR The *y*-coefficients also become opposites. Both variables are eliminated when you add the equations.

ASK *When you use elimination to solve any system where corresponding x- and y-coefficients are related by the same multiple, will both variables be eliminated? Why?*

LISTEN FOR Yes. If you multiply one equation by a value to make the *x*-coefficients opposites, then the same value will also make the *y*-coefficients opposites.

Develop Determining When a System Has Zero or Infinitely Many Solutions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the system of equations is the same in each representation. Explain that they will now use those representations to reason about how to determine the number of solutions a system has.

Before students begin to record and expand on their work in Model It, tell them that problems 1–3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The end statements in the first Model Its are false, indicating that there is no (x, y) pair that will make both equations true. Therefore, the system has no solution.
- The graph of the system will show two parallel lines. Because parallel lines never intersect, the system has no solutions.
- There is no way to eliminate one variable without eliminating the other.

Facilitate Whole Class Discussion

3 Look for understanding of how this system is different from the system in the Try It.

> **ASK** *What happens when you solve this system? How is this different from the solution of the system in the Try It?*
>
> **LISTEN FOR** Both variables are eliminated and you get a true statement. In the Try It system, the resulting statement is false.

> **ASK** *How do the graphs of the two systems compare?*
>
> **LISTEN FOR** For this system, the graphs of both equations are the same line. For the Try It system, they are parallel lines.

4 Look for understanding that when both variables are eliminated, the system either has no solutions or infinitely many solutions.

> **ASK** *When both variables are eliminated, how does the end statement indicate whether there are no solutions or infinitely many solutions?*
>
> **LISTEN FOR** When the end statement is false, there are no solutions. When the end statement is true, there are infinitely many solutions.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to determine the number of solutions a system of equations has.**

1 Look at the first **Model It**. What happens to the variables when you solve by either method? What kind of statements are $-2 = 3$ and $0 = 10$?
Both variables are eliminated; They are false statements.

2 Look at the second **Model It**. What will the graph of this system look like? How many solutions does this system have?
The graph would show parallel lines. Parallel lines never intersect, so the system has no solution.

3 **a.** Solve this system of equations by either substitution or elimination. What happens to the variables? What result do you get?

$5x - y = 8$

$10x - 2y = 16$

Both variables are eliminated; Possible answers: $16 = 16$ or $0 = 0$

b. Write both equations in slope-intercept form. Compare slopes and y-intercepts. What will the graph of this system look like? How many solutions does this system have?
Both are $y = 5x + 8$, so slopes and y-intercepts are the same; The graph is the same line. The system has infinitely many solutions.

c. Suppose the equations had different slopes. How many solutions would the system have? **only one**

4 Suppose you solve a system of equations and both variables are eliminated. What do you know about the number of solutions the system has? Do you need to solve the system to find the number of solutions? Explain.
The system has either no solutions or infinitely many solutions. No; Possible explanation: I can write the equations in $y = mx + b$ form and compare m and b.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to determine whether a system has no solution or infinitely many solutions.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use a graph to predict the number of solutions to a system.

If students are unsure about systems with no solution or infinitely many solutions, then use this activity to relate the graph of a system to its algebraic solution.

Materials For display: a large four-quadrant coordinate plane

- Display systems: A: $2x + 3y = 6$ and $6y - 12 = -4x$; B: $4y - x = 0$ and $2x - 8y = -16$. Ask volunteers to rewrite each equation in slope-intercept form. Have others come up and graph each line. Encourage students who are watching to check the work.
- When both systems have been graphed, ask: *How many solutions does System A have? System B? How do you know?* [System A has infinitely many solutions because the graphs are the same line. System B has no solution because the lines are parallel.]
- Ask students to make a prediction about whether the algebraic solution to each system would result in a true statement or a false statement.
- Divide students into two groups and have each solve a system algebraically. Have groups share results and assess predictions. [System A ends in a true statement and has infinitely many solutions; System B ends in a false statement and has no solution.]

Apply It

For all problems, encourage students to use a model to support their thinking.

6 Students may solve using substitution by replacing x in the first equation with -2. The end statement is $4 = 1$. Since this is a false statement, the system has no solution.

Students may use elimination by multiplying the second equation by 2 and adding the two equations. The end statement is $0 = -3$. This is a false statement, so the system has no solution.

7 **C** **is correct.** Students may see that if they multiply the first equation by -2 and the second by 3, only x will be eliminated.

A is not correct. Adding the equations eliminates both variables.

B is not correct. Multiplying the first equation by -2 and the second by 3 and adding eliminates both variables.

D is not correct. Multiplying the first equation by -2 and adding eliminates both variables.

Apply It

➤ **Use what you learned to solve these problems.**

6 How many solutions does the system of equations have? How do you know?

$-2x = 1$

$x = -2$

None; Possible explanation: The graphs of the equations are two different vertical lines, $x = -\frac{1}{2}$ and $x = -2$. All vertical lines are parallel, so there is no solution.

7 Which system of equations has exactly one solution?

A $3x + 6y = 5$
$-3x - 6y = 5$

B $3x - 6y = 0$
$-2x + 4y = 0$

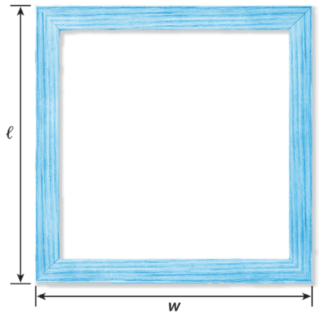
C $3x + 6y = 12$
$2x - 4y = -8$

D $-3x + 2y = 7$
$-6x + 4y = 14$

8 Alec is building a rectangular picture frame. He wants the sum of the length and width to be 12 in. and the perimeter to be 30 in. Use the system of equations to determine how many possibilities there are for the length and width of the frame.

$\ell + w = 12$

$2\ell + 2w = 30$



Possible work:

$-2(\ell + w = 12) \longrightarrow -2\ell - 2w = -24$

$2\ell + 2w = 30 \longrightarrow 2\ell + 2w = 30$

$0 = 6$

The system has no solution. There is no combination of length and width that satisfies both requirements.

CLOSE EXIT TICKET

8 See ***Connect to Culture*** to support student engagement. Students' solutions should show an understanding of:

- how to decide on a strategy to solve the system.
- how to analyze the solution result.

Error Alert If students find a solution, then they may have made a mistake in their solution steps. Encourage them to check their solution by substituting it back into both equations in the system. Then, have them check the steps of their solution process to try to identify their mistake.

Practice Determining When a System Has Zero or Infinitely Many Solutions

Assign **Practice Determining When a System Has Zero or Infinitely Many Solutions** as extra practice in class or as homework.

1 **a.** If students add the equations in order to eliminate the y-terms, they will get $2x = 32$, or $x = 16$. The solution of the system is $\left(16, \frac{1}{2}\right)$. ***Basic***

b. ***Basic***

LESSON 13 | SESSION 4 Name:

Practice Determining When a System Has Zero or Infinitely Many Solutions

➤ **Study the Example showing how to determine the number of solutions a system of equations has. Then solve problems 1–4.**

Example

How many solutions does each system of equations have? Explain how you can tell.

a. $4x - 12y = 13$
$2x - 6y = 9$

b. $-5x + 9y = 17$
$5x - 9y = -17$

a. No solution; Multiply the second equation by -2 and add it to the first equation. Both variables are eliminated, resulting in $0 = -5$, a false statement.

b. Infinitely many solutions; Add the equations together. Both variables are eliminated, resulting in $0 = 0$, a true statement.

1 How many solutions does each system of equations have? Explain how you can tell.

a. $x + 6y = 19$
$x - 6y = 13$

One solution; When the equations are added, the y-terms are eliminated but the x-terms are not, so you can find a numerical value for x.

b. $8x + y = 3$
$-8x - y = 3$

No solution; When the equations are added, you get the false statement $0 = 6$.

Vocabulary
system of linear equations
a group of related linear equations in which a solution makes all the equations true at the same time. A system of equations can have zero, one, or infinitely many solutions.

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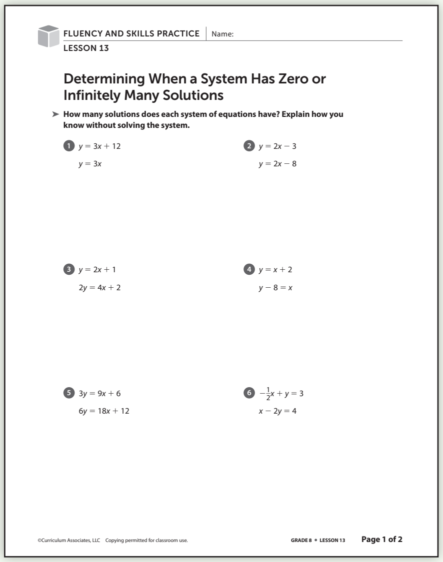
Fluency & Skills Practice

Determining When a System Has Zero or Infinitely Many Solutions

In this activity, students analyze systems by looking at the slopes and y-intercepts of the equations. They use this and other techniques, including substitution and elimination, to determine whether a system has one, zero, or infinitely many solutions.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 13

Determining When a System Has Zero or Infinitely Many Solutions

➤ How many solutions does each system of equations have? Explain how you know without solving the system.

1. $y = 3x + 12$, $y = 3x$
2. $y = 2x - 3$, $y = 2x - 8$
3. $y = 2x + 1$, $2y = 4x + 2$
4. $y = x + 2$, $y - 8 = x$
5. $3y = 9x + 6$, $6y = 18x + 12$
6. $-\frac{1}{2}x + y = 3$, $x - 2y = 4$

Page 1 of 2

Additional Practice

2 Students may also solve the second equation for either ℓ or w and use substitution to get an end statement of $50 = 50$. ***Medium***

3 **a.** Students may reason that 6 is 3 times 2 and so if g is 3 times 5 and h is 3 times 9, then multiplying the second equation by -3 and adding will result in the true statement $0 = 0$. ***Challenge***

b. Students may reason that 6 is 3 times 2 and so if g is 3 times 5 and h is any number *except* 3 times 9, then multiplying the second equation by -3 and adding will result in a false statement. ***Challenge***

c. Students may reason that 6 is 3 times 2 and so if g is any number *except* 3 times 5, then it will not be possible to eliminate both variables, and the system will have a solution. ***Challenge***

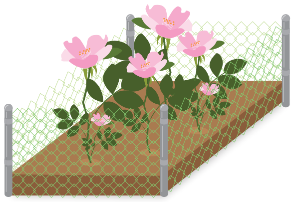
4 **a.** Students may multiply the first equation by 2, then add the equations to get the true end statement $0 = 0$. ***Medium***

b. Students may multiply the first equation by 2 and the second equation by 3 to eliminate y and get the false end statement $0 = -60$. ***Medium***

c. Students may substitute 5 for y into the first equation to get the true end statement of $1 = 1$. ***Medium***

d. Students may multiply the second equation by 4 and then add. This eliminates only y, so the system has one solution. ***Medium***

2 Mariko is fencing her garden. She wants to use 50 feet of fencing. Mariko also wants the sum of the length and the width of the garden to be 25 feet. Use the system of equations to confirm that there are infinitely many possibilities for the length and width. Are there any limits to what values the length and width can be? Explain your reasoning.



$2\ell + 2w = 50$

$\ell + w = 25$

Possible answer: I get the true statement 0 = 0 when I solve, so there are infinitely many solutions. Yes, there are limits on the length and width. For example, neither can be negative and neither can be 25 or larger.

3 Use the system of equations shown.

$gx - 6y = h$
$5x - 2y = 9$

a. What values could you substitute for g and h to create a system of equations with infinitely many solutions?

$g = 15$, $h = 27$

b. What values could you substitute for g and h to create a system of equations with no solution?

$g = 15$; any value for h other than 27

c. What values could you substitute for g and h to create a system of equations with exactly one solution?

any value for g other than 15; any value for h

4 Tell how many solutions each system of equations has.

a. $3x - 2y = -1$

$-6x + 4y = 2$

infinitely many solutions

b. $12x - 15y = 24$

$-8x + 10y = -36$

no solution

c. $\frac{1}{5}y = 1$

$y = 5$

infinitely many solutions

d. $-4y = 8x + 20$

$y = 2x + 5$

one solution

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Speaking/Writing

Before reading Apply It problem 7, use **Notice and Wonder** to help students discuss the system of equations and the illustration.

Read the problem aloud, pausing after each sentence to confirm understanding. Reread the third sentence. Have students use **Act It Out** with two number cubes to show how the girls might meet at a given distance or time. Have students complete this frame:

- *A solution for the variable ____ would show the ____ that the girls meet.*

Have partners solve the problem and craft their written responses together. Invite partners to share with the class. Reword any unclear statements to model use of academic English.

Levels 2–4: Speaking/Writing

Before reading Apply It problem 7, modify **Notice and Wonder** to help students discuss the system of equations and illustration. First, have partners turn and talk about what they notice and wonder. Have students share their ideas. Then read the problem aloud. Ask: *What would a solution for each variable show?* Have students reread the third sentence and respond using:

- *A solution for the variable ____ would show ____.*

Have partners solve the problem and craft their written responses together. Invite partners to share with the class. Call on volunteers to explain how they solved it.

Levels 3–5: Speaking/Writing

Before reading Apply It problem 7, have students use **Notice and Wonder** to discuss the equations and the visual representation. Call on students to share their ideas while you record them for reference.

Have students read the problem and discuss what a solution for each variable would show about the girls' ride to school.

Have students independently solve the problem and craft their written responses to answer the question. Then have them share and compare their answers with partners. Reinforce that students should refer to their work as they justify and explain their responses.

Refine Solving Systems of Linear Equations Algebraically

Purpose

- **Refine** strategies for solving systems of equations algebraically.
- **Refine** understanding of how to choose an efficient strategy to solve a system of linear equations, when it might be helpful to estimate a solution by graphing, and how to determine the number of solutions.

START CHECK FOR UNDERSTANDING

What is the solution to the system of equations?

$5x = 3y + 11$
$-5x = 2y - 1$

Solution

$(1, -2)$

WHY? Confirm students' understanding of solving systems of equations algebraically, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 13 | SESSION 5

Refine Solving Systems of Linear Equations Algebraically

➤ **Complete the Example below. Then solve problems 1–9.**

Example

What is the solution of the system of equations?

$3x = 4y - 20$

$3x = -y + 10$

Look at how you could solve the system by substitution.

$4y - 20$ and $-y + 10$ are both equal to $3x$.

$4y - 20 = -y + 10$	$3x = -y + 10$
$5y = 30$	$3x = -6 + 10$
$y = 6$	$3x = 4p$
	$x = \frac{4}{3}$

SOLUTION $\left(\frac{4}{3}, 6\right)$

CONSIDER THIS . . .
The equations show two different expressions that are equal to $3x$.

PAIR/SHARE
How could you use elimination to solve this problem?

Apply It

1. What is the solution of the system of equations? Show your work.

$y = x + 3$

$3x - 4y = -7$

Possible work:

$3x - 4y = -7$	$y = x + 3$
$3x - 4(x + 3) = -7$	$y = -5 + 3$
$3x - 4x - 12 = -7$	$y = -2$
$-x - 12 = -7$	
$-x = 5$	
$x = -5$	

SOLUTION $(-5, -2)$

CONSIDER THIS . . .
Look at how the equations are written. Does this make you want to use a particular strategy?

PAIR/SHARE
How can you check your answer?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$(-5, -12)$ or $(5, -12)$	have subtracted the right sides of the equations instead of adding when eliminating x.	Elicit from students that the terms $5x$ and $-5x$ are opposites. Prompt them to articulate that this means the equations should be added, not subtracted, in order to eliminate x.
$(-2, 1)$	have switched the x- and y-values.	Ask students to describe how they can use substitution to verify their solution.
no solution or infinitely many solutions	have thought that when adding the equations, both variables would be eliminated.	Prompt students to add the equations. Point out that only x is eliminated. Elicit from students that once x is eliminated, they can solve for y.

Example

Guide students in understanding the Example. Ask:

- *What are the expressions on the right side of each equation equal to?*
- *If two expressions are equal to the same quantity, how can substitution be used?*
- *After using substitution to solve for one of the variables, how can you find the value of the other variable?*

Help all students focus on the Example and responses to the questions by asking them to agree, disagree, or add on to classmates' responses.

Look for understanding that students can set the expressions for $3x$ equal to each other and solve for y.

Apply It

1. Students may also rewrite the first equation as $-x + y = 3$ and then multiply it by 3. Adding the resulting equations would eliminate the variable x. *DOK 1*

2. Students may also multiply the second equation by -2 and then add the equations to eliminate the variable x. *DOK 1*

3. **C is correct.** Multiplying the first equation by 2 and the second by 3 and then adding gives the false statement $0 = -12$.

   A is not correct. This system has one solution.

   B is not correct. This system has one solution.

   D is not correct. This system has infinitely many solutions.

   DOK 3

2 What is the solution of the system of equations? Show your work.

$10x + 16y = 6$

$5x - 8y = 5$

Possible work:

$$2(5x - 8y = 5) \rightarrow \begin{array}{r} 10x - 16y = 10 \\ + \; 10x + 16y = 6 \\ \hline 20x = 16 \\ x = \frac{4}{5} \end{array} \qquad \begin{array}{r} 5x - 8y = 5 \\ 5\left(\frac{4}{5}\right) - 8y = 5 \\ 4 - 8y = 5 \\ -8y = 1 \\ y = -\frac{1}{8} \end{array}$$

SOLUTION $\left(\frac{4}{5}, -\frac{1}{8}\right)$

CONSIDER THIS . . . How can you combine the two equations to make a one-variable equation?

PAIR/SHARE Does it matter which variable you solve for first?

3 Which of these systems of equations has no solution?

A $3x + 2y = -12$
$2x + 3y = -12$

B $2x + 3y = 12$
$2x - 3y = 12$

C $-3x + 3y = 12$
$2x - 2y = -12$

D $-3x - 2y = 12$
$3x + 2y = -12$

Mateo chose C as the correct answer. How might he have gotten that answer?

Possible answer: Mateo may have solved each equation for y to see that the lines have the same slope and different y-intercepts, so the lines are parallel. Parallel lines never cross, so there is no solution of the system.

CONSIDER THIS . . . How can the coefficients of the variables help you determine the number of solutions the system has?

PAIR/SHARE Do any of the systems have infinitely many solutions?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 1, 2, 8

Meeting Proficiency

- **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

Resources for Differentiation are found on the next page.

Refine Solving Systems of Linear Equations Algebraically

Apply It

4 **A, B, C, D, and F are correct.** These equations are the original equations in the system or are equivalent to those equations. Any of them can be used to find x.

E is not correct. This equation is the equation that resulted from eliminating x. It cannot be used to find x, because it does not include an x-term.

DOK 2

5 After solving the first equation in the system for x, the resulting expression for x must be substituted into the second equation, not the first equation. *DOK 3*

6 **a.** Students may write each equation in slope-intercept form in order to graph it. *DOK 2*

b. Students may also multiply the second equation by 2 and then add the equations to eliminate x. Or they may multiply the second equation by 4 and add the equations to eliminate y. *DOK 2*

---

4 The first part of a solution to a problem is shown. Which of these equations could you substitute the value of y into in order to find x?

$$\begin{array}{lcr} -2x + 5y = 10 & \longrightarrow & -6x + 15y = 30 \\ 3x - 2y = -4 & \longrightarrow & +\quad 6x - \;\;4y = -8 \\ & & \hline 11y = 22 \\ & & y = 2 \end{array}$$

A $-2x + 5y = 10$

B $3x - 2y = -4$

C $-6x + 15y = 30$

D $6x - 4y = -8$

E $11y = 22$

F $2y = 3x + 4$

5 Sebastián tried to solve the system of equations below. He says there are infinitely many solutions. Do you agree? Explain.

$$\begin{array}{lcl} x - 2y = 1 & \longrightarrow & x = 2y + 1 \\ 4x - 4y = 11 & & \end{array}$$

$$\begin{aligned} x - 2y &= 1 \\ (2y + 1) - 2y &= 1 \\ 2y + 1 - 2y &= 1 \\ 1 &= 1 \end{aligned}$$

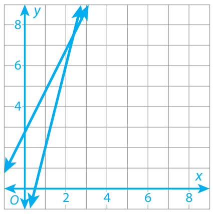
No; Possible explanation: After Sebastián solved the first equation for x, he substituted the expression back into the same equation. If he had substituted the expression into the other equation, he would have gotten a numerical value for y.

6 **a.** Use a graph to estimate the solution of the system.

$-8x + 4y = 11$

$4x - y = 2$

Possible answer: $x \approx 2\frac{1}{2}$, $y \approx 7\frac{1}{2}$



b. Find the exact solution of the system in problem 6a. Show your work.

Possible work:

$$\begin{aligned} 4x - y &= 2 \\ -y &= -4x + 2 \\ y &= 4x - 2 \end{aligned}$$

$$\begin{aligned} -8x + 4(4x - 2) &= 11 \\ -8x + 16x - 8 &= 11 \\ 8x &= 19 \\ x &= 2\tfrac{3}{8} \end{aligned}$$

$$\begin{aligned} y &= 4\left(2\tfrac{3}{8}\right) - 2 \\ y &= 9\tfrac{1}{2} - 2 \\ y &= 7\tfrac{1}{2} \end{aligned}$$

SOLUTION $\left(2\frac{3}{8}, 7\frac{1}{2}\right)$

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DIFFERENTIATION

RETEACH

Hands-On Activity

Use algebra tiles to solve a system of linear equations.

Students approaching proficiency with solving systems of linear equations algebraically will benefit from using a model.

Materials For each pair: algebra tiles (at least 10 each of x- and y-tiles and 20 1-tiles)

- Display this system: $3x - 2y = 3$ and $-2x + 2y = 2$.
- Have pairs collaborate to model both equations with algebra tiles, one below the other.
- Tell students that they will solve the system by elimination first. Ask: *What is a good first step to eliminate one of the variables? Why?* [Add the equations, because the y-terms are opposites and y will be eliminated.]
- Have pairs use the tiles to solve the system by eliminating y.
- Ask: *What is the equation for the sum?* [$x = 5$] Have them substitute 5 for x in either equation and solve for y. [$y = 6$]
- Tell students they will now solve the system by substitution. Have them rebuild the models for both equations with the tiles.
- Ask: *Which equation will you use to solve for one of the variables? Why?* [If you use the second equation, the expression will not include any fractions.]
- Have pairs use the tiles to solve the second equation for y.
- Ask: *What expression do you find for y?* [$x + 1$]
- Have students use their tiles to substitute $x + 1$ for y in the first equation and solve for x. [$x = 5$] Then have students substitute 5 for x in either equation and solve for y.

Refine

7 See ***Connect to Culture*** to support student engagement. The girls ride at the same speed (same slope) but Hasina starts first (different y-intercepts), so Jada will never catch up to Hasina. *DOK 2*

8 Students may think: *5y times what number is equal to the opposite of 15y? DOK 1*

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that when adding equations will eliminate a variable, use elimination. When one equation is already solved for a variable, use substitution.

Error Alert If students use substitution in part a, ask: *How are −8y and 8y related to each other?* Students should see that the terms are opposites, and so their sum is 0. This points toward elimination. If they use elimination in part b, ask: *What is the first step when using substitution?* Students should know that the first step is to solve for one variable in terms of the other. Since an equation is already solved for y, this points toward substitution.

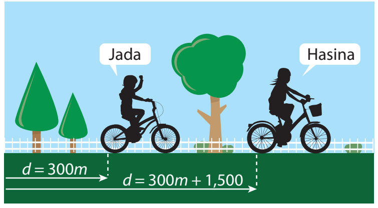
End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they work with a partner to review the Session 3 Model Its.

SELF CHECK Have students review and check off any new skills on the Unit 3 Opener.

LESSON 13 | SESSION 5

7 Hasina and her sister Jada ride their bikes to school. Hasina starts first. The equations show the distance d each girl is from home m minutes after Jada starts. What is the solution of the system of equations? What does this mean in this situation?



Hasina: $d = 300m + 1{,}500$

Jada: $d = 300m$

There is no solution of the system. Both girls ride at the same speed, but since Hasina left first, Jada never catches up to her along the route to school.

8 To eliminate the y-terms in the system of equations below, multiply the first equation by $\underline{\;-3\;}$ and add it to the second equation.

$9x + 5y = -3$

$11x + 15y = -1$

9 **Math Journal** For each system of equations, tell whether it would take fewer steps to solve by substitution or by elimination. Then use that strategy to solve the system.

a. $3x - 8y = 31$

$7x + 8y = 19$

$$\begin{array}{r} 3x - 8y = 31 \\ +\ 7x + 8y = 19 \\ \hline 10x = 50 \\ x = 5 \end{array} \qquad \begin{array}{r} 7(5) + 8y = 19 \\ 35 + 8y = 19 \\ 8y = -16 \\ y = -2 \end{array}$$

elimination; $(5, -2)$

b. $y = 2x$

$-5x + 3y = -4$

$$\begin{array}{r} -5x + 3(2x) = -4 \\ -5x + 6x = -4 \end{array} \qquad \begin{array}{l} y = 2(-4) \\ y = -8 \\ x = -4 \end{array}$$

substitution; $(-4, -8)$

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Write a new entry for *eliminate*. Tell what you do when you *eliminate* a variable to solve a system of equations.

☐ **SELF CHECK** Go back to the Unit 3 Opener and see what you can check off.

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REINFORCE

Problems 4–8

Solve systems of linear equations.

Students meeting proficiency will benefit from additional work with solving systems algebraically by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve problems by writing and solving a system of linear equations.

Students extending beyond proficiency will benefit from first writing and then solving a system.

- Have students work with a partner to solve this problem: *One line has a y-intercept of 4 and passes through (−3, −5). Another line has a y-intercept of −6 and passes through (2, −2). At what point do the lines intersect?*
- Students may find the slope of each line, write an equation for each line, and then solve the system using substitution or elimination. [(−10, −26)]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Lesson Quiz

TESTED SKILLS

Assesses 8.EE.C.8b

Problems on this assessment require students to be able to solve a system of linear equations by either substitution or elimination. They will need to determine the number of solutions to a system and write a system of linear equations to represent and solve a real-world situation. Students will also need to examine a solution and determine if an error is present. They will explain which solution method is more efficient given a system. Students will need to be familiar with simplifying algebraic expressions and performing integer operations.

Alternately, teachers may assign the **Digital Comprehension Check** online to assess student understanding of this material.

Error Alert Errors may result if students:

- multiply the factor outside the parentheses by only one of the addends when applying the distributive property.
- substitute an expression for the variable it does not represent when solving by substitution.
- incorrectly distribute a constant when solving by elimination, perhaps by not multiplying the constant by all terms or ignoring a negative sign.
- forget to solve for x or y after solving for one of the variables in the system.
- do not check the reasonableness of their solutions.

LESSON 13 • QUIZ | Name:

Solve the problems.

1 What is the solution of the system of equations? Write your answers in the blanks. *(2 points)*

$4x - 2y = 10$

$2x + y = -3 \longrightarrow y = -2x - 3$

$4x - 2(\underline{-2x - 3}) = 10$

$4x + 4x + 6 = 10$

$8x = 4$

$x = \underline{\tfrac{1}{2}}$

$y = -2(\underline{\tfrac{1}{2}}) - 3$

$y = \underline{-4}$

2 Michael bakes a soft pretzel and a loaf of bread. Use the system of equations to find c, the amount of flour, in cups, needed for the soft pretzel, and b, the amount of flour, in cups, needed for the loaf of bread. Show your work. *(2 points)*

$b = 24c$

$\frac{1}{4}b + 4c = \frac{5}{4}$

Possible student work:

$\frac{1}{4}(24c) + 4c = \frac{5}{4}$

$6c + 4c = \frac{5}{4}$

$c = \frac{1}{8}$

$b = 24\left(\frac{1}{8}\right)$

$b = 3$

SOLUTION $\frac{1}{8}$ c of flour for a soft pretzel and 3 c of flour for a loaf of bread

3 Which of these systems of equations has infinitely many solutions? *(1 point)*

A $\frac{1}{2}x - 5y = 10$; $-\frac{1}{2}x + 5y = -10$

B $7x + 2y = -14$; $-7x + 2y = -14$

C $x + 6y = -18$; $x - 6y = 18$

D $\frac{3}{4}x - 9y = 12$; $-\frac{3}{4}x + 9y = 12$

Problem Notes

1 (2 points)

DOK 1 | 8.EE.C.8b

Extended Response Scoring Rubric

Points	Expectations
4	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of concepts and/or procedures.
3	Response contains mostly correct solution(s) and demonstrates a strong understanding of concepts and/or procedures.
2	Response shows partial to limited understanding of concepts and/or procedures.
1	Response contains incorrect solution(s), shows poorly organized and incomplete work and explanations, and demonstrates limited understanding of concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of concepts and/or procedures.

Fill-in-the-Blank Scoring Rubric

Points	Expectations
2	All answers are correct
1	1 incorrect answer
0	2 or more incorrect answers

Short Response Scoring Rubric

Points	Expectations
2	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of concepts and/or procedures.
1	Response contains mostly correct solution(s) and shows partial understanding of concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of concepts and/or procedures.

2 Students could also solve the system by elimination.

(2 points)

DOK 2 | 8.EE.C.8b

3 **A is correct**. Students could solve the problem by using substitution or elimination and then determine that the result is a true statement: $0 = 0$.

B is not correct. This answer represents a system with one solution. The two equations have different slopes but the same y-intercept.

C is not correct. This answer represents a system with one solution. The two equations have different slopes but the same y-intercept.

D is not correct. This answer represents a system with no solution. The two equations have the same slope but different y-intercepts.

(1 point)

DOK 2 | 8.EE.C.8b

4 Students could also solve the problem by distributing -2 to each term in the second equation: $-4x - 4y = -48$. Add the equations and solve for $y = 6$. Substitute $y = 6$ into either equation to get $x = 6$.

(2 points)

DOK 2 | 8.EE.C.8b

5 Students could explain that elimination involves three steps: multiply the first equation by 3, eliminate x and solve for y, substitute y into an equation and solve for x.

(4 points)

DOK 3 | 8.EE.C.8b

LESSON 13 • QUIZ　　Name:

4 Kana tried to solve the system of equations below. She says there are no solutions. Is Kana correct? Explain your reasoning. *(2 points)*

$$\begin{aligned} 4x - 2y &= 12 \\ 2x + 2y &= 24 \end{aligned} \quad \longrightarrow \quad -2(2x + 2y = 24) \quad \longrightarrow \quad \begin{aligned} 4x - 2y &= 12 \\ + -4x + 2y &= 24 \\ \hline 0 &= 36 \end{aligned}$$

SOLUTION No; Possible explanation: Kana did not distribute -2 to all terms inside the parentheses. If she had eliminated the variable y using the original equations, she would have gotten $6x = 36$, $x = 6$. Substituting $x = 6$ into either equation results in $y = 6$, so the system of equations has one solution, (6, 6).

5 For the system of equations shown, tell whether it would take fewer steps to solve by substitution or elimination. Then use that strategy to solve the system. Explain your reasoning. *(4 points)*

$-3x - 4y = 15$

$9x + 7y = 25$

Possible student work:

$$3(-3x - 4y = 15) \longrightarrow \begin{aligned} -9x - 12y &= 45 \\ +9x + 7y &= 25 \\ \hline -5y &= 70 \\ y &= -14 \end{aligned} \qquad \begin{aligned} -9x - 12(-14) &= 45 \\ -9x + 168 &= 45 \\ -9x &= -123 \\ x &= 13\tfrac{2}{3} \end{aligned}$$

SOLUTION Elimination; The solution is $(13\frac{2}{3}, -14)$. Possible explanation: Substitution would take more steps because one equation needs to be solved for a variable, x or y. Then, substitute the result into the second equation, solve for the unknown variable, and then substitute the solution back into the equation and solve for the second variable.

DIFFERENTIATION

RETEACH Tools for Instruction

Students who require additional support for prerequisite or on-level skills will benefit from activities that provide targeted skills instruction.

GRADE 8
- **Lesson 13** TFI title TK

REINFORCE Math Center Activity

Students who require practice to reinforce concepts and skills and deepen understanding will benefit from small group collaborative games and activities (available in on-level, below-level, and above-level versions).

GRADE 8
- **Lesson 13** Center Activity title TK

EXTEND Enrichment Activity

Students who have achieved proficiency with concepts and skills and are ready for additional challenges will benefit from group collaborative games and activities that extend understanding.

GRADE 8
- **Lesson 13** Enrichment title TK

End of Unit 3

Overview | Linear Relationships and Systems of Equations

MATH FOCUS

Focus Standards

8.EE.B.5, 8.EE.C.7a, 8.EE.C.7b, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

This lesson provides support for:

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

Additional SMP

2, 4, 5, 7, 8

Lesson at a Glance

In this lesson, students apply multiple skills from the unit to solve real-world problems related to science. Problems involve solving systems of linear equations, writing linear equations in slope-intercept form, graphing linear equations, interpreting the slope, *y*-intercept, and intersection points of graphs, and solving one-variable equations.

Key Skills

- Write and solve systems of linear equations.
- Write linear equations in slope-intercept form.
- Graph linear equations.
- Interpret slopes, *y*-intercepts, and points of intersection shown on a graph within the context of problems.
- Solve one-variable equations with variables on both sides.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

Pacing Guide	MATERIALS
SESSION 1 (45–60 min)	
Coral Nursery	
Study an Example Problem and Solution (20–30 min) Example Problem (10–15 min) | One Student's Solution (10–15 min) **Try Another Approach** (25–30 min) Plan It (5 min) | Solve It (15–20 min) | Reflect (5 min)	**Math Toolkit** algebra tiles, graph paper, sticky notes For each student: Activity Sheet *Solution Sheet*
SESSION 2 (45–60 min)	
Analyzing Growth Data	
Discuss Models and Strategies (20–30 min) Plan It and Solve It (15–25 min) | Reflect (5 min)	**Math Toolkit** graph paper, straightedges For each student: Activity Sheet *Solution Sheet*
Designing an Experiment	
Persevere On Your Own (25–30 min) Solve It (20–25 min) | Reflect (5 min) **DIFFERENTIATION** **RETEACH** Extra Support — **Materials** For each student: straightedge **EXTEND** Challenge	**Math Toolkit** grid paper, sticky notes For each student: Activity Sheet *Solution Sheet*

Purpose
- **Examine** a problem that involves a system of linear equations.
- **Analyze** a sample solution to identify what makes it a good solution.
- **Demonstrate** that the problem has more than one approach and more than one solution.

Study an Example Problem and Solution

Coral Nursery

Make Sense of the Problem

Present the Coral Nursery problem and use **Three Reads** to help students make sense of it. For each read, have a different volunteer read aloud each section of information.

After the first read, be sure students recognize there are two sizes of blocks, two choices for each size, and that all blocks of the same size must hold the same number of samples.

ASK *What types of blocks will be used? What requirements does Maria need to think about when choosing blocks?*

LISTEN FOR Maria will use two types of blocks, small and large. There are two choices for each size, and each holds a different number of samples. Maria must choose one small block and one large block to use in the nursery.

After the second read, listen for students to articulate that they need to find the number of small blocks and the number of large blocks that Maria could use at the new location.

After the third read, have students identify the important quantities and relationships in the problem, including any quantities that involve student choice. Ensure that students understand that student choice means there can be multiple correct solutions to the problem.

Invite students to share their ideas about how they might use concepts from the unit to solve this problem. Allow them to describe different approaches, but do not implement any strategies.

ASK *How do you know that you need to use a system of equations to solve this problem?*

LISTEN FOR Maria wants to identify two different quantities—the number of small blocks and the number of large blocks. Problems with two unknowns are solved using systems of equations.

Math in Action | SESSION 1
Linear Relationships and Systems of Equations

Math IN Action

SMP 1 Make sense of problems and persevere in solving them.

Study an Example Problem and Solution

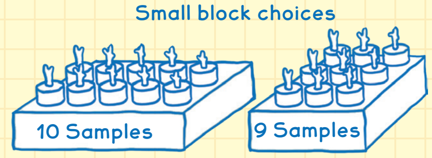
➤ **Read this problem involving systems of linear equations. Then look at one student's solution to this problem on the following pages.**

Coral Nursery

Maria is a marine biologist who grows coral in underwater nurseries to help restore coral reefs. The coral in some of the nurseries grows on reef blocks. Read her notes about a new underwater nursery location. Then write and solve a system of equations to determine the number of small reef blocks and the number of large reef blocks that Maria could use at this location.

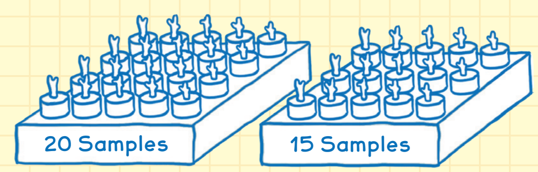
New Nursery Location:
- Coral will grow on 16 reef blocks at this location. Some blocks will be small and some will be large.
- All the small blocks used at this location must hold the same number of samples. There are two choices.

Small block choices

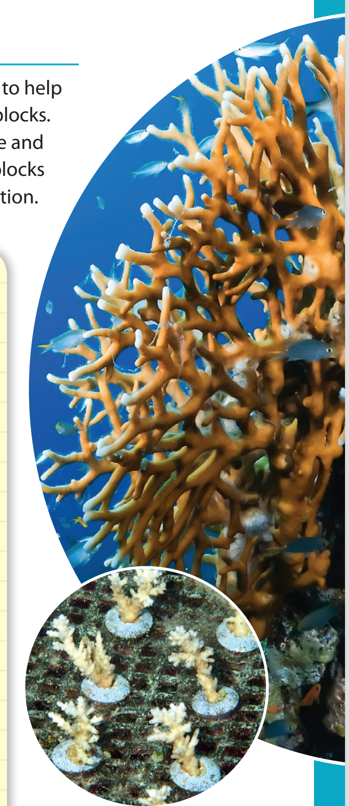


- All the large blocks used at this location must hold the same number of samples. There are two choices.

Large block choices



- This location needs to house exactly 210 samples.



There are many ways to grow coral in underwater nurseries. Some nurseries have coral samples attached to blocks or metal frames on the ocean floor. Once coral samples reach a certain size, they can be planted on coral reefs.

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CONNECT TO CONTEXT

Coral Nursery Students may be familiar with coral, a tiny soft-bodied animal that typically lives within a stony skeleton grouped in large colonies. Coral reefs form when free-swimming coral larvae attach to submerged rocks or other hard surfaces along the edges of islands or continents. The reefs are a critical habitat for fish and other organisms but are threatened from damage caused by shipping vessels, severe storms, earthquakes, disease, pollution, predators, overfishing, and more. To combat these threats, coral is grown in underwater nurseries. Coral clippings are attached to small concrete disks called pucks. The pucks are fastened to blocks on the ocean floor. These blocks lift the coral closer to the sunlight, which they need for survival. The coral is cared for until they can be moved to a natural reef. Invite students to share what they know about coral and coral reefs. Optionally, work with a biology teacher at your school to help students further explore coral.

Math in Action

One Student's Solution

Read through the sample solution to the Coral Nursery problem together, one section at a time. Point out that the Notice That boxes provide important information about this student's solution.

Problem-Solving Checklist

Explain to students that they will use the Problem-Solving Checklist on this page to identify what makes this a good solution.

- Have students look at the sample solution to find and circle where the student wrote known, or given, information.
- Then have students underline where the student wrote what they need to find.
- As a class, find and box where the student showed their work.
- Finally, ask students to put a checkmark next to the place where the student checked their solution.
- Tell students that they can use this checklist as a model when they write their own solutions for this problem and for other problems.

Facilitate Whole Class Discussion

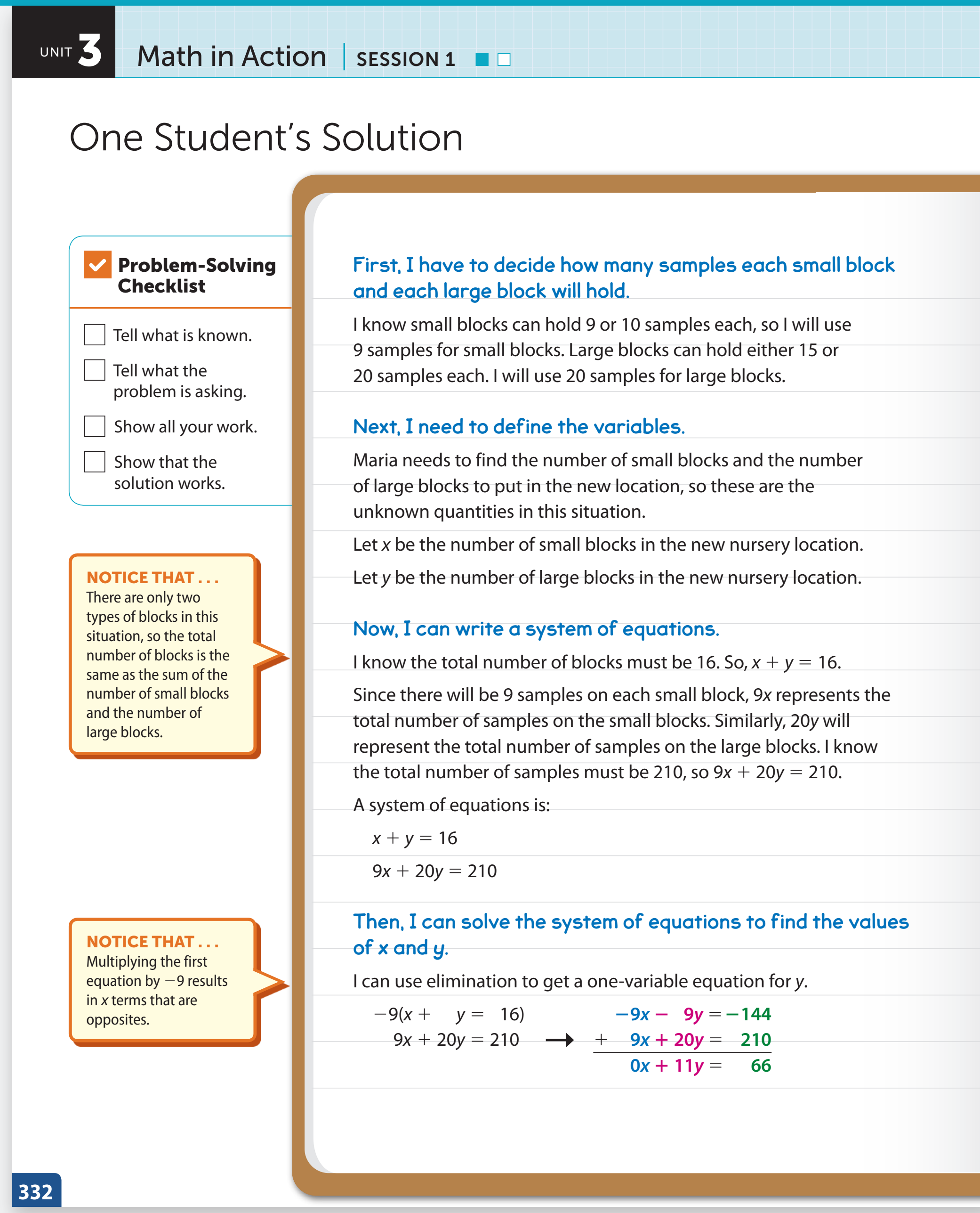
Support students in analyzing and discussing the steps of the sample solution in order to better understand it.

ASK *Why is it important to define the variables?*

LISTEN FOR Defining the variables is important because each unknown can be represented by a unique variable at each step of the solution process.

ASK *How do you think the student decided what each equation in the system should represent?*

LISTEN FOR Each equation in a system of equations must contain both variables. I think the student recognized that the total number of blocks and the total number of samples were given, so each equation should represent one of these totals.



UNIT 3 Math in Action | SESSION 1 ■ □

One Student's Solution

Problem-Solving Checklist

- □ Tell what is known.
- □ Tell what the problem is asking.
- □ Show all your work.
- □ Show that the solution works.

First, I have to decide how many samples each small block and each large block will hold.

I know small blocks can hold 9 or 10 samples each, so I will use 9 samples for small blocks. Large blocks can hold either 15 or 20 samples each. I will use 20 samples for large blocks.

Next, I need to define the variables.

Maria needs to find the number of small blocks and the number of large blocks to put in the new location, so these are the unknown quantities in this situation.

Let x be the number of small blocks in the new nursery location.

Let y be the number of large blocks in the new nursery location.

NOTICE THAT . . . There are only two types of blocks in this situation, so the total number of blocks is the same as the sum of the number of small blocks and the number of large blocks.

Now, I can write a system of equations.

I know the total number of blocks must be 16. So, $x + y = 16$.

Since there will be 9 samples on each small block, $9x$ represents the total number of samples on the small blocks. Similarly, $20y$ will represent the total number of samples on the large blocks. I know the total number of samples must be 210, so $9x + 20y = 210$.

A system of equations is:

$$x + y = 16$$
$$9x + 20y = 210$$

NOTICE THAT . . . Multiplying the first equation by −9 results in x terms that are opposites.

Then, I can solve the system of equations to find the values of x and y.

I can use elimination to get a one-variable equation for y.

$$\begin{array}{l} -9(x + y = 16) \\ 9x + 20y = 210 \end{array} \longrightarrow \begin{array}{rrcr} & -9x - 9y & = & -144 \\ + & 9x + 20y & = & 210 \\ \hline & 0x + 11y & = & 66 \end{array}$$

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Prompt students to discuss why the elimination method is an efficient strategy for solving the problem.

ASK *Why do you think the student chose to use the elimination method?*

LISTEN FOR Both equations are in the same form, so the x- and y-terms line up. This means the student will only need to use multiplication and then addition to eliminate one of the variables.

ASK *Why did the student multiply by −9? How else could the student have eliminated a variable?*

LISTEN FOR The student multiplied the first equation by −9 so that when the two equations are added, the x-terms are eliminated. The student could have also multiplied the first equation by −20 and then added the equations to eliminate the y-terms.

Have students discuss what the solution means mathematically and within the context of the problem.

ASK *What does a solution to a system of linear equations represent?*

LISTEN FOR A solution represents an (x, y) pair that makes all equations in the system true. On a coordinate plane, this is the point where the graphs of the equations intersect.

ASK *How can you interpret the solution within the context of the problem?*

LISTEN FOR I can go back to where I defined the variables and use these definitions to summarize what the solution means within the context of the problem.

After discussing the sample solution, have a volunteer summarize the steps taken to solve the problem.

ASK *How can you summarize the steps in the sample solution?*

LISTEN FOR Choose a small block and a large block so that you know how many samples each size block will hold. Define the variables and use the given information to write a system of equations. Choose an appropriate strategy such as substitution or elimination to solve the system of equations. Use the results to interpret the solution in terms of the problem context.

Math IN Action

Now, I can solve for y and then use substitution to solve for x.

$11y = 66$

$\frac{11y}{11} = \frac{66}{11}$

$y = 6$

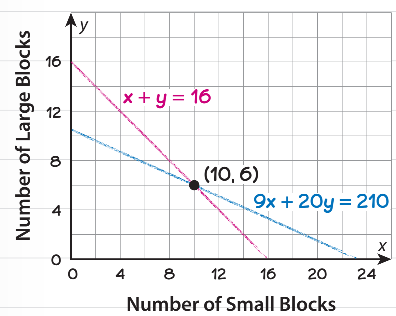
I will substitute $y = 6$ into the equation $x + y = 16$:

$x + 6 = 16$

$x + 6 - 6 = 16 - 6$

$x = 10$

Finally, I can check my work by graphing both equations in the same coordinate plane.



The point of intersection of the two lines confirms that the algebraic solution is correct.

So, Maria could use 10 small blocks with 9 samples each and 6 large blocks with 20 samples each to hold 210 total samples in the new nursery location.

NOTICE THAT . . . Substituting $y = 6$ into the other equation in the system, $9x + 20y = 210$, should result in the same value for x.

NOTICE THAT . . . Since small blocks are on the x-axis and large blocks are on the y-axis, the point of intersection (10, 6) indicates that both equations are true when there are 10 small blocks and 6 large blocks.

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Try Another Approach

Coral Nursery

Materials For each student: Activity Sheet *Solution Sheet*

Facilitate Whole Class Discussion

After reviewing the sample student solution on the previous pages, prompt students to look for different approaches to solve the problem.

ASK *What are some different steps you could use to solve the problem?*

LISTEN FOR Choose a different small block and/or large block. Use a table or substitution to solve the system of equations. Substitute the solution into both equations to check the answer.

Plan It

Facilitate Whole Class Discussion

Read the Plan It questions from the next page aloud. Prompt students to discuss how they will approach and solve this problem in another way.

ASK *How could you use graphing to estimate the solution to a system of equations? How could you use substitution to check the solution?*

LISTEN FOR I could graph each linear equation in the same coordinate plane. The coordinates of the point where the two graphs intersect is the solution. I could estimate the *x*- and *y*-coordinates of this point and substitute them into both equations to check that they result in true statements.

ASK *Will using the same blocks but a different solution method result in a different answer than the sample solution? Will choosing different blocks result in a different answer? Explain.*

LISTEN FOR If I only change the solution method, the answer will not change because the solution method does not change the system of linear equations I am solving. If I choose different blocks, the answer will change because the coefficients of the *x*- and *y*-terms will be different for one equation. I will be solving a different system of linear equations.

Try Another Approach

➤ **There are many ways to solve problems. Think about how you might solve the Coral Nursery problem in a different way.**

Problem-Solving Checklist

- ☐ Tell what is known.
- ☐ Tell what the problem is asking.
- ☐ Show all your work.
- ☐ Show that the solution works.

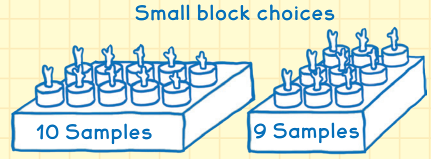
Coral Nursery

Maria is a marine biologist who grows coral in underwater nurseries to help restore coral reefs. The coral in some of the nurseries grows on reef blocks. Read her notes about a new underwater nursery location. Then write and solve a system of equations to determine the number of small reef blocks and the number of large reef blocks that Maria could use at this location.

New Nursery Location:

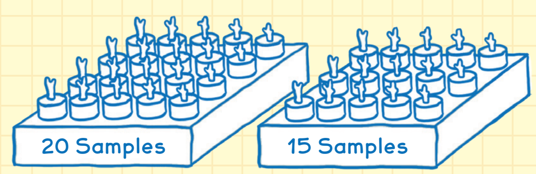
- Coral will grow on 16 reef blocks at this location. Some blocks will be small and some will be large.
- All the small blocks used at this location must hold the same number of samples. There are two choices.

Small block choices



- All the large blocks used at this location must hold the same number of samples. There are two choices.

Large block choices



- This location needs to house exactly 210 samples.

Solve It

Problem-Solving Tips

Introduce the Problem-Solving Tips as ideas students may use to explain their thinking. Remind them to also use the Problem-Solving Checklist to make sure their answer is complete.

Have students write their own complete solutions on a copy of Activity Sheet *Solution Sheet* or a blank sheet of paper.

Reflect

As they work, have students share their thinking with a partner and discuss the Reflect questions.

As time permits, have students explain their solutions to the class.

Possible Solution

I will use 10 samples for small blocks and 15 samples for large blocks. Let x be the number of small blocks and let y be the number of large blocks.

I know the total number of blocks must be 16, so $x + y = 16$. Since there will be 10 samples on each small block, $10x$ represents the total number of samples on the small blocks. Since there will be 15 samples on each large block, $15y$ represents the total number of samples on the large blocks.
I know the total number of samples must be 210, so $10x + 15y = 210$.

A system of equations is:

$$\begin{aligned} x + y &= 16 \\ 10x + 15y &= 210 \end{aligned}$$

I can solve the first equation for either variable and then substitute into the second equation to solve.

$$y = -x + 16$$

$$\begin{aligned} 10x + 15(-x + 16) &= 210 \\ 10x - 15x + 240 &= 210 \\ -5x + 240 &= 210 \\ -5x &= -30 \\ x &= 6 \end{aligned}$$

To solve for y, I will substitute $x = 6$ into the equation $x + y = 16$.

$$\begin{aligned} 6 + y &= 16 \\ y &= 10 \end{aligned}$$

Maria could use 6 small blocks with 10 samples each and 10 large blocks with 15 samples each to hold 210 total samples in the new nursery location.

Math IN Action

Plan It

➤ **Answer these questions to help you start thinking about a plan.**

a. What is another approach you could use to solve or check a system of equations?

b. Do you expect to get the same answer or a different answer than the sample solution?

Solve It

➤ **Find a different solution for the Coral Nursery problem. Show all your work on a separate sheet of paper. You may want to use the Problem-Solving Tips to get started.**

PROBLEM-SOLVING TIPS

Math Toolkit algebra tiles, graph paper, sticky notes

Key Terms

system of equations	linear equation	elimination
substitution	variable	coefficient
constant	slope-intercept form	

Models You may want to use . . .

- inverse operations to solve an equation in one variable.
- slope-intercept form to graph an equation.
- a table, graph, or algebraic method to solve or check a system of equations.

Reflect

Use Mathematical Practices As you work through the problem, discuss these questions with a partner.

- **Make Sense of Problems** How will you reflect your choice for the number of samples each block holds in the system of equations?
- **Use Structure** How could you change one of your equations so that the system would have no solution? Infinitely many solutions?

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Scoring Rubric (4 points)

Points	Expectations
4	The solution and explanation are clear and complete, with no errors. All work is shown. The solution is correctly interpreted within the context of the problem.
3	The solution is complete, with one or two errors. Solution steps are correct, but not all work is shown or is disorganized. The solution may not be correct due to calculation errors but is correctly interpreted as the number of small and large blocks Maria could use.
2	The solution is partially complete. Solution steps may not be appropriate for the problem. There are several calculation errors. The solution is not correct but is correctly interpreted as the number of small and large blocks Maria could use.
1	The solution is not complete. The calculations may be incorrect, incomplete, or inaccurate. The number of small and large blocks is missing or incorrect.

Math in Action

Purpose

- **Understand** two new open-ended, multi-step problems involving linear relationships.
- **Choose** appropriate models and strategies to plan for and solve the problems.

Discuss Models and Strategies

Analyzing Growth Data

Materials For each student: Activity Sheet *Solution Sheet*

Make Sense of the Problem

Present the Analyzing Growth Data problem and use **Three Reads** to help students make sense of it. Have different volunteers take turns reading aloud each section of information.

ASK *What are some clarifying questions you could ask about the details of the problem?*

LISTEN FOR Student responses may include questions about which specimens to analyze, how to graph the information for a specimen, or how to determine growth rates.

Invite volunteers to point out what is known and what they need to figure out.

ASK *What do you need to include in your analysis and solution?*

LISTEN FOR The analysis needs to include a graph showing the growth of two specimens. The solution needs to include statements interpreting the slopes, y-intercepts, and any points of intersection, and a statement that compares the growth rates of the two specimens.

Ensure that students understand the given information and the tasks on Maria's to-do list.

ASK *Compare the information given for each specimen. How will this affect your solution strategy?*

LISTEN FOR For two specimens, the branch length is given after two different time periods. For each of these, I can use the (x, y) pairs to calculate the slope, substitute into slope-intercept form to find the y-intercept, and then graph the line. For the other two specimens, the branch length after a certain time period is given along with a rate of change. For these specimens, I can write an equation of the line in slope-intercept form, substitute to find the y-intercept, and then graph the line.

Discuss Models and Strategies

Problem-Solving Checklist

- ☐ Tell what is known.
- ☐ Tell what the problem is asking.
- ☐ Show all your work.
- ☐ Show that the solution works.

➤ **Read the problem. Write a solution on a separate sheet of paper. Remember, there can be lots of ways to solve a problem.**

Analyzing Growth Data

Maria recorded data about the growth of several coral samples. Read Maria's notes about the branch length of the coral samples and her to-do list. Then help complete Maria's analysis.

Aa Notes

Notes ▾
Calendar
Search
Meetings
Data

Coral Growth Data

Specimen A:

Time (weeks)	Branch Length (cm)
10	12
50	28

Specimen B:
Length after 20 weeks was 17 cm.
Length after 50 weeks was 35 cm.

Specimen C:
Grew at a constant rate of 7 cm every 20 weeks.
It was 20.5 cm long after 30 weeks.

Specimen D:
Grew 0.4 cm each week to reach a length of 21 cm after 40 weeks.

To Do: ▾

- Make a graph showing growth over time for two specimens.
- Interpret the slopes, y-intercepts, and any points of intersection shown on the graph.
- Write a statement that compares the growth rates of the two specimens.

Discuss possible steps, ways to organize the information, and techniques that might be helpful. Invite students to take notes.

ASK *Will you write linear equations to help solve the problem? Why?*

LISTEN FOR Some students may say that writing a linear equation for each specimen would be a helpful way to find and interpret the slopes and y-intercepts. Other students may prefer to work with a different representation of the data.

CONNECT TO CONTEXT

Analyzing Growth Data Staghorn coral is classified as a *threatened* species. It is one of the most common corals grown in nurseries. It is shaped like the antlers of a deer and grows about 4 to 8 inches per year. Have students share other animals or plants they know are threatened or endangered and what is being done to help restore the species.

Assumptions Alert Tell students that for Specimens A and B, the growth rates are assumed to be constant because only two lengths are given. Point out that it is sometimes necessary to make assumptions as part of modeling with mathematics.

Appropriate Precision Have students identify what each axis of the graph represents. Discuss the importance of choosing an appropriate scale for each axis. Ask: *How could using a scale that is too big or too small make the graph difficult to interpret?*

Plan It and Solve It

Problem-Solving Checklist & Tips

Have students use various resources, including the Problem-Solving Checklist and the Problem-Solving Tips, as they begin to plan a solution.

Support Student Work

Have students work in pairs to discuss their preliminary solution strategies and the Reflect questions. Discuss a variety of approaches as a class. Let students revise their plans and discuss again with a partner.

When students are confident that their plans make sense, have them write a complete solution on a copy of Activity Sheet *Solution Sheet* or a blank sheet of paper.

Reflect

As they work, have students share their thinking with a partner and discuss the Reflect questions.

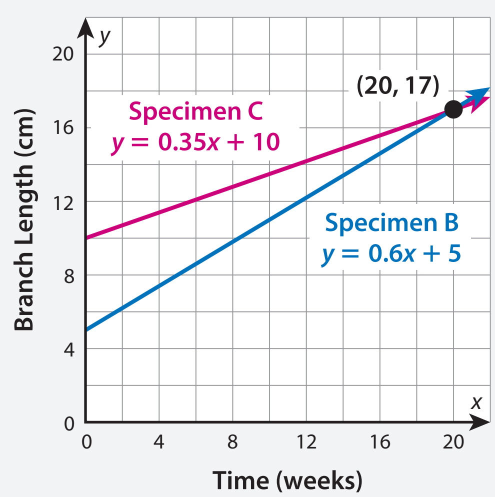
As time permits, have students explain their solutions to the class.

Possible Solution

I chose Specimens B and C for my solution. For Specimen B, I used the two given pairs of values to find m, and then I calculated b. I got $y = 0.6x + 5$.

For Specimen C, I used the given rate for m and then I calculated b. I got $y = 0.35x + 10$.

Then I graphed the lines.



Specimen B had a starting length of 5 cm and grew at a rate of 0.6 cm each week. Specimen C had a starting length of 10 cm and grew at a rate of 0.35 cm each week. The point of intersection (20, 17) means that after 20 weeks, both specimens were 17 cm long. Specimen C grows at approximately half the rate of Specimen B.

Math IN Action

Plan It and Solve It

Find a solution to the Analyzing Growth Data problem.

Write a detailed plan and support your answer. Be sure to include:

- a graph showing growth over time for two specimens. Show the data for both specimens in the same coordinate plane. Label each line with its equation.
- your interpretation of the slopes, *y*-intercepts, and any points of intersection shown on the graph.
- a statement that compares the growth rates of the two specimens.

PROBLEM-SOLVING TIPS

Math Toolkit graph paper, straightedges

Key Terms

slope	rate of change	*y*-intercept
coefficient	constant	coordinate

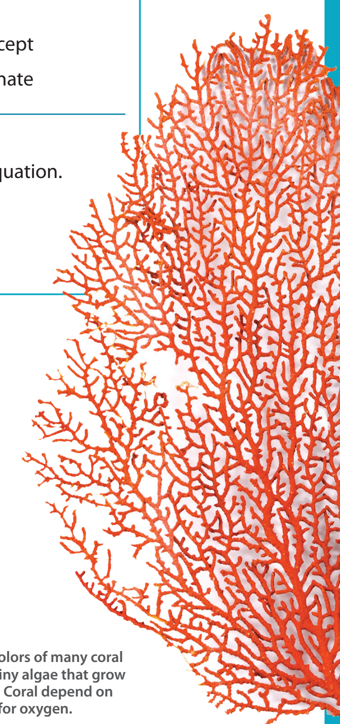
Models You may want to use . . .

- the coordinates of two points to make a graph and write the equation.
- the slope formula to calculate the slope of a line.
- the slope-intercept form of an equation to identify the *y*-intercept and make a graph.

Reflect

Use Mathematical Practices As you work through the problem, discuss these questions with a partner.

- **Be Precise** How will you determine what scale to use to label each of the axes on your graph?
- **Critique Reasoning** Do you agree with your partner's interpretation of the slopes, *y*-intercepts, and points of intersection shown in his or her graph? Explain.



The bright colors of many coral come from tiny algae that grow inside them. Coral depend on these algae for oxygen.

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Scoring Rubric (4 points)

Points	Expectations
4	The solution and explanation are clear and complete, with no errors. All work is shown. The solution correctly interprets the slopes, *y*-intercepts, and point of intersection for two specimens and accurately compares their growth rates.
3	All parts of the task are complete, with one or two errors. Solution steps are correct, but calculation errors result in incorrect equations and graphs. The solution correctly interprets the slopes, *y*-intercepts, and point of intersection for the equations that are graphed and accurately compares these growth rates.
2	All parts of the task are attempted. The equations for the specimens may be calculated or graphed incorrectly. The interpretation of the slopes, *y*-intercepts, and point of intersection, and the comparison of the growth rates, are partially accurate.
1	The solution is not complete. The calculations may be incorrect, incomplete, or inaccurate. The interpretation of the slopes, *y*-intercepts, and point of intersection is incomplete or inaccurate. The growth rates are not compared, or an inaccurate comparison is made.

Persevere On Your Own

Designing an Experiment

Materials For each student: Activity Sheet *Solution Sheet*

Make Sense of the Problem

Present the Designing an Experiment problem and use **Three Reads** to help students make sense of it. Have different volunteers take turns reading aloud each section of information. Make sure students recognize that this problem requires them to pick two structures and then find the length of the structures that will result in the same area of wire mesh. The wire mesh covers all faces of each structure except the bottom.

CONNECT TO CONTEXT

Designing an Experiment Explain that Maria wants to complete an experiment to answer the question: *Does the shape of a wire mesh structure impact coral growth?* Maria will make a prediction or hypothesis about what she thinks will happen and then use the data from the experiment to determine if she was correct. Invite students to tell about any science experiments they have done. What question were they trying to answer? Was their hypothesis or prediction correct?

UNIT 3 Math in Action | SESSION 2 ■ ■

Persevere On Your Own

➤ **Read the problem. Write a solution on a separate sheet of paper.**

Designing an Experiment

Read an email from Maria's coworker about designing an experiment to test different ideas for growing coral at an underwater nursery. Then help her respond to the email.

Problem-Solving Checklist

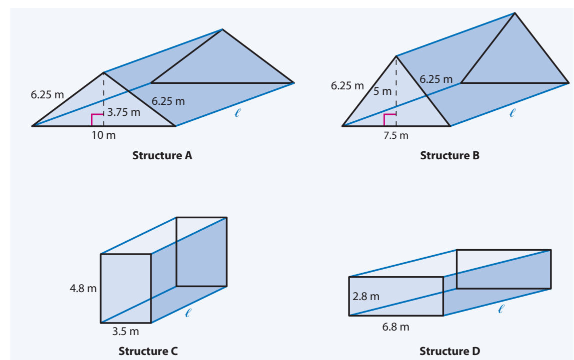
- ☐ Tell what is known.
- ☐ Tell what the problem is asking.
- ☐ Show all your work.
- ☐ Show that the solution works.

Delete Archive Reply Reply All Forward

To: Maria
Subject: Testing Wire Mesh Structures for Growing Coral

Hi Maria,

I want to do an experiment to compare how coral grows on two of the wire mesh structures shown. Both structures need to have the same area of wire mesh and the same length. The faces of each structure are wire mesh, except for the bottom of each structure, which will be open.



PLEASE PROVIDE:

- your suggestion of which two structures to use in the experiment.
- the length for which the two structures chosen will have the same area of mesh.

Thanks,

Vin

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DIFFERENTIATION | Use with Designing an Experiment

RETEACH

Extra Support

Find the total area of wire mesh for a prism.

- Have students look at Structure B.
- Ask: *Which face of the structure should not be included in the area expression?* [the bottom rectangle with side lengths 7.5 m and ℓ]
- Ask: *Which faces of the structure should be included in the area expression?* [the two triangles with base length 7.5 m and height 5 m, and the two rectangles with side lengths 6.25 m and ℓ]
- Ask: *Which area formulas will be needed to find the total area of these faces?* [the formula for area of a triangle, $\frac{1}{2}$ times base times height; the formula for area of a rectangle, length times width]
- Ask: *What expression gives the total area of these faces?*

$\left[2\left(\frac{1}{2}\right)(7.5)(5) + 2(6.25)\ell\right]$

EXTEND

Challenge

Solve a different problem.

Vin emails Maria to tell her they changed one of the dimensions of Structure D from 6.8 m to 3 m. He asks her to find the length ℓ for which Structures B and D will have the same area of wire mesh. Find this length and explain what the solution means within the context of the problem.

Possible Solution

Structure B: $2\left(\frac{1}{2}\right)(7.5)(5) + 2(6.25)\ell = 37.5 + 12.5\ell$

Structure D: $2(3)(2.8) + 2(2.8)\ell + 3\ell = 16.8 + 8.6\ell$

Solve for ℓ: $37.5 + 12.5\ell = 16.8 + 8.6\ell$; $\ell \approx -5.3$

It is not possible for length to be negative. There is no length for which the two structures have the same area of wire mesh.

Solve It

Remind students that there are many different ways to solve a problem.

Invite them to look back at the Problem-Solving Checklist to help them get started. They might also want to look at the Problem-Solving Tips on other pages to get some ideas for how to start.

Have students write their complete solution on a copy of Activity Sheet *Solution Sheet* or a blank sheet of paper.

Reflect

Have students work with a partner to share their thinking and discuss the Reflect questions.

As time permits, have students explain their solutions to the class.

Possible Solution

I suggest using Structures B and D. First, I need to write an expression that represents the total area of all sides of each structure, excluding the bottom.

Structure B: The total area of this structure is the sum of the areas of both triangular faces and two of the rectangular faces (the other rectangle is the bottom). The area of a triangle is half the base times the height. The area of a rectangle is the base times the height.

(area of triangles) + (area of rectangles) = total area

$2\left(\frac{1}{2}\right)(7.5)(5) + 2(6.25)\ell = 37.5 + 12.5\ell$

Structure D: The total area of this structure is the sum of the areas of the front and back faces, the left and right faces, and the top face (the bottom face is not included). Each face is a rectangle.

(area of front/back) + (area of left/right) + (area of top) = total area

$2(6.8)(2.8) + 2(2.8)\ell + 6.8\ell = 38.08 + 12.4\ell$

For the two structures to have the same area of wire mesh, the two expressions must be equal.

$37.5 + 12.5\ell = 38.08 + 12.4\ell$

$0.1\ell = 0.58$

$\ell = 5.8$

Structures B and D will have the same area of wire mesh when the length is 5.8 m.

Math IN Action

Solve It

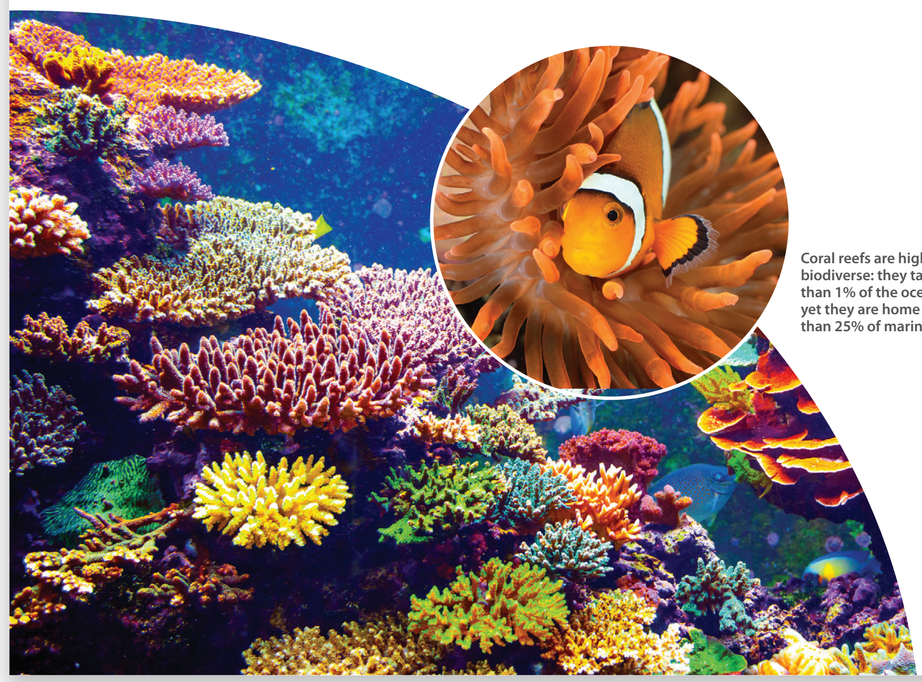
➤ **Find a solution to the Designing an Experiment problem.**

- Choose two structures to use for the experiment.
- Write and solve an equation to determine the length for which the two structures have the same area of wire mesh.

Reflect

Use Mathematical Practices After you complete the problem, choose one of these questions to discuss with a partner.

- **Persevere** What was your first step? What did you do next?
- **Reason Mathematically** How do you know when to combine like terms, use the distributive property, or use inverse operations when solving a linear equation in one variable?



Coral reefs are highly biodiverse: they take up less than 1% of the ocean floor, yet they are home to more than 25% of marine life.

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Scoring Rubric (4 points)	
Points	**Expectations**
4	The solution and explanation are clear and complete, with no errors. All work is shown. The length of the structures is correct and is correctly interpreted within the context of the problem.
3	The solution is complete, with one or two errors. The expressions for total area and the resulting equation are correct, but there may be small calculation errors. The length of the structures may not be correct but is correctly interpreted within the context of the problem.
2	The solution is partially complete. The expressions for total area and the resulting equation are partially correct and there may be calculation errors. The length of the structures may not be given or is unreasonable.
1	The solution is not complete. The expressions for total area and the resulting equation are incorrect or incomplete. There are several calculation errors. The length of the structures is not given.

Self Reflection

Have students review the skills on the **Self Reflection** page and work in pairs to respond to the prompts. Encourage students to revisit the work they did in each lesson in order to help develop growth mindset.

- Remind students that this is the same list of skills that they saw on the **Self Check** page at the beginning of the unit.
- Tell students that revisiting the list of skills is an opportunity for them to reflect on their learning and progress during the unit.
- Have students read through the list of skills independently and then work in pairs to respond to the prompts. Encourage students to revisit the work they did in each lesson as they think about how to respond to the prompts.
- Discuss students' responses to the prompts as a class if time permits. Tell students that they will build on these skills in later lessons during the year and/or in other grade levels.

UNIT 3 Self Reflection

In this unit you learned to . . .

Skill	Lesson(s)
Define *slope* and show that the slope of a line is the same between any two points on the line.	8
Find the slope of a line and graph linear equations given in any form.	8, 9
Derive the linear equations $y = mx$ and $y = mx + b$.	9
Represent and solve one-variable linear equations with the variable on both sides of the equation.	10
Determine whether one-variable linear equations have one solution, infinitely many solutions, or no solutions, and give examples.	11
Solve systems of linear equations graphically and algebraically.	12, 13
Represent and solve systems of linear equations to solve real-world and mathematical problems.	14
Justify solutions to problems about linear equations by telling what you noticed and what you decided to do as a result.	8–14

Think about what you have learned.

➤ **Use words, numbers, and drawings.**

1 The math I could use in my everyday life is ____________ because . . .

2 A mistake I made that helped me learn was . . .

3 I still need to work on . . .

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Vocabulary Self-Assessment

- Students have interacted with and used unit math and academic vocabulary throughout the unit in listening, speaking, reading, and writing. Use this activity to review the terms and help students check their understanding of the definitions.
- Have students read the vocabulary terms and put a check mark by the terms they can use in speaking and writing. Students can look up words they do not know in the Interactive Glossary or the Academic Glossary in the Teacher Toolbox.

Problem Notes

1. Check student understanding that *eliminate* or *elimination* are both appropriate responses. Support student understanding by providing the example that *substitute* is the verb form of *substitution*.
2. Students may also identify m as the rate of change or the entire equation as a linear equation.
3. Students may also identify $\frac{1}{2}$ as the slope or rate of change and 4 as the y-intercept in both equations. Encourage students to use complete sentences in their responses.

If time allows, this is a good opportunity to have students work in pairs to read and provide feedback on responses. Feedback should focus on the word meaning, accuracy, and clarity of the response.

UNIT 3 Vocabulary Review

➤ **Review the unit vocabulary. Put a check mark by terms you can use in speaking and writing. Look up the meaning of any terms you do not know.**

Math Vocabulary		Academic Vocabulary
☐ linear equation	☐ slope-intercept form	☐ eliminate
☐ rate of change	☐ systems of linear equations	☐ infinitely many
☐ slope	☐ *y*-intercept	☐ substitution

➤ **Use the unit vocabulary to complete the problems.**

1 Which math or academic vocabulary terms could you use to describe ways to solve systems of linear equations?

Possible answers: eliminate (elimination), substitution

2 Use at least three math vocabulary terms to label parts of the equation.

Possible answers:

$$\underbrace{y = \overset{\text{slope}}{m}x + \overset{y\text{-intercept}}{b}}_{\text{slope-intercept form}}$$

3 Use two math vocabulary terms and two academic vocabulary terms to describe the system of equations. Underline each term you use.

$y = \frac{1}{2}x + 4$

$2y - x = 8$

Possible answers: Both equations are linear equations. If you add x to both sides of the second equation and divide by 2, it will be in slope-intercept form. The system has infinitely many solutions. You can start to solve this system with substitution by replacing y with $\frac{1}{2}x + 4$ in the second equation.

4 Choose a math or an academic vocabulary term from the box that you have not used as an answer in problems 1–4. If you have used all the terms, choose one you only used once. Explain what the term means using your own words.

Answers will vary. Check for understanding of the term.

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Problem Notes

1 **C is correct.** Students could solve the problem by looking at the axis labels and determining that the slope, $\frac{\text{rise}}{\text{run}}$, corresponds to the unit rate $\frac{\text{scoop of bananas}}{\text{scoop of strawberries}}$.

A is not correct. This answer is not true because the graph of a proportional relationship is a line.

B is not correct. This answer is not true because it represents the reciprocal of the unit rate.

D is not correct. This answer is not true because $\frac{\text{rise}}{\text{run}}$ is $m = \frac{1-0}{5-0} = \frac{1}{5}$.

DOK 1 | **8.EE.B.5**

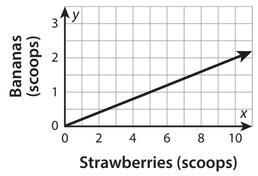
2 Students could solve the equation to show $x = \frac{1}{2}$.

DOK 2 | **8.EE.C.7a**

UNIT **3** Unit Review

➤ **Use what you have learned to complete these problems.**

1 Trevor makes this graph of the amounts of bananas and strawberries to mix to make a smoothie. Which statement is true about the line?



A The line does not show a proportional relationship.

B The unit rate for the proportional relationship shown in the graph is 5.

C The slope tells how many scoops of banana for each scoop of strawberry.

D The $\frac{\text{rise}}{\text{run}}$ is greater than 1.

2 How many solutions does $\frac{3}{4}(8x - 4) = 3 - 6x$ have? Show your work.

Possible student work:

$$\frac{3}{4}(8x - 4) = 3 - 6x$$

$$\frac{24}{4}x - \frac{12}{4} = 3 - 6x$$

$$6x - 3 = 3 - 6x$$

SOLUTION **One solution; Possible answer: The coefficients for the variable are different on each side of the equation.**

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SMP 1, 2, 5, 6, 7

Unit Game

8.EE.C.8a, 8.EE.C.8b

It's Systematic

Materials For each player: Recording Sheet, graph paper (optional); For each pair: Equation Cards, 2 number cubes (1–6)

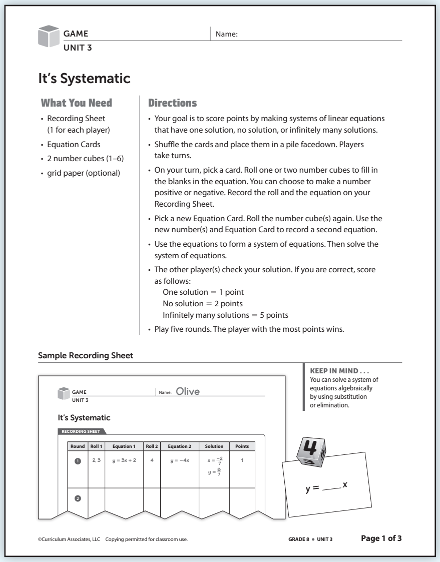
WHY Reinforce solving a system of equations and identifying how many solutions the system has.

HOW Players roll number cubes to complete linear equations and build a system of equations. They solve the system to determine if there is one solution, no solution, or infinitely many solutions.

- Model one round for students before they play. Show how to choose how many number cubes to roll based on the number of blanks on the Equation Card. Point out that they should complete Equation 1 before selecting, rolling, and completing Equation 2.
- After students finish the game, ask them to share strategies they used.

Vary the Game On any turn, allow players the option of rolling one or both number cubes a second time. .

ELL Support Have students graph the equations to help them see the solution(s) and understand what it means to have one solution, no solution, or infinitely many solutions.


GAME
UNIT 3
Name:

It's Systematic

What You Need
- Recording Sheet (1 for each player)
- Equation Cards
- 2 number cubes (1–6)
- grid paper (optional)

Directions
- Your goal is to score points by making systems of linear equations that have one solution, no solution, or infinitely many solutions.
- Shuffle the cards and place them in a pile facedown. Players take turns.
- On your turn, pick a card. Roll one or two number cubes to fill in the blanks in the equation. You can choose to make a number positive or negative. Record the roll and the equation on your Recording Sheet.
- Pick a new Equation Card. Roll the number cube(s) again. Use the new number(s) and Equation Card to record a second equation.
- Use the equations to form a system of equations. Then solve the system of equations.
- The other player(s) check your solution. If you are correct, score as follows:
  One solution = 1 point
  No solution = 2 points
  Infinitely many solutions = 5 points
- Play five rounds. The player with the most points wins.

Sample Recording Sheet

GAME
UNIT 3
Name: Olive

It's Systematic

RECORDING SHEET

Round	Roll 1	Equation 1	Roll 2	Equation 2	Solution	Points
1	2, 3	$y = 3x + 2$	4	$y = -4x$	$x = \frac{-2}{7}$, $y = \frac{8}{7}$	1
2						

KEEP IN MIND . . . You can solve a system of equations algebraically by using substitution or elimination.

$y = \_\_\_x$

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GRADE 8 • UNIT 3 Page 1 of 3

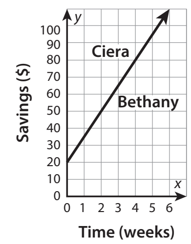
3 Another possible situation with a different number of solutions occurs when Ciera and Bethany deposit different amounts to start and then deposit different amounts each week. The number of solutions would be 1.

***DOK 3* | 8.EE.C.8a**

4 Students could also solve the equation $r + b = 16$ for either r or b and substitute that expression into the equation $6r + 18b = 216$. Then they can solve the system for r and b.

***DOK 2* | 8.EE.C.8c**

3 Ciera and Bethany save money using savings accounts opened on the same day. Both deposit $20 to start and an additional $15 each week. The graph represents the total amount, y, in each savings account after x weeks. Describe the number of solutions modeled in the graph and what it means for the situation. How can you change the situation so that the graphs model a different number of solutions? Explain your reasoning.



SOLUTION Possible description: Since the lines for Bethany and Ciera are the same, the number of solutions is infinitely many. This means that at any given time, both accounts have the same amount in them. Possible example: Ciera and Bethany deposit different amounts to start and then deposit the same amount each week. The lines would be parallel, and the number of solutions would be 0.

4 Jackie runs and rides her bike. She runs 6 miles in an hour. She bikes 18 miles in an hour. Last week she ran and biked a total of 216 miles. It took her 16 hours. How many hours did Jackie run and bike last week? Show your work.

Possible student work:

r = the number of hours Jackie ran last week

b = the number of hours Jackie biked last week

The system that represents the situation is $\begin{aligned} 6r + 18b &= 216 \\ r + b &= 16 \end{aligned}$.

$$6r + 18b = 216 \longrightarrow 6r + 18b = 216 \qquad r + b = 16$$

$$-6(r + b = 16) \longrightarrow \underline{-6r - 6b = -96} \qquad r + 10 = 16$$

$$12b = 120$$

$$b = 10 \qquad r = 6$$

SOLUTION Jackie spent 6 hours running and 10 hours biking.

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Literacy Connection

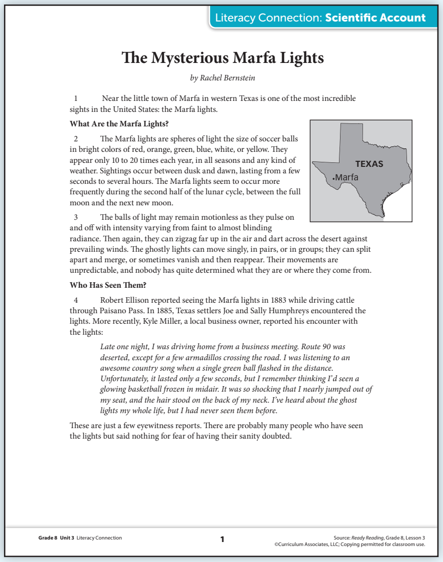
Scientific Account

Materials "The Mysterious Marfa Lights," Literacy Connection Problems

Summary In "The Mysterious Marfa Lights," students learn about a strange phenomenon: basketball-size colored lights that appear regularly in Marfa, Texas night sky.

Math Connection The purpose of a scientific account is to describe research on a particular topic. This account analyzes the movements of the Marfa lights. After reading this passage, students will use linear equations to solve art and film application problems related to Marfa, Texas.

- Have students read the passage.
- istribute the literacy connection problems. After reading the directions aloud, direct students to turn and talk about problem 1. Check for understanding.
- Have students work independently to complete the remaining problems. Encourage them to use manipulatives or to draw pictures to solve each problem.
- Circulate and monitor while students work.
- Ask volunteers to share and discuss their solutions with the class.



Literacy Connection: **Scientific Account**

The Mysterious Marfa Lights

by Rachel Bernstein

1 Near the little town of Marfa in western Texas is one of the most incredible sights in the United States: the Marfa lights.

What Are the Marfa Lights?

2 The Marfa lights are spheres of light the size of soccer balls in bright colors of red, orange, green, blue, white, or yellow. They appear only 10 to 20 times each year, in all seasons and any kind of weather. Sightings occur between dusk and dawn, lasting from a few seconds to several hours. The Marfa lights seem to occur more frequently during the second half of the lunar cycle, between the full moon and the next new moon.

TEXAS
•Marfa

3 The balls of light may remain motionless as they pulse on and off with intensity varying from faint to almost blinding radiance. Then again, they can zigzag far up in the air and dart across the desert against prevailing winds. The ghostly lights can move singly, in pairs, or in groups; they can split apart and merge, or sometimes vanish and then reappear. Their movements are unpredictable, and nobody has quite determined what they are or where they come from.

Who Has Seen Them?

4 Robert Ellison reported seeing the Marfa lights in 1883 while driving cattle through Paisano Pass. In 1885, Texas settlers Joe and Sally Humphreys encountered the lights. More recently, Kyle Miller, a local business owner, reported his encounter with the lights:

> *Late one night, I was driving home from a business meeting. Route 90 was deserted, except for a few armadillos crossing the road. I was listening to an awesome country song when a single green ball flashed in the distance. Unfortunately, it lasted only a few seconds, but I remember thinking I'd seen a glowing basketball frozen in midair. It was so shocking that I nearly jumped out of my seat, and the hair stood on the back of my neck. I've heard about the ghost lights my whole life, but I had never seen them before.*

These are just a few eyewitness reports. There are probably many people who have seen the lights but said nothing for fear of having their sanity doubted.

Grade 8 Unit 3 Literacy Connection 1 Source: *Ready Reading*, Grade 8, Lesson 3 ©Curriculum Associates, LLC; Copying permitted for classroom use.

Unit Review

Purpose

Connect understanding of writing and solving equations with rational number coefficients to real-world problems in order to solve escape room clues.

MATH FOCUS

DOK 3
Standards 8.EE.C.7, 8.EE.C.7b

MONITOR & GUIDE SMP 1, 2, 4, 6

Make Sense of the Problem

Before students work on the Performance Task, use **Notice and Wonder** to help them make sense of the problem. Highlight things students notice, such as the table, combination blanks, and clues. Then discuss things students wonder, such as *How many hours are in a day on other planets?* or *How will these numbers compare to how many hours are in a day on Earth?* Call attention to how the first two clues are about the relationship between the number of hours in a day on Mars and Jupiter, while the third and fourth clues deal with the relationship between Saturn and Neptune. Confirm that students understand they will be using equations to represent these situations.

Facilitate Problem Solving

Have students complete the task independently, in partners, or in small groups.

If students need additional support as they get started, have them read the first clue aloud. Point out that the number of hours on Mars is described in terms of the number of hours on Jupiter. Ask: *What variable can you use? What does the variable represent in the expression?*

When students read the second clue, guide them to notice that it also describes the number of hours on Mars in terms of Jupiter. Ask them how the variable they used in the first clue can help them model the second clue.

After students have solved the clues describing Mars and Jupiter, ask: *How can you use a similar approach to solve the clues about Neptune and Saturn?*

If students need additional support to solve the equation that models the clues about Neptune and Saturn, guide them to rewrite the coefficients as decimals to make their calculations more efficient.

After students find the number of hours in a day for each planet, remind them that the combination is made up of these numbers, written in order from least to greatest.

UNIT 3 Unit Review

Performance Task

➤ **Answer the questions and show all your work on separate paper.**

You are in a space-themed escape room, trying to solve the final problem. The combination to the door is made up of four numbers in least to greatest order. Each number is the number of hours in a day on a different planet, when rounded to the nearest whole number. You are given clues to find these numbers.

Use the clues to write and solve equations that help you find each missing number. Use a calculator to make sure your solutions are correct and record them in the table. Then write the numbers in least to greatest order to identify the combination to the door.

Checklist

Did you . . .

- ☐ Use the clues to find the missing numbers in the table?
- ☐ Reread the clues to make sure your equations are accurate?
- ☐ List the numbers in order from least to greatest to identify the combination to the door?

Planet	Number of Hours in a Day
Jupiter	
Mars	
Neptune	
Saturn	
Combination to the Door: ____ ____ ____ ____	

- The number of hours in a day on Mars is 2.5 times the number of hours in a day on Jupiter.
- A day on Mars lasts 15 hours longer than a day on Jupiter.
- The number of hours in a day on Saturn is 3 more than half the number of hours in a day on Neptune.
- A day on Saturn lasts 0.6875 times as long as a day on Neptune.

Reflect

Use Mathematical Practices After you complete the task, choose one of the following questions to answer.

- **Model** How do you know your equations match the information given in the clues?
- **Be Precise** How could you test your solutions to see if they satisfy the clues?

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Problem Notes

Students should demonstrate understanding that they need to define and use variables to translate clues into expressions.

Students should recognize that the first two and last two expressions represent the same value, so each pair of expressions can be set as equal to form equations.

Students should be able to describe the meaning of each term in their equations and connect the equations to the clues.

Students should demonstrate the ability to solve each equation.

Student responses should include a value that represents the number of hours in a day for each planet and the combination to the door.

Reflect

Model Look for understanding that variables represent the number of hours in a day on Jupiter and Neptune, and information from clues, such as *times*, hours *longer*, *more than*, and *half*, is represented in the equations.

Be Precise Look for explanations that include verifying solutions by substituting values of variables into equations.

4-Point Solution

Let J = the number of hours in a day on Jupiter.

The number of hours in a day on Mars is 2.5 times the number of hours in a day on Jupiter: $2.5J$.

A day on Mars lasts 15 hours longer than a day on Jupiter: $J + 15$.

The expressions both represent the number of hours in a day on Mars. I set them as equal in an equation to solve for J.

$2.5J = J + 15$

$1.5J = 15$

$J = 10$ A day on Jupiter lasts 10 hours, and a day on Mars lasts $10 + 15 = 25$ hours.

Let N = the number of hours in a day on Neptune.

The number of hours in a day on Saturn is 3 more than $\frac{1}{2}$, or 0.5, the number of hours in a day on Neptune: $0.5N + 3$.

A day on Saturn lasts 0.6875 times as long as a day on Neptune: $0.6875N$.

The expressions both represent the number of hours in a day on Saturn. I set them as equal in an equation to solve for N.

$0.5N + 3 = 0.6875N$

$3 = 0.1875N$

$16 = N$ A day on Neptune lasts 16 hours, and a day on Saturn lasts $\frac{1}{2}(16) + 3 = 8 + 3 = 11$ hours.

Planet	Number of Hours in a Day
Jupiter	10
Mars	25
Neptune	16
Saturn	11
Combination to the Door: 10, 11, 16, 25	

Scoring Rubric

Points	Expectations
4	The student's response is accurate and complete. The student translated the clues into expressions, found a value that represents the number of hours in a day for each planet, and correctly found the combination to the door.
3	The student's strategy and process are correct, but there are minor errors in calculations. All clues have been addressed, and some of the values that represent the number of hours in a day for each planet are correct.
2	The response contains several mistakes in the calculations. The student may have attempted to address all parts of the problem, but the response is incorrect and/or incomplete.
1	The student's response is incorrect and incomplete. The response does not address all the clues given in order to solve the problem.

DIFFERENTIATION | EXTEND

If students have more time to spend on this problem or require an additional challenge, then use this extension to have them find the combination to a bonus door.

The combination to the bonus door is the sum of the number of hours in a day on Mercury and Venus. Use the clues to find each missing number and the combination.

- The number of hours in a day on Venus is 152 hours less than 4.25 times the number of hours in a day on Mercury.
- A day on Venus lasts 4,424 hours longer than a day on Mercury.

Solution

Let M = the number of hours in a day on Mercury.

$4.25M - 152 = 4{,}424 + M$

$3.25M = 4{,}576$

$M = 1{,}408$ There are 1,408 hours in a day on Mercury.

The number of hours in a day on Venus is $4{,}424 + 1{,}408 = 5{,}832$.

The combination to the door is the sum: $1{,}408 + 5{,}832 = 7{,}240$.

Problem Notes

Form A is shown. *See **Teacher Toolbox** for Form B.*

1 **C is correct.** Students could solve the problem by using $m = \frac{\text{rise}}{\text{run}}$ to find the slope. The unit rate, or cost of each customized T-shirt, is the same as the slope of the graph of the linear equation.

A is not correct. This answer represents $\frac{x_2 - x_1}{y_2 - y_1}$ instead of $\frac{y_2 - y_1}{x_2 - x_1}$.

B is not correct. This answer represents $\frac{y_2 - x_2}{y_1 - x_1}$ instead of $\frac{y_2 - y_1}{x_2 - x_1}$.

D is not correct. This answer represents $y_2 - y_1$ and does not include dividing by the difference $x_2 - x_1$.

(1 point)

DOK 1 | **8.EE.B.5**

2 (1 point)

DOK 2 | **8.EE.C.7b**

UNIT 3 • UNIT ASSESSMENT Name:
FORM A

➤ **Solve the problems.**

1 Seth owns a printing shop that makes customized T-shirts. The cost to make T-shirts is proportional to the number of T-shirts he makes. Seth graphs a line that shows the cost per customized T-shirt. Two points on the line are (2, 8) and (4, 16). What does the slope of the line mean in this situation? *(1 point)*

A The cost is \$0.25 per T-shirt.

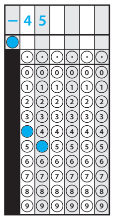
B The cost is \$2 per T-shirt.

C The cost is \$4 per T-shirt.

D The cost is \$8 per T-shirt.

2 What is the solution of the equation? Record your answer on the grid. Then fill in the bubbles. *(1 point)*

$\frac{1}{5}(4x - 2.5) = 5(0.4x + 10) + 3\frac{1}{2}$



3 (2 points)

DOK 1 | **8.EE.C.8b**

4 Students could use a table to organize *x*- and *y*-values from the graph.

(2 points)

DOK 3 | **8.EE.B.6**

UNIT 3 • UNIT ASSESSMENT Name:
FORM A continued

3 Show how to solve the system of equations by substitution. Write your answers in the blanks. *(2 points)*

$y = 2x - 1$

$6x - 2y = 24$

$6x - 2y = 24$

$6x - 2(\underline{2x - 1}) = 24$

$6x - \underline{4x} + \underline{2} = 24$

$2x + 2 = 24$

$2x = 22$

$x = 11$

$y = 2x - 1$

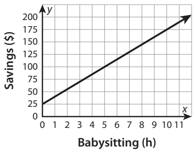
$y = 2(\underline{11}) - 1$

$y = 22 - 1$

$y = 21$

(11, 21)

4 Kaya babysits to add money to her savings. She draws a graph to show how much she can earn by babysitting. What is the equation of Kaya's line in slope-intercept form? What do the slope and *y*-intercept represent in the situation? Show your work. *(2 points)*



Possible student work:

(0, 25) and (5, 100) are two points on the line. $m = \frac{100 - 25}{5 - 0} = \frac{75}{5} = 15$; the slope is 15. The line intersects the *y*-axis at (0, 25), so the *y*-intercept is 25. The equation of the line is $y = 15x + 25$.

SOLUTION $y = 15x + 25$; Possible explanation: The slope represents the amount of money Kaya earns babysitting per hour. The *y*-intercept represents the amount of Kaya's savings to start, in dollars.

5 **D is correct.** Students could solve the problem by distributing 3 on the left side of the equation to get $12x - 24 = -24 + 12x$ and recognizing that the two sides are equivalent.

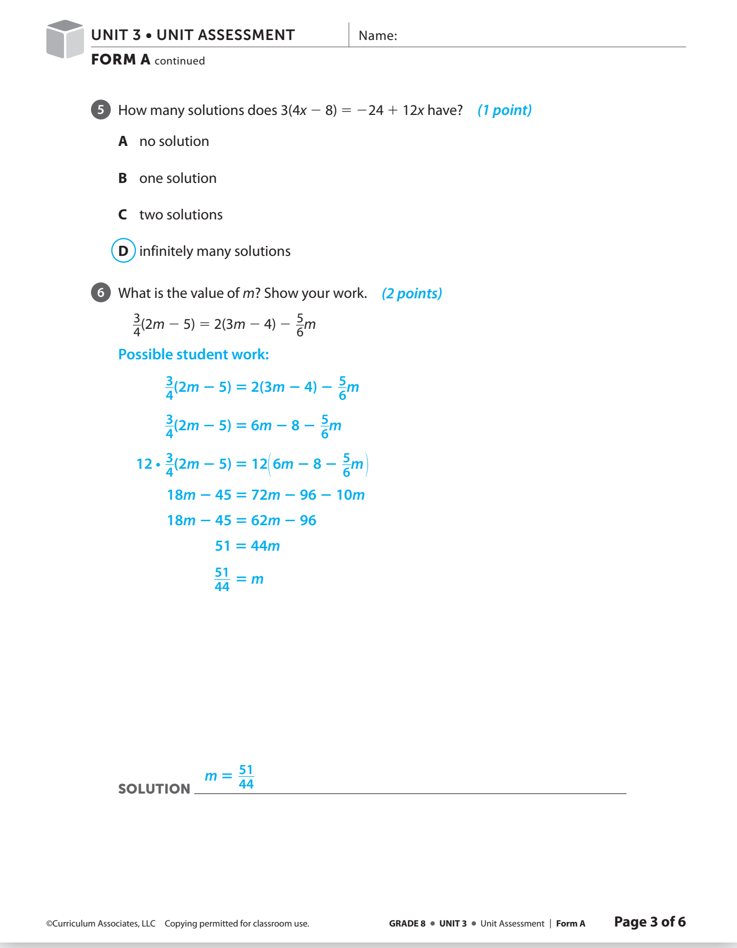
A is not correct. This answer could represent rewriting the equation as $12x - 12x = -24 + 24$ and interpreting $0 = 0$ as no solution.

B is not correct. This answer could represent dividing the expressions $\frac{12x - 24}{12x - 24}$ to get 1 and interpreting the result as one solution.

C is not correct. This answer could represent solving $12x = 24$ and interpreting $x = 2$ as two solutions.

(1 point)

***DOK 2* | 8.EE.C.7a**



UNIT 3 • UNIT ASSESSMENT | Name:

FORM A continued

5 How many solutions does $3(4x - 8) = -24 + 12x$ have? *(1 point)*

A no solution

B one solution

C two solutions

D infinitely many solutions

6 What is the value of m? Show your work. *(2 points)*

$\frac{3}{4}(2m - 5) = 2(3m - 4) - \frac{5}{6}m$

Possible student work:

$\frac{3}{4}(2m - 5) = 2(3m - 4) - \frac{5}{6}m$

$\frac{3}{4}(2m - 5) = 6m - 8 - \frac{5}{6}m$

$12 \cdot \frac{3}{4}(2m - 5) = 12\left(6m - 8 - \frac{5}{6}m\right)$

$18m - 45 = 72m - 96 - 10m$

$18m - 45 = 62m - 96$

$51 = 44m$

$\frac{51}{44} = m$

SOLUTION $m = \frac{51}{44}$

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6 Students could solve the problem by distributing $\frac{3}{4}$ and 2 and then multiplying each term by the least common denominator.

(2 points)

***DOK 2* | 8.EE.C.7b**

7 **a.** The lines are not parallel, so they will intersect at one point. The intersection point is the solution to the system.

b. If the graph is extended far enough to the right, the lines will intersect and the intersection point would be visible. The intersection point is the solution to the system.

c. If two lines are not parallel, there will be one point of intersection, which is the solution to the system.

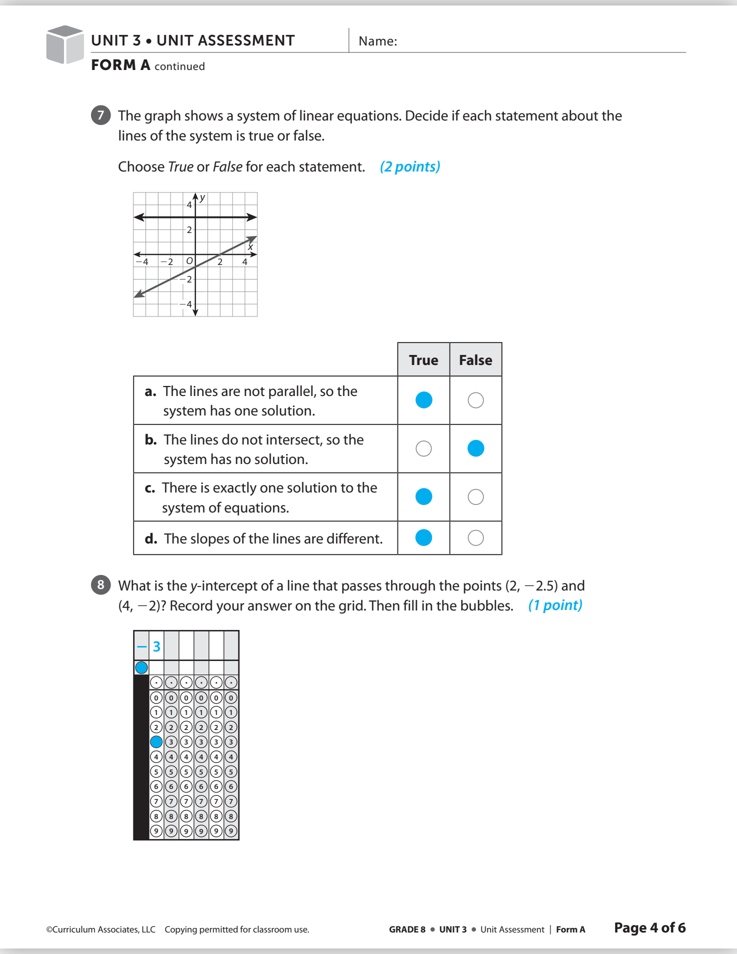
d. The equations for the lines are $y = \frac{1}{2}x - 1$ and $y = 3$, so the slopes are $\frac{1}{2}$ and 0.

(2 points)

***DOK 2* | 8.EE.C.8a**

8 (1 point)

***DOK 2* | 8.EE.B.6**



UNIT 3 • UNIT ASSESSMENT | Name:

FORM A continued

7 The graph shows a system of linear equations. Decide if each statement about the lines of the system is true or false.

Choose *True* or *False* for each statement. *(2 points)*

	True	False
a. The lines are not parallel, so the system has one solution.	●	○
b. The lines do not intersect, so the system has no solution.	○	●
c. There is exactly one solution to the system of equations.	●	○
d. The slopes of the lines are different.	●	○

8 What is the y-intercept of a line that passes through the points $(2, -2.5)$ and $(4, -2)$? Record your answer on the grid. Then fill in the bubbles. *(1 point)*

-3

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Problem Notes

9 (2 points)

DOK 2 | **8.EE.C.8c**

10 Students could solve the problem by multiplying the second equation by 6 to get a first term of $2x$. Then they could subtract the second equation from the first equation. This will give a one-variable equation in terms of y. The result, $y = 4$, can be substituted into either equation to get $x = 3$.

(2 points)

DOK 2 | **8.EE.C.8b**

9 Yuki and Roman are photographers for the yearbook. Yuki starts with 20 photos. She then takes 60 photos each week. Roman starts with 110 photos. He then takes 35 photos each week. After how many weeks will Yuki and Roman have taken the same number of photos? Write a system of equations that can be used to solve the problem. Write your answers in the blanks. *(2 points)*

$p =$ 60 $w +$ 20

$p =$ 35 $w +$ 110

10 What is the solution to the system of equations? Show your work. *(2 points)*

$2x - 2y = -2$

$\frac{1}{3}x + y = 5$

Possible student work:

$2x - 2y = -2 \longrightarrow 2x - 2y = -2$

$2\left(\frac{1}{3}x + y = 5\right) \longrightarrow \frac{2}{3}x + 2y = 10$

$\frac{8}{3}x = 8$

$x = 3$

$2(3) - 2y = -2$

$6 - 2y = -2$

$-2y = -8$

$y = 4$

SOLUTION (3, 4)

11 Students could substitute 1 for x in each equation to get $y = 14$. This will confirm algebraically that (1, 14) is a point on both lines.

(4 points)

DOK 3 | **8.EE.C.8b**

12 **a.** $7x - 3 = 4x + 3$ has a solution of $x = 2$. This means there is one solution.

b. Substituting any value for x in $6x + 2 = 2 + 6x$ gives a true statement. So, the equation has infinitely many solutions.

c. Substituting any value for x in $4x - 5 = 4x - 5$ gives a true statement. So, the equation has infinitely many solutions.

d. $-x - 8 = -x + 8$ is equivalent to $0 = 16$. This is not a true statement. No value of x makes the equation true, so there is no solution.

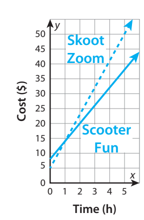
(2 points)

DOK 1 | **8.EE.C.7a**

11 Scooter Fun and Skoot Zoom are electronic scooter rental companies. Scooter Fun charges an \$8 deposit plus \$6 per hour rental fee. Skoot Zoom charges a \$5 deposit plus \$9 per hour rental fee. The system of equations represents the cost, y, for renting a scooter for x hours. Graph the system and label each line with the scooter company it represents. What is the solution to the system of equations? What does it mean in the context of the problem? Explain your reasoning. *(4 points)*

$y = 6x + 8$

$y = 9x + 5$



SOLUTION (1, 14); Possible explanation: At 1 hour, the cost of renting from either scooter company is the same, \$14.

12 Decide if each equation has no solution, one solution, or infinitely many solutions.

Choose *No Solution*, *One Solution*, or *Infinitely Many Solutions* for each equation. *(2 points)*

	No Solution	One Solution	Infinitely Many Solutions
a. $7x - 3 = 4x + 3$	○	●	○
b. $6x + 2 = 2 + 6x$	○	○	●
c. $4x - 5 = 4x - 5$	○	○	●
d. $-x - 8 = -x + 8$	●	○	○

Scoring Guide

For the problems in the Unit 3 Unit Assessments (Forms A and B), the table shows:

- depth of knowledge (DOK) level
- points for scoring
- lesson assessed by each problem
- standard addressed

Problem	DOK	Points	Lesson	Standard
1	1	1	8	8.EE.B.5
2	2	1	10	8.EE.C.7b
3	1	2	13	8.EE.C.8b
4	3	2	9	8.EE.B.6
5	2	1	11	8.EE.C.7a
6	2	2	10	8.EE.C.7b
7	2	2	12	8.EE.C.8a
8	2	1	9	8.EE.B.6
9	2	2	14	8.EE.C.8c
10	2	2	13	8.EE.C.8b
11	3	4	12	8.EE.C.8b
12	1	2	11	8.EE.C.7a

Scoring Rubrics

Extended Response Scoring Rubric

Points	Expectations
4	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of mathematical concepts and/or procedures.
3	Response contains mostly correct solution(s) and demonstrates a strong understanding of mathematical concepts and/or procedures.
2	Response shows partial to limited understanding of mathematical concepts and/or procedures.
1	Response contains incorrect solution(s), shows poorly organized and incomplete work and explanations, and demonstrates limited understanding of mathematical concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of mathematical concepts and/or procedures.

Short Response Scoring Rubric

Points	Expectations
2	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of mathematical concepts and/or procedures.
1	Response contains mostly correct solution(s) and shows partial understanding of mathematical concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of mathematical concepts and/or procedures.

Fill-in-the-Blank and Choice Matrix Scoring Rubric

Points	Expectations
2	All answers are correct
1	1 incorrect answer
0	2 or more incorrect answers

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