## Mathematics



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## Student Sample, Lesson 18

Ready Classroom Mathematics lessons begin with a Family Letter that provides activities and instructional supports to foster school, family, and community involvement and partnerships.
Each multiday lesson includes three types of sessions: Explore, Develop, and Refine.

The following pages, 373-388, represent a complete student lesson.

## Fractions as Division

## Dear Family,

## This week your child is learning how fractions and division are related.

He or she might see a problem like the one below.
Three family members equally share 4 granola bars. How much does each family member receive?

This word problem can be represented as a division problem. The family equally shares 4 granola bars among 3 people, so the division problem to solve is $4 \div 3$.

A model is a useful way to show the problem.
The model below shows 4 wholes. Each whole is divided into 3 parts.


Each family member receives $\frac{1}{3}$ of each of 4 whole bars. So, the answer to the division problem $4 \div 3$ is $\frac{4}{3}$. You can say that the fraction $\frac{4}{3}$ represents the division problem $4 \div 3$.

This shows how fractions and division are related. You can think of fractions as the division of two numbers.

Another way to write the fraction $\frac{4}{3}$ is to show it as a mixed number. So, each family member receives $\frac{4}{3}$, or $1 \frac{1}{3}$, granola bars.

Invite your child to share what he or she knows about how fractions and division are related by doing the following activity together.


## ACTIUITY FRACTIONS RS DIVISION

## Do this activity with your child to explore fractions as division.

Work with your child to find opportunities to practice modeling a division situation as a fraction.

- Together with your child, think of things that can be shared equally among family members, such as boxes of crackers or bags of grapes.
- Choose one idea. Work together with your child to show how to equally divide a number of the items among the people in your family.

Example: 4 family members equally share 7 bags of trail mix.

- Have your child write the idea as a division problem.

Example: $7 \div 4=\frac{7}{4}$

- Have your child explain how much of the item each family member will get.

Example: Each person will get $\frac{7}{4}$, or $1 \frac{3}{4}$, bags of trail mix.


## Explore Fractions as Division

You know that division is used for equal sharing and that fractions represent a number of equal parts of a whole. In this lesson, you will learn how division and fractions are related. Use what you know to try to solve the problem below.

## Mrs. Tatum needs to share 4 fluid ounces of red paint equally among 5 art students. How many ounces of red paint will each student get?

## Learning Target

- Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b}=a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.


## TRY IT

## Math Toolkit

- fraction circles or tiles
- fraction bars
- tenths grids
- number lines
- index cards


DISCUS5 IT
Ask your partner: Why did you choose that strategy?
Tell your partner: At first, I thought

## CONNECTIT



## (1) LOOK BACK

Explain how to find the amount of paint each student gets.

## 2) LOOK AHEAD

Suppose Mrs. Tatum wants to share 8 fluid ounces of paint equally among the 5 students. You can think about this quotient in two ways.
a. Think of each student getting $\frac{1}{5}$ of each ounce. Shade $\frac{1}{5}$ of each whole in the model below to show one student's share.

1 ounce

1 ounce

1 ounce

1 ounce

1 ounce

1 ounce

1 ounce

8 ounces $\div 5=8 \times$ $\qquad$ $=$ $\qquad$ ounces
b. Think of 8 ounces as 5 ounces +3 ounces. Explain how the shaded part of the model below shows one student's share.


c. Write the quotient $8 \div 5$ as a fraction and as a mixed number. $\qquad$

## (3) REFLECT

How would you write the fraction $\frac{2}{5}$ as a division expression? Write a word problem that can be represented by your expression and by the fraction $\frac{2}{5}$.

## Prepare for Fractions as Division

1 Think about what you know about division. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

| Word | In My Own Words | Example |
| :--- | :--- | :--- |
| fraction |  |  |
| division <br> expression |  |  |
| quotient |  |  |

(2) Write the fraction $\frac{3}{4}$ as a division expression.

How could you use multiplication to check your answer?
(3) Solve the problem. Show your work.

Mrs. Tatum needs to share 3 grams of glitter equally among 8 art students. How many grams of glitter will each student get?


Solution
(4) Check your answer. Show your work.

$$
\therefore \quad{ }^{+} d
$$

## Develop Fractions as Division

Read and try to solve the problem below.
Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?


- fraction circles or tiles
- fraction bars
- tenths grids
- number lines
- index cards

Explore different ways to understand fractions as quotients.

## Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?

## PICTURE IT

You can use a fraction model to picture how the students divide up the work.
There are 5 hallways for 3 students to decorate, which is $5 \div 3$.
If they share the work equally, each student can decorate $\frac{1}{3}$ of each hallway.


## MODEL IT

## You can use a number line to model each student's share of the work.

The number line is numbered from 0 to 5 because there are 5 hallways. It is divided into thirds because each student can decorate one third of each hallway.


The thirds can be rearranged to show each student's share of the work.


## CONNEET IT

## Now you will use the problem from the previous page to help you understand fractions as quotients.

(1) How many thirds of a hallway are there to decorate in 5 hallways? .................. thirds
(2) How many thirds of a hallway will each student decorate? .................. thirds

Write this as a fraction. ................... of a hallway
(3) Write a division equation that shows the quotient as a fraction.

Write a multiplication equation to check this equation.
(4) Both Picture It and Modell It show each student decorating $\frac{1}{3}$ of each hallway. Here is another way to think about dividing up 5 hallways among 3 people.

How many whole hallways can each student decorate?
How many hallways remain after those are done?
How much of the 2 remaining hallways will each student decorate?
Write a mixed number to show how many hallways each student will decorate. hallways
(5) Write a division equation that shows the quotient as a mixed number.

6 How does the bar in a fraction represent division?

## (7) REFLECT

Look back at your Try It, strategies by classmates, and Picture It and Model It. Which models or strategies do you like best for finding fraction quotients? Explain.
$\qquad$
$\qquad$
$\qquad$

## APPLY IT

## Use what you just learned to solve these problems.

8 Five friends are equally sharing 3 packs of football cards. How many packs of cards will each friend get? Use a visual model to support your answer.

Solution
9 Elena made 10 ounces of apple chips. She puts the same amount of apple chips into each of 4 containers. How many ounces of apple chips are in 1 container? Write a division expression to represent the problem and solve. Use a visual model to support your answer.

## Solution

(10) Which expression is equivalent to $\frac{12}{7}$ ?
(A) 12-7
(B) 7-12
(C) $12 \div 7$
(D) $7 \div 12$

## Practice Fractions as Division

## Study the Example showing whole-number division with a fraction quotient. Then solve problems 1-5.

## EXAMPLE

There are 4 packages of printer paper to be divided equally among 6 classrooms. How much paper will each classroom get?

There are 4 packages for 6 classrooms to share, which is $4 \div 6$.
If you divide each package into sixths, each classroom would get one sixth of each package. So, $\frac{1}{6}$ of each package from 4 packages is the same as $\frac{4}{6}$ of a package.

4

$\times$

$\frac{1}{6}$

$=\square$
$=\frac{\square}{\square}$

$$
4 \div 6=\frac{4}{6}
$$

Each classroom gets $\frac{4}{6}$ of a package.

1) Circle the number line you would use to solve the problem in the Example.


Number Line B


2 Look at the Example. Suppose only 5 classrooms share 4 packages. How would the model in the Example change? How would the answer change?

3 Trish is taking care of the Han family's dogs. The Hans leave 7 cans of dog food for the 3 days they will be away. How much food will the dogs get each day if Trish feeds them an equal amount each day? Show your work.


## Solution

4. Raul plans to run 30 miles this week. He wants to run the same number of miles each day of the week. He says he will run $\frac{7}{30}$ mile each day. Is he correct? Explain.

5 Gus makes 48 fluid ounces of spiced cider. If he serves an equal amount to each of 7 people, will each person get more than 1 cup of cider or less than 1 cup? ( 1 cup $=8$ fluid ounces) Show your work.

## Solution

## Refine Fractions as Division

Complete the Example below. Then solve problems 1-9.

## EXAMPRE

Luke, Carter, and Ava have 2 quarts of juice. They want to share it equally. How many quarts of juice will each of them get?

Look at how you could show your work using a model and equations.

$$
\begin{aligned}
2 \div 3 & =2 \times \frac{1}{3} \\
& =\frac{2}{3}
\end{aligned}
$$



## Solution

## APPLY IT

(1) Erica has 7 square feet of space in her rectangular garden to plant carrots, beans, peppers, and lettuce. Suppose she gives each vegetable an equal amount of space. How much space will each vegetable get? Show your work.

2 quarts are shared equally by 3 friends, sol know that each friend will have less than 1 quart of juice. That means the quotient is a fraction.


## PAIR/SHARE

Model the problem for 3 quarts of juice divided equally among Luke, Carter, Ava, and Ava's little brother.

Each vegetable will get at least 1 square foot of garden space. How will the rest of the space be divided up?
(2) Deon needs to make 6 pizza crusts. He has 20 ounces of dough and wants to use the same amount of dough for each crust. He weighs a portion of dough for 1 crust on a scale. The weight, in ounces, should fall between what two whole numbers? Show your work.

## Solution

3 Jonas is doing a science experiment with his class. The teacher has 21 fluid ounces of pond water to share equally among 10 pairs of students. How much pond water will Jonas and his science partner receive?
(A) $\frac{10}{21}$ fluid ounce
(B) $1 \frac{1}{10}$ fluid ounces
(C) 2 fluid ounces
(D) $\frac{21}{10}$ fluid ounces

Olivia chose (A) as the correct answer. How did she get that answer?

How many whole ounces of dough will each crust get? What will happen with the remaining ounces?


## PAIR/SHARE

Create a different division story to represent $\frac{20}{6}$.

About how much water will each pair of students receive? Will it be more or less than 2 fluid ounces?


## PAIR/SHARE

Does Olivia's answer make sense?

4 Teddy makes 32 fluid ounces of hot cocoa. He pours equal amounts of cocoa into 5 cups. The amount of hot cocoa in each cup will fall between which two amounts?
(A) 3 and 4 fluid ounces
(B) 4 and 5 fluid ounces
(C) 5 and 6 fluid ounces
(D) 6 and 7 fluid ounces

5 Pierce swims 10 laps in a pool in 8 minutes. He spends the same amount of time on each lap. How much time does each lap take him?
(A) $\frac{2}{10}$ minute
(B) $\frac{8}{10}$ minute
(C) $\frac{10}{8}$ minutes
(D) $1 \frac{2}{8}$ minutes

6 Dani needs 8 equal sections from a board that is 13 meters long. Does the expression represent the largest possible length of 1 section of the board, in meters?

|  | Yes | No |
| :--- | :---: | :---: |
| $1 \frac{5}{8}$ | (A) | (B) |
| $\frac{8}{13}$ | © | (D) |
| $\frac{13}{8}$ | () | © |
| $8 \div 13$ | (G) | (H) |
| $13 \times \frac{1}{8}$ | (I) | (ㄷ) |

(7) Which situations can be represented by $\frac{25}{9}$ ?
(A) Melanie equally shares 25 meters of paper to make 9 banners.
(B) Quill gives away 9 baseball cards from a pack of 25 cards.
(C) George invites 25 kids and 9 adults to his birthday party.
(D) Becca makes 9 rows with 25 buttons each.
(E) Joe makes 9 equal servings from a 25 -ounce bag of peanuts.
(8) Paco is trying to explain to his friend that $7 \div 2=\frac{7}{2}$.

Part A Draw a model or number line showing $7 \div 2=\frac{7}{2}$.

Part B Explain the equivalence of $7 \div 2$ and $\frac{7}{2}$ using words.

## 9 MATH JOURNAL

Write a division word problem that can be represented by the expression $12 \div 5$. Then explain how to solve your problem.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

## Teacher Sample, Lesson 18

The Ready Classroom Mathematics Teacher's Guide includes support for planning, differentiation, and facilitating meaningful mathematical discourse.

The following pages, 373a-388b, represent a complete teacher lesson that corresponds to the student lesson included in this sample.

## CCSS Focus

## Domain

Number and Operations-Fractions

## Cluster

B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

## Standard

5.NF.B. 3 Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b}=a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3 , and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

## Standards for Mathematical Practice (SMP)

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
7 Look for and make use of structure.

## Lesson Objectives

## Content Objectives

- Use visual fraction models to represent a fraction as division.
- Solve word problems involving division of whole numbers in which the quotient is a fraction or mixed number.
- Understand a fraction as a way to represent division where the numerator is divided by the denominator.


## Language Objectives

- Make a visual fraction model to represent a fraction as a division of two whole numbers and explain the relationship of the model to the fraction.
- Draw a visual model and write an equation to represent word problems involving a quotient of whole numbers where the quotient is a fraction.


## Prerequisite Skills

- Understand division as equal groups or sharing.
- Understand that multiplication and division are inverse operations.
- Divide whole numbers.
- Multiply a fraction by a whole number.


## Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- denominator the number below the line in a fraction that tells the number of equal parts in the whole.
- fraction a number that names equal parts of a whole. A fraction names a point on the number line and can also represent the division of two numbers.
- numerator the number above the line in a fraction that tells the number of equal parts that are being described.
- quotient the result of division.


## Learning Progression

In previous grades students have understood the meaning of division as equal groups or equal shares. They have interpreted fractions as equal parts of a whole or equal parts of a group.
In this lesson students extend their understanding of both division and fractions to see that when a whole number is divided by another whole number the quotient may be a fraction or a mixed number, e.g., $2 \div 5=\frac{2}{5}$ and $23 \div 7=\frac{23}{7}$. From exploring problems like this they recognize that a fraction can be interpreted as representing a
division expression, one where the numerator is divided by the denominator. They further connect this idea to the inverse relationship of multiplication and division, e.g., $2 \div 5=\frac{2}{5}$ and $5 \times \frac{2}{5}=2$. In later lessons students will learn to divide whole numbers by unit fractions and divide unit fractions by whole numbers. In Grade 6 students will apply and extend their understanding of division with fractions to divide a whole number by a fraction and to divide a fraction by a fraction.

## Lesson Pacing Guide

## Whole Class Instruction

| SESSION 1 <br> Explore <br> 45-60 min | Fractions as Division <br> - Start 5 min <br> - Try It 10 min <br> - Discuss It 10 min <br> - Connect It 15 min <br> - Close: Exit Ticket 5 min | Additional Practice Lesson pages 377-378 |
| :---: | :---: | :---: |
| SESSION 2 <br> Develop <br> 45-60 min | Fractions as Division <br> - Start 5 min <br> - Try It 10 min <br> - Discuss It 10 min <br> - Picture It \& Model It 5 min <br> - Connect It \& Apply It 10 min <br> - Close: Exit Ticket 5 min | Additional Practice <br> Lesson pages 383-384 <br> Fluency <br> Fractions as Division |
| SESSION 3 <br> Refine <br> 45-60 min | Fractions as Division <br> - Start 5 min <br> - Example \& Problems 1-3 15 min <br> - Practice \& Small Group Differentiation 20 min <br> - Close: Exit Ticket 5 min | Lesson Quiz |

## Lesson Materials

Lesson Activity Sheet: Number Lines
(Required)
Activities Per student: base-ten blocks (1 tens rods, 2 ones units), scissors Activity Sheets: Fraction Bars, Digit Cards, 1-Inch Grid Paper

Math Toolkit fraction circles, fraction tiles, fraction bars, tenths grids, number lines, index cards

[^0]
## Small Group Differentiation

## PREPARE

Ready Prerequisite Lessons
Grade 4

- Lesson 23 Understand Fraction Multiplication


## RETEACH

Tools for Instruction

## Grade 4

- Lesson 23 Understand Fraction Multiplication


## Grade 5

- Lesson 18 Interpreting Fractions as Division


## REINFORCE

## Math Center Activities

## Grade 5

- Lesson 18 Fractions as Quotients
- Lesson 18 Relate Situations to Fractional Quotients


## EXTEND

## Enrichment Activity

Grade 5

- Lesson 18 Pizza Party


## Tiin-Ready

Independent Learning
PERSONALIZE
i-Ready Lesson*
Grade 5

- Fractions as Division

Teacher pages have been reduced. Actual book size is $101 / 4^{\prime \prime} \times 12^{\prime \prime}$.

## Connect to Family, Community, and Language Development

The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

## Connect to Family

Use the Family Letter-which provides background information, math vocabulary, and an activityto keep families apprised of what their child is learning and to encourage family involvement.


Dear Family,
This week your child is learning how fractions and division are related.

He or she might see a problem like the one below.
Three family members equally share 4 granola bars. How much does each family member receive?

This word problem can be represented as a division problem. The family equally shares 4 granola bars among 3 people, so the division problem to solve is $4 \div 3$.

A model is a useful way to show the problem.
The model below shows 4 wholes. Each whole is divided into 3 parts.


Each family member receives $\frac{1}{3}$ of each of 4 whole bars. So, the answer to the Each family member receives $\frac{1}{3}$ of each of 4 whole bars. So, the answer to th
division problem $4 \div 3$ is $\frac{4}{3}$. You can say that the fraction $\frac{4}{3}$ represents the division problem $4 \div 3$.
This shows how fractions and division are related. You can think of fractions as the division of two numbers.
Another way to write the fraction $\frac{4}{3}$ is to show it as a mixed number. So, each family member receives $\frac{4}{3^{\prime}}$, or $1 \frac{1}{3^{\prime}}$ granola bars.
Invite your child to share what he or she knows about how fractions and division are related by doing the following activity together.


## ACTIVITY PRACTIONS AS DIVIIION

Do this activity with your child to explore fractions as division.

Work with your child to find opportunities to practice modeling a division situation as a fraction.

- Together with your child, think of things that can be shared equally among family members, such as boxes of crackers or bags of grapes.

Choose one idea. Work together with your child to show how to equally divide a number of the items among the people in your family. Example: 4 family members equally share 7 bags of trail mix.

Have your child write the idea as a division problem.
Example: $7 \div 4=\frac{7}{4}$
Have your child explain how much of the item each family member will get.
Example: Each person will get $\frac{7}{4}$, or $1 \frac{3}{4}$, bags of trail mix.


## Math Talk at Home

Encourage students to look for objects at home that can be divided into equal parts to show relationships between fractions and division.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- What are some objects or items that can be shared equally among our family members?
- How can we represent that as a division? How can we represent it as a fraction?


## Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

## Sessions 1 and 3 Use anytime during these sessions.

- Many problems in this lesson discuss dividing labor or items into equal parts. Ask students to make a personal connection to how they divide chores or activities around their home. Ask if the chores are divided by the days of the week when they are done, the number of people doing the chores, etc.
- Sharing objects or items equally is an example of division that students encounter in their daily life. An equal number of students assigned to each classroom is another example of dividing with whole numbers.
- Division situations do not always involve dividing a given quantity by a number less than the given quantity. Dividing one whole (e.g., a snack) between two or more people can be represented by a fraction. As students think of how to represent this division situation as a fraction, encourage them to make personal connections to how they use fractions and division in their daily lives.


## Session 2 Use with Try It.

- Decorating school hallways is a way to show school pride and encourage collaboration between different classrooms. The collaboration may include developing a theme and organizing how students will work together to complete the work. In the same way, cities and communities make murals and landscape public spaces to show pride. Ask students to share any community work they might have been a part of and how the work was divided.


## Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

English Language Learners:
Differentiated Instruction

Prepare for Session 1
Use with Connect lt.

## Levels 1-3

Speaking/Writing Read Connect It problem 3 with students. Ask students to work with a partner. Provide two small cups to each pair of students and ask them to describe to their partner one way they would divide 2 cups of paint among 5 students. Provide the sentence frames below and have students complete them:

- Ineed to divide 2 cups of paint among students.
- I can write this as the division $\underline{2 \div 5}$.


## Levels 2-4

Speaking/Writing Have students read Connect It problem 3 with a partner. Ask them to share their ideas about what the word expression means. Remind students that an expression does not have an equal sign. Provide a sentence frame to guide students' discussions:

- We can write the fraction $\frac{2}{5}$ as the division expression $2 \div 5$.
Ask partners to discuss possible scenarios for their word problems. They may want to use the context about sharing paint. Provide a vocabulary list: equally, paint, cups, ounces.
Have students provide feedback to each other before they write the sentence.


## Levels 3-5

Speaking/Writing Have students read Connect It problem 3 with a partner. Ask them to write a complete sentence to describe a scenario for the division expression.
When complete, have students share their work with another set of partners. Encourage each pair of students to explain and defend their answer.

## LESSON 18

## LESSON 18

SESSION 1 •••
Purpose In this session, students draw on their understanding of division as equal sharing and fractions as equal parts of a whole. They share models to explore how to represent the division of a whole number by a greater whole number. They will look ahead to think about how division of whole numbers can lead to quotients that are fractions or mixed numbers.

## Start

## Connect to Prior Knowledge

Why Supports students in relating repeated addition of a fraction to multiplying the fraction by a whole number.

How Have students complete the addition equation and the multiplication equation to represent the number of sixths shown in the fraction model.


## TRY IT

## Make Sense of the Problem

To support students in making sense of the problem, have them identify that the number of students sharing the paint, 5 , is greater than the number of ounces of paint, 4.

## D15CUS5 IT

## Support Partner Discussion

Encourage students to use the Discuss It question and sentence starter on the Student Worktext page as part of their discussion.
Look for, and prompt as necessary for, understanding of:

- 4 ounces as the amount to be shared equally
- 5 students to get equal shares
- the amount of each share is the unknown


## Explore Fractions as Division

You know that division is used for equal sharing and that fractions represent a number of equal parts of a whole. In this lesson, you will learn how division and fractions are related. Use what you know to try to solve the problem below.

Mrs. Tatum needs to share 4 fluid ounces of red paint equally among 5 art students. How many ounces of red paint will each student get?

Learning Target - Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b}=a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

TRY HT
Possible student work:
Sample A

| 1 ounce |  |  |  |  |  | 1 ounce |  |  |  |  |  | 1 ounce |  |  |  |  | 1 ounce |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  | B | C | D | E | A | B | C | C | D | E | A | B | C | D | E | A | B | C | D | E |

Each student gets $\frac{1}{5}$ of each ounce. Since $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{4}{5}$, each student gets $\frac{4}{5}$ of an ounce.

Sample B
4 ounces $=40$ tenths of an ounce


40 tenths $\div 5=8$ tenths
Each student gets 0.8 ounce of red paint.

Math Toolkit - fraction circles or tiles - fraction bars - tenths grids - number lines - index cards


Ask your partner: Why did you choose that strategy? Tell your partner: At first, I thought.

Common Misconception Look for students who are not comfortable with the idea of dividing a whole number by a greater whole number. As students present solutions, have them specify why they represented the 4 ounces of paint the way they did.

## Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models showing 5 groups of 4 fifths
- drawings to represent the problem
- number lines marked in fifths
- equations showing 4 as an equivalent fraction with numerator divisible by 5


## Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.
Ask How do [student name]'s and [student name]'s models show the ounces of paint divided? Each student's share of paint?

Listen for Each ounce can be divided into 5 equal parts. Each student gets the same number of equal parts for their fair share. They each get four $\frac{1}{5} \mathrm{~s}$, or $\frac{4}{5}$, or 8 tenths.

## CONNECT IT

## (1) LOOK BACK

Look for understanding that each whole ounce of paint can be divided into equal parts and each student's equal share is the sum of the equal parts that is his share in each ounce.

## Hands-On Activity

Use fraction bars to divide.
If . . . students are unsure about dividing 4 wholes into equal parts that are fractions,
Then . . . use this activity to model Try It.
Materials For each student: scissors, Activity Sheet Fraction Bars (4 bars for fifths)

- Explain to students that they will model the Try It problem.
- Explain that each fraction bar represents 1 ounce of paint. Have students write the letters A through E in the sections of each fraction bar, each letter representing one of the 5 art students sharing the ounces of paint. Prompt students to identify each art student's share of each ounce as $\frac{1}{5}$ of an ounce.
- Have students cut the fraction bars into fifths and collect each art student's shares in a separate pile. Have them count fifths to find the size of one share. Discuss the results.
- Repeat activity, representing other situations such as 4 students sharing 3 bags of popcorn.


## (2) LOOK AHEAD

Point out that sometimes when you divide whole numbers, the quotient may be in the form of a fraction greater than 1 or a mixed number.
Students should be able to show or interpret the equal parts comprising one share for both ways of dividing the 4 ounces of paint into 5 equal shares. Have students think about using multiplication to explain why the quotient of two whole numbers can be represented by a fraction.
Ask Your work in problem 2 shows that $8 \div 5=\frac{8}{5}$. What is the related multiplication equation that shows this same relationship?
Listen for $5 \times \frac{8}{5}=8 ; 5 \times \frac{8}{5}=\frac{(5 \times 8)}{5}=\frac{40}{5}=8$

## CONNECT IT

(1) LOOK BACK

Explain how to find the amount of paint each student gets.
Possible answer: I drew a picture to show that each student gets $\frac{1}{5}$ of each
ounce. I added $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ to get $\frac{4}{5}$ ounce for each student.
(2) LOOK AHEAD

Suppose Mrs. Tatum wants to share 8 fluid ounces of paint equally among the 5 students. You can think about this quotient in two ways.
a. Think of each student getting $\frac{1}{5}$ of each ounce. Shade $\frac{1}{5}$ of each whole in the model below to show one student's share.


8 ounces $\div 5=8 \times \ldots \frac{\overline{5}}{5} \ldots \ldots . . \quad \frac{8}{5} \ldots \ldots$ ounces
b. Think of 8 ounces as 5 ounces +3 ounces. Explain how the shaded part of the model below shows one student's share.


Possible explanation: Each student gets 1 full ounce of paint and $\frac{1}{5}$ of each of the remaining 3 ounces.
c. Write the quotient $8 \div 5$ as a fraction and as a mixed number. $\frac{8}{5}=1 \frac{3}{5}$

REFLECT
How would you write the fraction $\frac{2}{5}$ as a division expression? Write a word problem that can be represented by your expression and by the fraction $\frac{2}{5}$.
$2 \div 5$; Possible problem: How much paint will each student get if 2 ounces of
paint are shared equally among 5 students?

## Close: Exit Ticket

## (3) REFLECT

Look for understanding that a fraction can represent division of the numerator by the denominator. The problem situation students describe should have 2 as the quantity being divided and 5 as the number of equal shares it is divided into.

Common Misconception If students confuse which number in a fraction represents the dividend and which represents the divisor in the equivalent division expression,
then write the three division expressions from the Student Worktext page for this session: $4 \div 5,8 \div 5,2 \div 5$. Have students pair each expression with the fraction they found that represents it. For each expression/fraction pair have students circle the numerator in the fraction and identify where that number is in the expression.

## Real-World Connection

Encourage students to think about everyday places or situations in which dividing a quantity into equal shares may result in an amount that includes a fractional part. Have volunteers share ideas. Examples: cooking 5 eggs for breakfast to feed 3 people, making 10 place mats out of 6 feet of fabric, needing 3 cups of yogurt to make 24 servings of fruit smoothie.

Teacher pages have been reduced. Actual book size is $101 / 4^{\prime \prime} \times 12^{\prime \prime}$.

## LESSON 18 SESSION - Additional Practice

## Solutions

## Support Vocabulary Development

(1) Have students say the terms: fraction, division expression, and quotient. Guide a conversation with students in which they discuss what they know about division. Then ask them to discuss the definition for each term and write it in the In My Own Words column.

Call on volunteers to share what they wrote for each term. Correct any misconceptions and ask students to revise their answers if necessary.

As students share their graphic organizer, encourage them to use this sentence frame:

- My explanation for $\qquad$ is $\qquad$ .

(2)
Read the problem. Have students discuss with a partner how you use multiplication to check an answer to a division problem. Then ask students to confirm their work with another set of partners.

## Supplemental Math Vocabulary

- dividend
- divisor
- quotient


## Prepare for Fractions as Division

(1) Think about what you know about division. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. Possible answers:

| Word | $\ln$ My Own Words | Example |
| :--- | :--- | :---: |
| fraction | a number that names equal parts of <br> a whole | $\frac{4}{5}$ |
| division <br> expression | an expression that has the <br> operation of division | $4 \div 5$ |
| quotient | the result of division | $16 \div 8=2$ |

(2) Write the fraction $\frac{3}{4}$ as a division expression. ....... $3 \div 4$

How could you use multiplication to check your answer?
Possible answer: If $3 \div 4=\frac{3}{4}$, then $4 \times \frac{3}{4}$ should be 3 .
$4 \times \frac{3}{4}=\frac{4 \times 3}{4}$
$=\frac{12}{4}$
$=3$

## ESSON 18 SESSION 1

(3)
Assign problem 3 to provide another look at fractions as division.

This problem is very similar to the problem about Mrs. Tatum sharing red paint among her art students. In both problems, students are given a word problem that requires them to divide one whole number by a greater whole number. The question asks how many grams of glitter each student will get.
Students may want to use fraction bars or grid paper.
Suggest that students read the problem three times, asking themselves one of the following questions each time:
-What is this problem about?
-What is the question I am trying to answer?

- What information is important?


## Solution:

Each student gets $\frac{1}{8}$ of each gram. Since $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$, each student gets $\frac{3}{8}$ gram of glitter.

## Medium

4 Have students solve the problem a different way, or use multiplication, to check their answer.
(3) Solve the problem. Show your work. Mrs. Tatum needs to share $\mathbf{3}$ grams of glitter equally among 8 art students. How many grams of glitter will each student get?


Possible student work using pictures:

| 1 gram | 1 | I |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A | B | C | D | E | F | G | H | A | B | C | D | E | F | G | H | A | B | C | D | E | F | G | H |

Each student gets $\frac{1}{8}$ of each gram.
$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

Solution Each student gets $\frac{3}{8}$ gram of glitter.
(4)

Check your answer. Show your work.
Possible student work:
$8 \times \frac{3}{8}=\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$
$=\frac{24}{8}$
$=3$
So, each student gets $\frac{3}{8}$ gram.


English Language Learners:
Differentiated Instruction

## Prepare for Session 2

Use with Apply lt.

## Levels 1-3

## Listening/Speaking Read Apply It

 problem 9 to students. Ask students to underline the phrase into each. Have students discuss why they think that a division expression is needed to solve the problem. Provide a sentence frame to guide students' discussions: I need a division expression because objects are separated, shared, or distributed into equal groups.
## Levels 2-4

## Listening/Speaking Read Apply It

 problem 9 with students. Ask them to state the problem in their own words. In partners, have them decide on the first step they will use to solve the problem.When complete, have students work with another set of partners to compare their first steps. Then ask them to solve the problem individually. Encourage students to justify their answer.

## Levels 3-5

Reading/Writing Have students read Apply It problem 9 individually. Ask students to write the steps that are needed to solve the problem, and to also draw a visual model. Have students trade their steps with another student and compare the steps and visual models they plan to use to solve the problem. Ask them to solve the problem individually.

## LESSON 18 <br> session 2 Develop

LESSON 18

Purpose In this session students solve a problem that requires finding the quotient $5 \div 3$ and representing it as both a fraction and a mixed number. Students model the division either on paper or with manipulatives. The purpose of this problem is to develop strategies for finding quotients that are fractions or mixed numbers.

## Start

## Connect to Prior Knowledge

Materials For each student: Activity Sheet Number Lines

Why Support students' facility with writing fractions greater than 1 as mixed numbers.
How Have students label a number line to show fourths, locate the fractions $\frac{7}{4}$ and $\frac{14}{4}$ on the number line, and then write the equivalent mixed numbers.


Solutions
Check students' number lines.
$\frac{7}{4}=1 \frac{3}{4}$
$\frac{14}{4}=3 \frac{2}{4}$, or $3 \frac{1}{2}$

## Develop Language

Why Review the terms dividend, divisor, and quotient to facilitate student discussions.
How Write the terms on the board and explain that they are used to identify the parts of a division equation. Write a division equation on the board and have students work to label each of the numbers with the term dividend, divisor, or quotient. Discuss and correct any misconceptions. Have students revise their work as needed, and copy the labeled equation onto a class chart for students to use as a reference.

## TRY IT

## Make Sense of the Problem

To support students in making sense of the problem, help them recognize that 3 students are decorating 5 hallways.
Ask The problem asks you to find how much each student will decorate. What does how much mean in this situation? What will be the unit for your answer?

## Develop Fractions as Division



$$
\begin{array}{cr}
\vdots & \text { Sample B } \\
\vdots & 5 \div 3=\frac{5}{3} \\
\vdots & =1 \frac{2}{3}
\end{array}
$$

洨 Each person decorates $1 \frac{2}{3}$ hallways.


## DISCUS5 IT

## Support Partner Discussion

Encourage students to name the model they used as they discuss their solutions.
Support as needed with questions such as:

- How would you describe your model?
- What was it about this problem that made you think of using that model?

Common Misconception Look for students who accurately model the problem but have difficulty identifying what constitutes one equal share from all the equal parts represented. As students present solutions, ask them to identify Jared's share in the model.

## Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models showing 3 groups of 5 thirds
- drawings to represent the problem
- number lines marked in thirds
- equations showing $5 \div 3$ can be represented by $\frac{5}{3}$


## Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask Where does your model show the number of hallways? The part of the hallways Jared decorates? Monica decorates? Heather decorates?

Listen for Students should recognize that accurate representations show 5 wholes divided into thirds, with each student's share identified. Responses may include each student's share is equivalent to $\frac{5}{3}$.

## PICTURE IT \& MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the 5 wholes
- the number of equal parts
- the number of parts decorated by each student

Ask What is the total number of equal parts shown in the fraction model? On the number line? Is it the same or different? Why?
Listen for 15 is the total number of equal parts shown in both the fraction model and on the number line. It is the same number because each model shows 5 wholes divided into thirds, so there are $5 \times 3$ thirds, or 15 thirds in all.

For the fraction model, prompt students to identify how color is used to help represent the problem.

- Why are three different colors used to shade the thirds in each whole?
For the number line model, prompt students to recognize that the same information is shown on both number lines.
- How many thirds are part of Jared's share in the top number line? The bottom number line?
- How is the first number line similar to the fraction model? How is it different?
- Think of each whole on the number line as representing 1 hallway. How does each way of shading the number line show a different way the students can divide up the work?

Explore different ways to understand fractions as quotients
Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?

## PICTURE IT

You can use a fraction model to picture how the students divide up the work.
There are 5 hallways for 3 students to decorate, which is $5 \div 3$.
If they share the work equally, each student can decorate $\frac{1}{3}$ of each hallway.


## MODEL IT

You can use a number line to model each student's share of the work.
The number line is numbered from 0 to 5 because there are 5 hallways. It is divided into thirds because each student can decorate one third of each hallway.


The thirds can be rearranged to show each student's share of the work.


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## Deepen Understanding Remainders <br> SMP 2 Reason abstractly and quantitatively.

When discussing the number line model, have students use a mixed number to identify what is shown as each student's share of the work. [ $1 \frac{2}{3}$ hallways]

Ask If the 3 students were sharing 5 paintbrushes instead of 5 hallways, would it make sense for them to each get $1 \frac{2}{3}$ paintbrushes? How many paintbrushes would each student get? How many paintbrushes would be left over?
Listen for $\frac{2}{3}$ of a paintbrush does not make sense. Each student would get 1 paintbrush with 2 brushes left over.

Prompt students to consider how in the past when they have divided whole numbers that did not divide evenly, they wrote the part left over as a remainder. Ask them how $5 \div 3$ would be written using this method. [1 R 2]
Discuss division situations where they would want to show a quotient as a mixed number, and situations where they would want to show it as a whole number and a remainder.

## LESSON 18

## SESSION 2 Develop

## CONNECT IT

- Remind students that one thing that is alike about all the representations is they show whole-number division that results in a quotient that is a fraction.
- Explain that on this page they will look at two different ways to think about the division and two different ways to show the quotient.


## Monitor and Confirm

(1)-3 Check for understanding that:

- there are 15 thirds in all
- 15 thirds $\div 3=5$ thirds
- the quotient can be written as the fraction $\frac{5}{3}$
- as with whole number division and multiplication, you can write related multiplication and division equations


## Support Whole Class Discussion

4-5 Have students think about modeling the way of dividing up the work described in problem 4.

Ask How would you change the fraction model in Picture It to show this way of dividing up the work? What would a number line model of this way look like?

Listen for In Picture It, each of the first three rectangles would be fully shaded in a single color, pink for Jared, blue for Monica, and green for Heather. On a number line, you could shade from 0 to 1 in pink, from 1 to 2 in blue, and from 2 to 3 in green. For the sections from 3 to 4 and 4 to 5, shade $\frac{1}{3}$ of each section pink, $\frac{1}{3}$ blue, and $\frac{1}{3}$ green.

Ask How would these models show the equation you wrote in problem 5?
Listen for For each person's share of the work you see 1 whole shaded and 2 thirds of the other wholes shaded. That shows $1 \frac{2}{3}$ as one person's equal share of the work.

6 Look for the idea that the bar in a fraction can be interpreted as meaning divided by-the numerator is divided by the denominator-just as the division symbol in an expression does.

## (7) REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

## CONNECT IT

Now you will use the problem from the previous page to help you understand fractions as quotients.
(1) How many thirds of a hallway are there to decorate in 5 hallways?
(2)

How many thirds of a hallway will each student decorate? ...... 5 ......thirds Write this as a fraction..... $\frac{5}{3}$ of a hallway
3 Write a division equation that shows the quotient as a fraction. $5 \div 3=\frac{5}{3}$ Write a multiplication equation to check this equation

(4) Both Picture It and Model It show each student decorating $\frac{1}{3}$ of each hallway. Here is another way to think about dividing up 5 hallways among 3 people.

How many whole hallways can each student decorate? ........ 1
How many hallways remain after those are done? ....... 2
How much of the 2 remaining hallways will each student decorate?
$\frac{2}{3}$
Write a mixed number to show how many hallways each student will decorate. $1 \frac{2}{3}$

## hallways

Write a division equation that shows the quotient as a mixed number. $5 \div 3=1 \frac{2}{3}$
(6)

How does the bar in a fraction represent division?
The bar means that the number in the numerator is divided by the number in the denominator.
(7) REFLECT

Look back at your Try It, strategies by classmates, and Picture It and Model It. Which models or strategies do you like best for finding fraction quotients? Explain. Students may respond that they like using fraction models or number lines to visualize dividing an amount into equal shares, or that they like representing a problem as a division equation that shows the quotient as a fraction.

## Hands-On Activity

## Connect fractions to equivalent division expressions.

If . . . students are unsure about how to interpret a fraction as division,
Then . . . use this activity to rewrite fractions as equivalent division expressions.
Materials For each student: base-ten blocks ( 1 tens rod, 2 ones units), Activity Sheet Digit Cards $(3,4,5)$

- Distribute materials to students. Have students use the digit cards and baseten blocks to "build" the fraction used to solve the Try It problem, $\frac{5}{3}$, using the rod as the fraction bar and placing a digit card for 5 above the rod and a digit card for 3 below it.
- Ask students to alter the fraction they built to show the division expression used to represent the problem, $5 \div 3$, moving the digit cards and using the ones units along with the rod to make a division symbol ( $\div$ ). Discuss where students placed the numerator and denominator to make the expression.
- Repeat activity, using the situation from the Explore Try It: 4 ounces of paint shared equally by 5 students. This time, have students first show the division expression that models the problem and then turn it into the fraction quotient.


## APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; dividing wholes and number lines into equal parts accurately can be very difficult.
(8) $\frac{3}{5}$ of a pack; Students may use a model to see that there are 15 fifths in 3 packs, so each friend gets 3 of the fifths. See possible model on the Student Worktext page. Students may also show a number line divided into fifths marked to show 5 shares of 3 fifths each.
(9) $10 \div 4=\frac{10}{4}$; Each container has $2 \frac{2}{4}$, or $2 \frac{1}{2}$, ounces of apple chips. See possible visual model on the Student Worktext page.

## Close: Exit Ticket

10 c; The expression $12 \div 7$ shows the numerator, 12 , divided by the denominator, 7 .

Error Alert If students choose A, B, or D, then remind students that the bar in a fraction can mean the numerator is divided by the denominator. Have them read the fraction $\frac{12}{7}$, inserting the words divided by for the fraction bar.

## APPLY IT

Use what you just learned to solve these problems.
8) Five friends are equally sharing 3 packs of football cards. How many packs of cards will each friend get? Use a visual model to support your answer.
Possible student work:


## Solution $\quad \frac{3}{5}$ of a pack

(9) Elena made 10 ounces of apple chips. She puts the same amount of apple chips into each of 4 containers. How many ounces of apple chips are in 1 container? Write a division expression to represent the problem and solve. Use a visual model to support your answer.
Possible student work:


$$
10 \div 4=2 \frac{2}{4}
$$

Solution $2 \frac{2}{4}$ or $2 \frac{1}{2}$ ounces of apple chips
(10) Which expression is equivalent to $\frac{12}{7}$ ?
(A) $12-7$
(B) $7-12$
(C) $12 \div 7$
(D) $7 \div 12$

## LESSON 18

## Solutions

1 Number Line A; This number line shows 4 wholes divided into sixths so it can be used to solve $4 \div 6$.

## Basic

(2) The model would change to show 5 equal parts in each rectangle; the answer would change to $\frac{1}{5} \times 4$, or $\frac{4}{5}$ of a package.
Medium

## Practice Fractions as Division

## Study the Example showing whole-number division with a fraction quotient.

 Then solve problems 1-5.
## EXAMPLE

There are 4 packages of printer paper to be divided equally among 6 classrooms. How much paper will each classroom get?

There are 4 packages for 6 classrooms to share, which is $4 \div 6$.
If you divide each package into sixths, each classroom would get one sixth of each package. So, $\frac{1}{6}$ of each package from 4 packages is the same as $\frac{4}{6}$ of a package.


$$
4 \div 6=\frac{4}{6}
$$

Each classroom gets $\frac{4}{6}$ of a package.

1) Circle the number line you would use to solve the problem in the Example.

(2) Look at the Example. Suppose only 5 classrooms share 4 packages. How would the model in the Example change? How would the answer change? Each rectangle would be divided into 5 equal sections instead of 6; the answer would change to $\frac{4}{5}$ of a package.

## Fluency \& Skills Practice Teacher Toolbox s

Assign Fractions as Division


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LESSON 18 SESSION 2
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3. $\frac{7}{3}$, or $2 \frac{1}{3}$ cans; Students may use fraction models or number lines divided into thirds, or the expression $7 \div 3$, to show how much food the dogs will get each day.

## Medium

4 No; See possible explanation on the Student Worktext page.
Challenge
(5) Less than 1 cup; Students may use fraction models divided into sevenths, the expression $48 \div 7$, or some other method to find each person will get $\frac{48}{7}$, or $6 \frac{6}{7}$ ounces. That amount is less than 8 ounces.

## Medium

(3) Trish is taking care of the Han family's dogs. The Hans leave 7 cans of dog food for the 3 days they will be away. How much food will the dogs get each day if Trish feeds them an equal amount each day? Show your work.
Students might use number lines, equations, or some other method to show the quotient of $7 \div 3$.


Solution $\frac{7}{3}$, or $2 \frac{1}{3}$ cans
Raul plans to run 30 miles this week. He wants to run the same number of miles each day of the week. He says he will run $\frac{7}{30}$ mile each day. Is he correct? Explain. No; Possible explanation: Raul divided the number of days by the number of miles, $7 \div 30$. He needed to divide the number of miles by the number of days, $30 \div 7$. Raul will run $\frac{30}{7}$, or $4 \frac{2}{7}$, miles a day.

Gus makes 48 fluid ounces of spiced cider. If he serves an equal amount to each of 7 people, will each person get more than 1 cup of cider or less than 1 cup? ( 1 cup $=8$ fluid ounces) Show your work.

Students might use fraction models, equations, or some other method to show that $48 \div 7=\frac{48}{7}$ or $6 \frac{6}{7} \cdot 6 \frac{6}{7}$ ounces $<8$ ounces

Solution less than 1 cup

English Language Learners: Differentiated Instruction

## Prepare for Session 3

Use with Apply It.

## Levels 1-3

Listening/Speaking Read Apply It problem 1 to students. Ask them to point to the words equal amount. Ask students to discuss with a partner what operation they will need to solve the problem and why.
Provide the following sentence frame: When you break up a bigger number into equal smaller amounts, you need to divide .

## Levels 2-4

Reading/Writing Read Apply It problem 1. Have students read the problem with a partner and work together to decide on a strategy to use to solve for the amount of space Erica will give each vegetable.
Have students work together to solve the problem using the strategy they chose.

## Levels 3-5

Reading/Writing Have students read Apply It problem 1 with a partner and work together to decide on a strategy to use to solve for the amount of space Erica will give each vegetable. Ask partners to solve the problem.

Have students work with a different group and discuss the different approaches each group selected to solve the problem.
Provide a sentence starter and encourage students to justify their thinking: We know our answer is correct because $\qquad$ .

## LESSON 18 <br> SESSION 3 Refine

Purpose In this session students solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the Check for Understanding to determine those who will benefit from additional support.
As students complete the Example and problems 1-3, observe and monitor their reasoning to identify groupings for differentiated instruction.

## Start

## Check for Understanding

Materials For remediation: Activity Sheet Fraction Bars ( 7 bars for thirds) 3 sheets of paper, scissors
Why Confirm understanding of fractions as division.
How Have students solve the problem using any strategy and write a division equation to show the solution.


## Solution

$\frac{7}{3}$, or $2 \frac{1}{3}$, lawns each;
$7 \div 3=\frac{7}{3}$, or $2 \frac{1}{3}$

Refine Fractions as Division

Complete the Example below. Then solve problems 1-9.

## EXAMPLE

Luke, Carter, and Ava have $\mathbf{2}$ quarts of juice. They want to share it equally. How many quarts of juice will each of them get?

Look at how you could show your work using a model and equations.


$$
\begin{aligned}
2 \div 3 & =2 \times \frac{1}{3} \\
& =\frac{2}{3}
\end{aligned}
$$

Solution $\frac{2}{3}$ quart

## APPLY IT

(1) Erica has 7 square feet of space in her rectangular garden to plant carrots, beans, peppers, and lettuce. Suppose she gives each vegetable an equal amount of space. How much space will each vegetable get?
Show your work.
Possible student work using a model:


Solution $\frac{7}{4}$, or $1 \frac{3}{4}$, square feet

2 quarts are shared equally by 3 friends, sol know that each friend will have less than 1 quart of juice. That means the quotient is a fraction.

## PAIR/SHARE

Model the problem for 3 quarts of juice divided equally among Luke, Carter, Ava, and Ava's little brother.

Each vegetable will get at least 1 square foot of garden space. How will the rest of the space be divided up?

## PAIR/SHARE

What are some ways you can check your solution?

## Error Alert

If the error is ... |  | Students may ... |
| :---: | :---: |
|  |  |
|  |  |
| $3 \div 7$ or $\frac{3}{7}$ | have reversed the numerator | and the denominator.

$$
7 \times 3=21,
$$

$$
7-3=4
$$

$$
\text { or } 7+3=10
$$

not understand that they need to divide to solve the problem.

## To support understanding ...

First, have students solve a simpler problem: 3 students mow 6 lawns and share the work equally. How many lawns does each person mow?
[ $6 \div 3=2$ ] Then have students apply this same reasoning to the given problem.

Have students cut out fraction bars to model 7 lawns and label a sheet of paper with the name of each person in the problem. Ask: How many lawns does each person get? Have students share the lawns equally by placing fraction bars or equal portions of fraction bars on each sheet of paper. Point out that this is an equal sharing problem and guide students to recall that division is used for equal sharing.

## EXAMPRLE

$\frac{2}{3}$ quart; the fraction model and equations show one way to solve the problem. Students could also solve the problem by drawing a number line from 0 to 2 that is divided into 3 equal sections of $\frac{2}{3}$. Look for You divide 2 wholes into 3 equal shares. When the divisor is greater than the dividend, the quotient is a fraction.

## APPIY IT

(1) $\frac{7}{4}$, or $1 \frac{3}{4}$ square feet; Students could solve the problem by drawing a model showing 1 square foot of the garden given to each of the 4 vegetables, and the remaining 3 square feet of garden divided into fourths, with a total of 3 fourths given to each vegetable.

## DOK 2

Look for The solution is a mixed number since there is enough garden space for each vegetable to get at least 1 square foot of space but not enough for each to get 2 square feet.

2 Between 3 and 4 ounces; See possible work on the Student Worktext page. Students could also solve the problem by reasoning that there are 6 sixths in 1 whole, so there are 18 sixths in 3 wholes and 24 sixths in 4 wholes.

## DOK 2

Look for The solution requires two steps: the first is to find the mixed number quotient of $20 \div 6$ and the second is to identify the two whole numbers that the quotient falls between.

3 D; Students could solve the problem using the equation $21 \div 10=\frac{21}{10}$.
Explain why the other two answer choices are not correct:
B is not correct because $\frac{21}{10}=2 \frac{1}{10}$, not $1 \frac{1}{10}$. C is not correct because $\frac{21}{10}>2$.
DOK 3
(2) Deon needs to make 6 pizza crusts. He has 20 ounces of dough and wants to use the same amount of dough for each crust. He weighs a portion of dough for 1 crust on a scale. The weight, in ounces, should fall between what two whole numbers? Show your work.
Possible student work using an equation:
20 ounces $\div 6$ pizza crusts $=\frac{20}{6}$ or $3 \frac{2}{6}$ ounces per pizza crust $3 \frac{2}{6}$ is between 3 and 4 .

Solution between 3 and 4 ounces
(3) Jonas is doing a science experiment with his class. The teacher has 21 fluid ounces of pond water to share equally among 10 pairs of students. How much pond water will Jonas and his science partner receive?
(A) $\frac{10}{21}$ fluid ounce
(B) $1 \frac{1}{10}$ fluid ounces
© 2 fluid ounces
(D) $\frac{21}{10}$ fluid ounces

Olivia chose (A) as the correct answer. How did she get that answer?
Possible explanation: She wrote a fraction to show 10 divided by 21 instead of a fraction to show 21 divided by 10.

How many whole ounces of dough will each crust get? What will
happen with the remaining
ounces?

PAIR/SHARE
Create a different division story to represent $\frac{20}{6}$

About how much water will each pair of students receive? Will it be more or less than 2 fluid ounces?


## PAIR/SHARE

 sense?Does Olivia's answer make

## LESSON 18 <br> SEsSION 3 Refine

(4) D; 32 fluid ounces $\div 5=\frac{32}{5}$, or $6 \frac{2}{5}$ fluid ounces. $6 \frac{2}{5}$ is between 6 and 7 .
DOK 2
(5)

B; Each lap takes almost 1 minute.
8 minutes $\div 10=\frac{8}{10}$ minute.
DOK 2
(6) A (Yes);

D (No);
E (Yes);
H (No);
I (Yes)
DOK 2
Error Alert Students may not recognize that the board length can be expressed as a multiplication expression.

Teddy makes 32 fluid ounces of hot cocoa. He pours equal amounts of cocoa into 5 cups. The amount of hot cocoa in each cup will fall between which two amounts?
(A) 3 and 4 fluid ounces
(B) 4 and 5 fluid ounces
(C) 5 and 6 fluid ounces
(D) 6 and 7 fluid ounces

Pierce swims 10 laps in a pool in 8 minutes. He spends the same amount of time on each lap. How much time does each lap take him?
(A) $\frac{2}{10}$ minute
(B) $\frac{8}{10}$ minute
(C) $\frac{10}{8}$ minutes
(D) $1 \frac{2}{8}$ minutes

Dani needs 8 equal sections from a board that is 13 meters long. Does the expression represent the largest possible length of 1 section of the board, in meters?

|  | Yes | No |
| :---: | :---: | :---: |
| 1 $\frac{5}{8}$ | (A) | (B) |
| $\frac{8}{13}$ | ( | (D) |
| $\frac{13}{8}$ | (E) | ® |
| $8 \div 13$ | ( ${ }^{\text {c }}$ | (H) |
| $13 \times \frac{1}{8}$ | (I) | (3) |

## RETEACH

## Hands-On Activity

Model dividing two whole numbers with a fraction quotient.
Students struggling with the concept of dividing a lesser number by a greater number Will benefit from additional work with concrete models of fraction quotients
Materials For each student: 8 squares of paper, each one a $3 \times 3$ array of squares cut from Activity Sheet 1-Inch Grid Paper, scissors

- Pose the following problem: 7 cups of juice are shared equally among 9 students. How much juice does each student get?
- Have students use 7 paper squares to model 7 cups. Set aside the remaining square.
- Have students shade $\frac{1}{9}$ in each square to show that each student gets $\frac{1}{9}$ from each cup.
- Have students cut out the shaded $\frac{1}{9}$ s and reassemble them on the remaining square to show how much juice one student gets. $\left[\frac{7}{9}\right.$ cup $]$
- Reassemble the squares and repeat the activity for numbers of cups less than 7, such as 3 cups and 5 cups, and sharing them equally among 9 students.


## EXTEND

## Challenge Activity

Model fractions as quotients.
Students who have achieved proficiency Will benefit from deepening understanding of fractions as division

- In groups of 3, students pass their work to the next person after each round.
- Round 1: Each student writes a division word problem in which the quotient is a fraction or a mixed number.
- Round 2: Students draw a visual model to represent the problem they receive.
- Round 3: Students write an equation that represents the model they receive.
- The group checks each other's work.
(1) A; Divide 25 meters of paper by the number of banners, 9 , to find how much paper is used for each banner.
E; Divide 25 ounces by the number of equal servings, 9 , to find how many ounces in each serving.
DOK 2


## 8 Part A

See possible number line on the Student Worktext page.

## Part B

See possible explanation on the Student Worktext page. Students may also explain that to model $7 \div 2$, you can show 7 divided into 2 equal parts. Each part is equal to $3 \frac{1}{2}$, which is equivalent to $\frac{7}{2}$.

## DOK 3

(7) Which situations can be represented by $\frac{25}{9}$ ?
(A) Melanie equally shares 25 meters of paper to make 9 banners.
(B) Quill gives away 9 baseball cards from a pack of 25 cards.
© George invites 25 kids and 9 adults to his birthday party.
(D) Becca makes 9 rows with 25 buttons each.
(E)Joe makes 9 equal servings from a 25 -ounce bag of peanuts.
(8) Paco is trying to explain to his friend that $7 \div 2=\frac{7}{2}$.

Part A Draw a model or number line showing $7 \div 2=\frac{7}{2}$.
Possible student work using a number line:

$7 \div 2=\frac{7}{2}$
$=3 \frac{1}{2}$
Part B Explain the equivalence of $7 \div 2$ and $\frac{7}{2}$ using words.
Possible explanation: Seven halves is equivalent to seven divided into two equal sections because both equal three and one half.

MATH JOURNAL
Write a division word problem that can be represented by the expression $12 \div 5$. Then explain how to solve your problem.
Possible problem: Five friends want to share 12 ounces of juice
equally. How many ounces of juice should each friend get?
Possible explanation: You can solve the equation $12 \div 5=n$ :
$n=\frac{12}{5}$. Each friend gets $\frac{12}{5}$, or $2 \frac{2}{5}$, ounces of juice.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

## REINFORCE

## Problems 4-9

## Interpret fractions as division.

All students will benefit from additional work with using fractions to represent division of whole numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.


## PERSONALIZE

## i-Ready

Provide children with opportunities to work on their personalized instruction path with i-Ready Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills


## Close: Exit Ticket

## (9) MATH JOURNAL

Student responses should include a word problem with 12 as the number of wholes to be shared and 5 as the number of equal shares. Students should explain that the quotient $12 \div 5$ can be represented by the fraction $\frac{12}{5}$.
Error Alert If students reverse the numerator and denominator in the fraction quotient, then have them use reasoning to determine which two whole numbers the quotient of $12 \div 5$ falls between and assess which of the two possible fractions, $\frac{12}{5}$ or $\frac{5}{12^{\prime}}$ is between those two numbers.
$\checkmark$ SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener page.

## LESSON 18

## Lesson Ouiz Traserforobax.

## Lesson 18 Quiz

## Tested Skills

## Assesses 5.NF.B. 3

Problems on this assessment form require students to be able to use visual fraction models to represent a fraction as division, solve word problems involving division of whole numbers in which the quotient is a fraction or mixed number, and write equations and expressions to represent word problems involving the division of whole numbers with a fractional quotient. Students will also need to be familiar with whole number division and writing fractions greater than 1 as mixed numbers and improper fractions.

Error Alert Students may:

- confuse the dividend and divisor or what each number in the problem represents.
- reverse the numerator and denominator.
- interpret an addition, subtraction, or multiplication situation as a division situation.


## Solutions

1 A (Yes);
D (No);
E (Yes);
H (No);
I (Yes);
K (Yes)
6 points
DOK 2
2 A, F
2 points
DOK 2

## Solve the problems.

1 Sara will use 7 cups of apples to make 4 batches of applesauce. Decide whether each expression shows the number of cups of apples in one batch.

Choose Yes or No for each expression.

|  | Yes | No |
| :---: | :---: | :---: |
| $7 \div 4$ | (A) | (B) |
| $4 \div 7$ | ( | (D) |
| $\frac{7}{4}$ | (E) | © |
| $\frac{4}{7}$ | (a) | $\oplus$ |
| $7 \times \frac{1}{4}$ | (1) | (3) |
| 1 $\frac{3}{4}$ | ® | (L) |

2 Which of the following situations can be represented by $\frac{14}{5}$ ? Select all the correct answers.
(A) Renee has 14 feet of ribbon that she will cut into 5 pieces of equal length.
(B) Michael has 14 packs of trading cards with 5 cards in each pack.
(C) Logan opens 5 bags of trail mix, which she will split equally into 14 bowls.
(D) Patrick takes 5 oranges from a bag containing 14 oranges.
(E) Tim walks 14 blocks from the library to a friend's house and then walks another 5 blocks to home.
(F) Arianna pours 5 equal servings of lemonade from a bottle containing 14 ounces.

| Short Response Scoring Rubric (2 points) |  |
| :---: | :--- |
| Points | Expectations |
| $\mathbf{2}$ | Response has the correct solution(s) and includes <br> well-organized, clear, and concise work demonstrating <br> thorough understanding of mathematical concepts <br> and/or procedures. |
| $\mathbf{1}$ | Response contains mostly correct solution(s) and <br> shows partial understanding of mathematical <br> concepts and/or procedures. |
| $\mathbf{0}$ | Response shows no attempt at finding a solution <br> and no effort to demonstrate an understanding of <br> mathematical concepts and/or procedures. |


| Multiple Select Scoring |  |  |
| :---: | :---: | :---: |
| $\mathbf{2}$ points | $\mathbf{1}$ point | $\mathbf{0}$ points |
| All answers <br> are correct | 1 incorrect answer | 2 or more <br> incorrect answers |

Teacher pages have been reduced. Actual book size is 10 1/4" $\times 12^{\prime \prime}$.
$3 \frac{3}{5}$ gallon 2 points DOK 2

4 Part A
A
1 point

## Part B

Possible answer: $7 \div 3=\frac{7}{3}$
1 point

## Part C

$\frac{7}{3}$, or $2 \frac{1}{3}$, pounds of peaches 1 point
DOK 2

## Lesson 18 Quiz continued

Ava pours 3 gallons of paint equally into 5 cans. How many gallons of paint will there be in each can? Show your work.
## ___ gallon(s)

The model shows how Micah is dividing some pounds of peaches to make three pies.

1 pound

1 pound

1 pound

1 pound

1 pound

1 pound

1 pound

## Part A

Which statement is true?
(A) Micah is dividing 7 pounds of peaches into 3 equal groups.
(B) Micah is dividing 3 pounds of peaches into 7 equal groups.
(C) Micah is dividing 7 pounds of peaches into groups of $\frac{1}{3}$ pound.
(D) Micah is dividing 3 pounds of peaches into groups of $\frac{1}{3}$ pound.

Part B
Write a division equation that represents how Micah divides the peaches.

Part C
How many pounds of peaches will Micah use in each pie?
___ pounds of peaches

## Differentiated Instruction Teacher Toolbox

## RETEACH

## Tools for Instruction

Students who require additional support for prerequisite or on-level skills
Will benefit from activities that provide targeted skills instruction

- Grade 5, Lesson 18


## REINFORCE

## Math Center Activities

Students who require additional practice to reinforce concepts and skills and deepen understanding Will benefit from small group collaborative games and activities (available in three versions-on-level, below-level, and above-level)

- Grade 5, Lesson 18


## EXTEND

## Enrichment Activities

Students who have achieved proficiency with concepts and skills and are ready for additional challenges
Will benefit from group collaborative games and activities that extend understanding

- Grade 5, Lesson 18


## Curriculum Associates

## sampler


[^0]:    *We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

