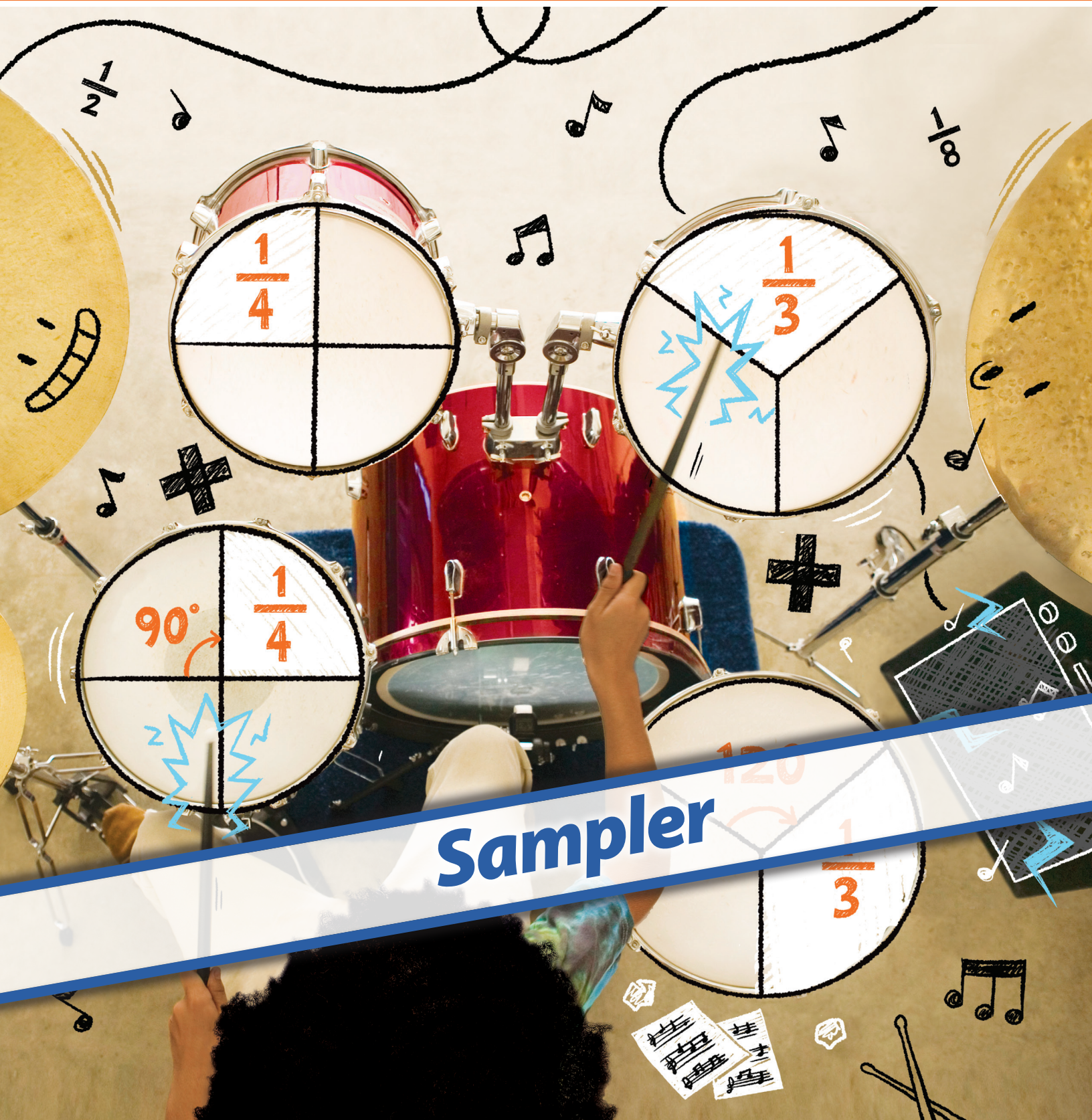
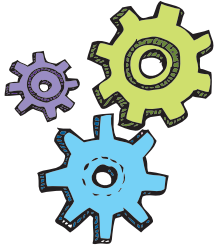


Mathematics



Sampler

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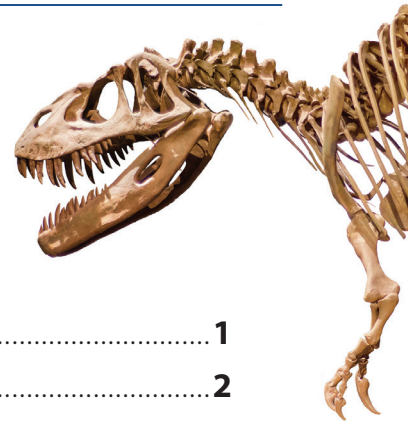


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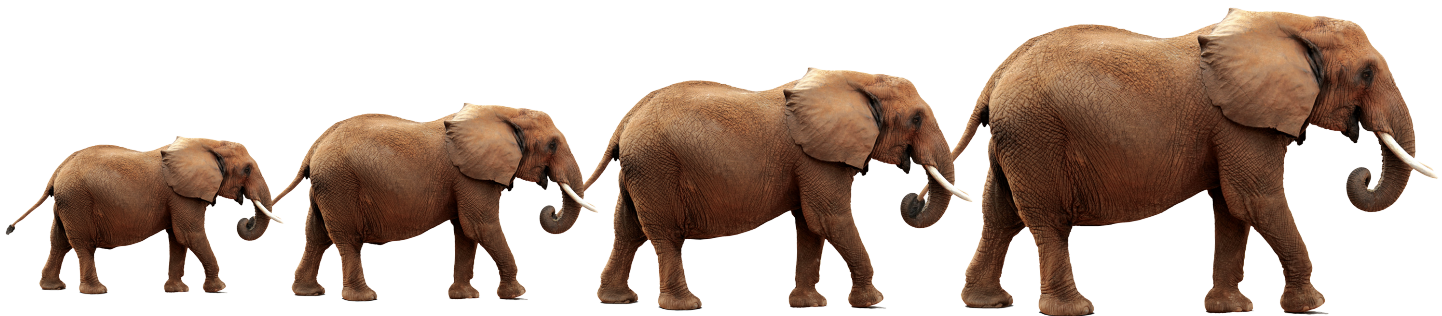
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..... UNIT
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Student Sample, Lesson 20

Ready Classroom Mathematics lessons begin with a Family Letter that provides activities and instructional supports to foster school, family, and community involvement and partnerships.

Each multiday lesson includes three types of sessions: Explore, Develop, and Refine.

The following pages, 413–440, represent a complete student lesson.

Add and Subtract Fractions



Dear Family,

This week your child is learning how to add and subtract fractions with like denominators.

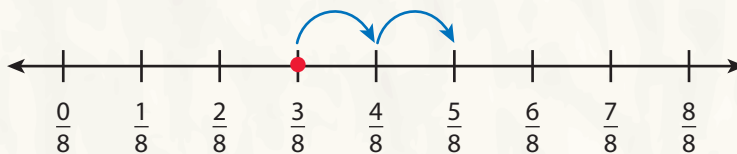
Fractions with the same number below the line have like denominators.

like denominators: $\frac{1}{4}$ and $\frac{3}{4}$ unlike denominators: $\frac{1}{2}$ and $\frac{3}{4}$

To find the sum of fractions with like denominators, understand that you are adding like units. Just as 3 apples plus 2 apples is 5 apples, 3 eighths plus 2 eighths is 5 eighths. Similarly, when you take away, or subtract, 2 eighths from 5 eighths, you have 3 eighths left.



You can also use a number line to understand adding and subtracting like fractions.



Remember that the denominator names units the same way that “apples” names units.

So, when you add two fractions with like denominators, the sum of the numerators tells how many of those units you have.

When you subtract two fractions with like denominators, the difference of the numerators tells how many of those units you have.

Invite your child to share what he or she knows about adding and subtracting fractions by doing the following activity together.

ACTIVITY ADDING AND SUBTRACTING FRACTIONS

Do this activity with your child to add and subtract fractions.

Materials bowl, measuring cup, ingredients shown in the recipe

Follow the recipe below to make a creamy cracker spread or veggie dip.

Creamy Spread

Ingredients

$\frac{5}{8}$ cup cream cheese

$\frac{2}{8}$ cup sour cream

herbs

crackers or veggies

Directions

Mix the cream cheese, sour cream, and herbs together in a medium bowl. Serve immediately with crackers or sliced fresh veggies. Enjoy!



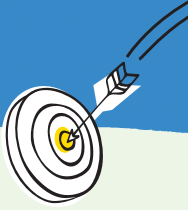
After you have made the spread, ask your child questions such as these:

- What fraction of a cup is the total amount of spread?
- If you spread $\frac{1}{8}$ of a cup on crackers or veggies, how much spread is left?

Make up a simple recipe using fractions for someone else in the family to make!



Explore Adding and Subtracting Fractions



Learning Targets

- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

Previously, you learned that adding fractions is similar to adding whole numbers. Use what you know to try to solve the problem below.

Lynn, Paco, and Todd share a pack of 12 cards. Lynn gets 4 cards, Paco gets 3 cards, and Todd gets the rest of the cards. What fraction of the pack does Todd get?



TRY IT



Math Toolkit

- counters
- fraction circles
- fraction tiles
- fraction bars
- number lines



DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: At first, I thought . . .

CONNECT IT

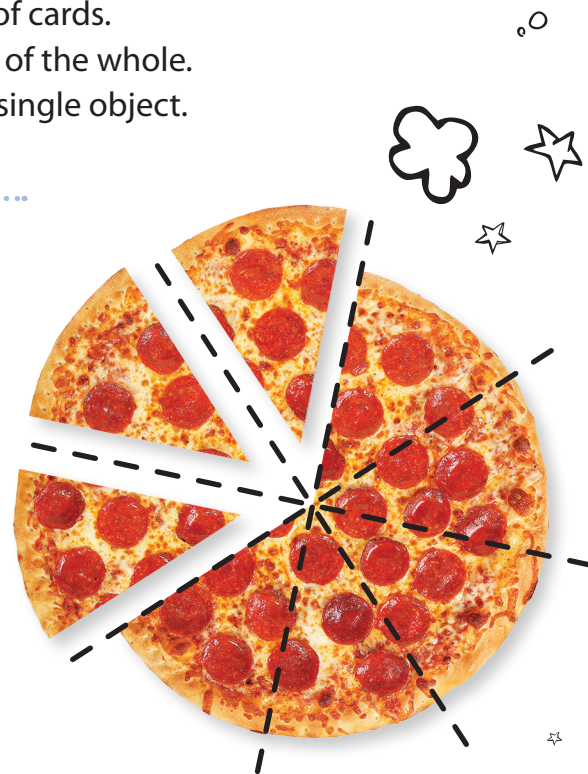
1 LOOK BACK

Explain how you can find the fraction of the pack that Todd gets.

2 LOOK AHEAD

In the problem on the previous page, the whole is the pack of cards. Since there are 12 cards in the pack, each card represents $\frac{1}{12}$ of the whole. Look at the whole shown here. The whole is the pizza. It is a single object.

- a. How many equal parts are shown in the pizza?
- b. What fraction can you use to describe each piece of pizza?
- c. What fraction can you use to describe the whole?
- d. What fraction can you use to describe the 3 pieces being taken away?



3 REFLECT

How does knowing about equal parts help you add and subtract fractions?

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Prepare for Adding and Subtracting Fractions

- 1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

<p>What Is It?</p>	<p>What I Know About It</p>	
<p>fraction</p>		
<p>Examples</p>	<p>Examples</p>	<p>Examples</p>

- 2 Does the model below show eighths? Why or why not?

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------

3 Solve the problem. Show your work.

Maria, Jon, and Kara share a set of 10 animal stickers. Maria gets 2 stickers, Jon gets 4 stickers, and Kara gets the rest of the stickers. What fraction of the stickers does Kara get?



Solution

4 Check your answer. Show your work.

Develop Adding Fractions

Read and try to solve the problem below.

Josie and Margo are painting a fence green. Josie starts at one end and paints $\frac{3}{10}$ of the fence. Margo starts at the other end and paints $\frac{4}{10}$ of it. What fraction of the fence do they paint altogether?

TRY IT



Math Toolkit

- fraction circles
- fraction tiles
- fraction bars
- number lines
- index cards



DISCUSS IT

Ask your partner: How did you get started?

Tell your partner:
A model I used was ...
It helped me ...

Explore different ways to understand adding fractions.

Josie and Margo are painting a fence green. Josie starts at one end and paints $\frac{3}{10}$ of the fence. Margo starts at the other end and paints $\frac{4}{10}$ of it. What fraction of the fence do they paint altogether?

PICTURE IT

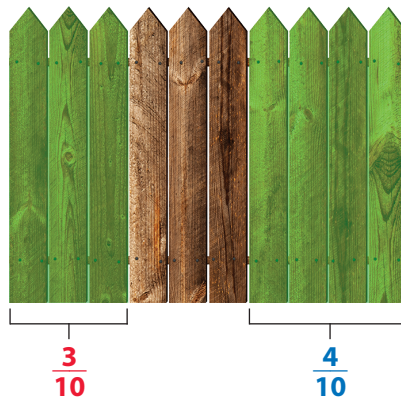
You can use a picture to help understand the problem.

Think what the fence might look like if it has 10 equal-sized parts.



Each part is $\frac{1}{10}$ of the whole.

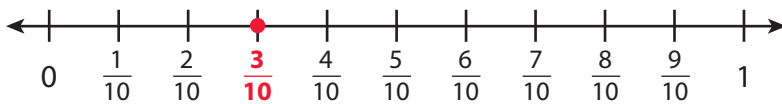
The girls paint 3 tenths and 4 tenths of the fence.



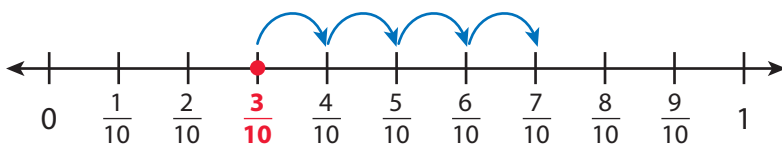
MODEL IT

You can also use a number line to help understand the problem.

The number line below is divided into tenths with a point at $\frac{3}{10}$.



Start at $\frac{3}{10}$ and count 4 tenths to the right to **add** $\frac{4}{10}$.



CONNECT IT

Now you will use the problem from the previous page to help you understand how to add any two fractions that have the same denominator.

1 Look at **Picture It**. How do you know that each section of fence is $\frac{1}{10}$ of the whole fence?

2 What do the numerators, 3 and 4, tell you?

3 How many tenths of the fence do Josie and Margo paint altogether?

4 Complete the equations to show what fraction of the fence Josie and Margo painted altogether.

Use words: **3 tenths** + **4 tenths** = tenths

Use fractions: $\frac{3}{10}$ + $\frac{4}{10}$ = $\frac{\square}{10}$

5 What would be the sum if the fractions were $\frac{3}{10}$ and $\frac{5}{10}$?

6 Explain how you add fractions that have the same denominator.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for adding fractions? Explain.

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APPLY IT

Use what you just learned to solve these problems.

- 8 Lita and Otis help their mom clean the house. Lita cleans $\frac{1}{3}$ of the house. Otis cleans $\frac{1}{3}$ of the house. What fraction of the house do Lita and Otis clean altogether? Show your work.

Solution

- 9 Mark and Imani use string for a project. Mark's string is $\frac{1}{5}$ of a meter long. Imani's string is $\frac{3}{5}$ of a meter long. How long are the two strings combined? Show your work.

..... of a meter

- 10 Paola makes a fruit smoothie. She uses $\frac{2}{8}$ of a pound of strawberries and $\frac{4}{8}$ of a pound of blueberries. How many pounds of fruit does she use? Show your work.

Solution



Practice Adding Fractions

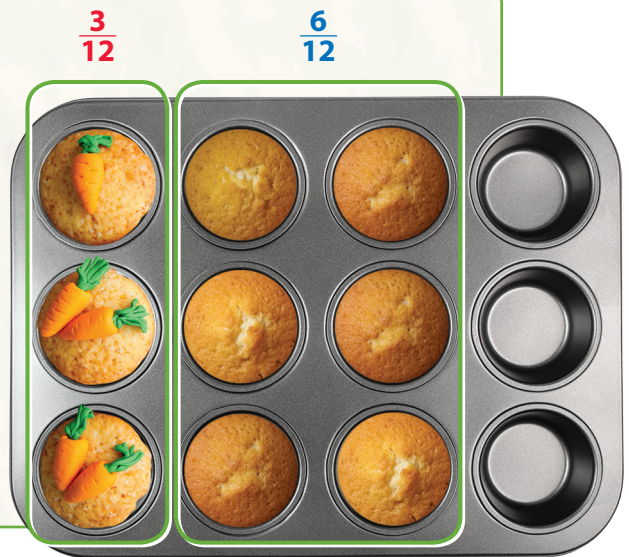
Study the Example showing one way to add fractions.
Then solve problems 1–9.

EXAMPLE

Shrina has a muffin pan that holds 12 muffins.
She fills $\frac{3}{12}$ of the pan with carrot muffin batter.
Then she fills $\frac{6}{12}$ with pumpkin muffin batter.
What fraction of the pan does she fill?

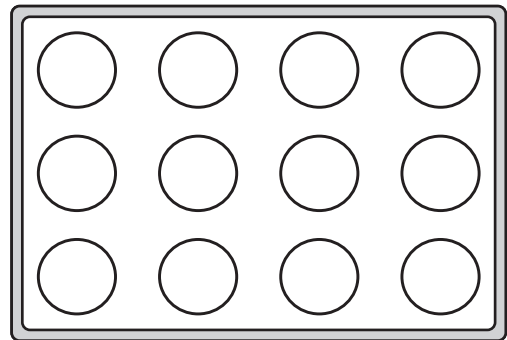
$$\frac{3}{12} + \frac{6}{12} = \frac{9}{12}$$

So, she fills $\frac{9}{12}$ of the muffin pan.



- 1 Sam fills $\frac{2}{12}$ of another pan with banana muffin batter.
Shade $\frac{2}{12}$ of the muffin pan diagram at the right.

- 2 Then Sam fills $\frac{6}{12}$ with lemon muffin batter.
Shade $\frac{6}{12}$ of the diagram to show this.

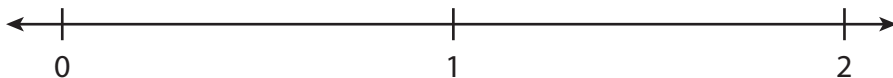


- 3 In problem 2, what fraction of the pan in all is filled now?
Write an equation for this problem that includes your answer.

Kay runs $\frac{6}{8}$ of a mile and rests. Then she runs another $\frac{6}{8}$ of a mile.



- 4 Divide the number line below to show eighths.



- 5 Label $\frac{6}{8}$ on the number line above.

- 6 Use arrows to show $\frac{6}{8} + \frac{6}{8}$ on the number line.

- 7 What is the total distance Kay runs?

- 8 Write an equation for this problem that includes your answer.

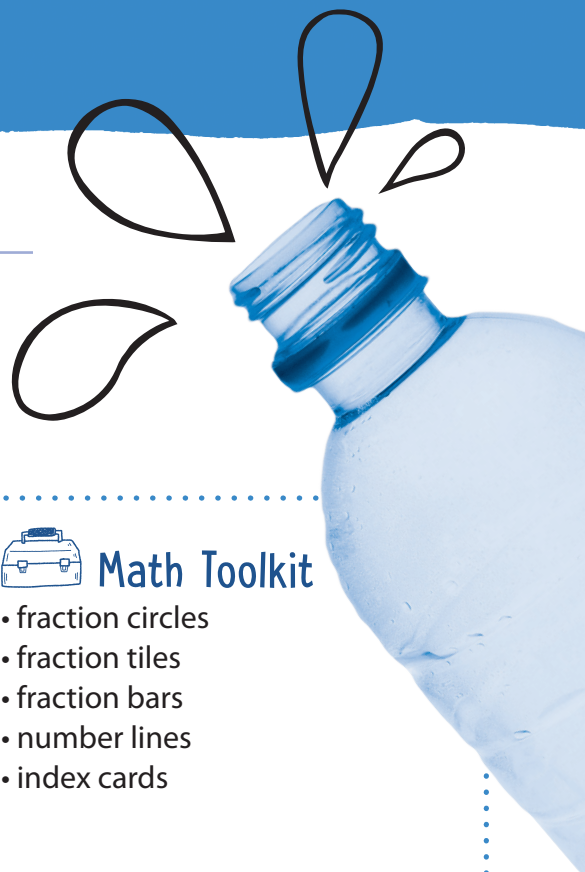
- 9 Jin cleans $\frac{1}{10}$ of the patio before lunch and $\frac{9}{10}$ of the patio after lunch. What fraction of the patio does Jin clean altogether? Show your work.

Solution

Develop Subtracting Fractions

Read and try to solve the problem below.

Alberto's water bottle has $\frac{5}{6}$ of a liter of water in it.
He drinks $\frac{4}{6}$ of a liter. What fraction of a liter of water
is left in the bottle?



TRY IT



Math Toolkit

- fraction circles
- fraction tiles
- fraction bars
- number lines
- index cards



DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I disagree with this part because . . .

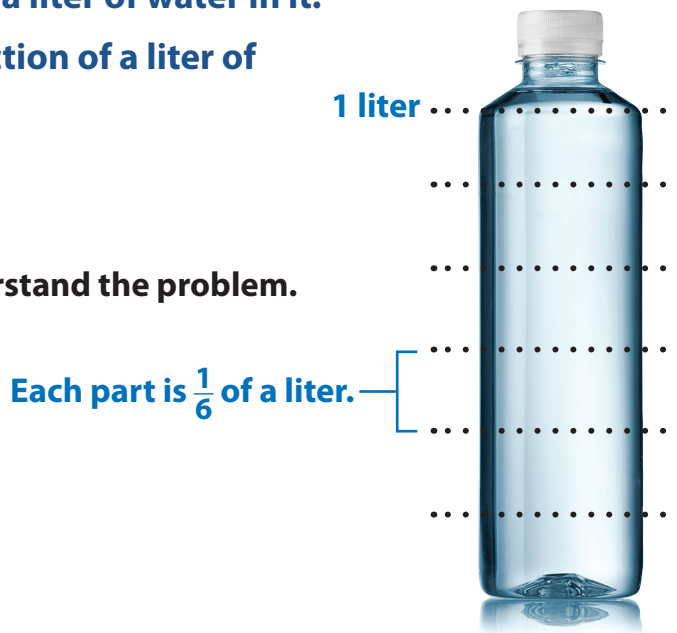
Explore different ways to understand subtracting fractions.

Alberto's water bottle has $\frac{5}{6}$ of a liter of water in it. He drinks $\frac{4}{6}$ of a liter. What fraction of a liter of water is left in the bottle?

PICTURE IT

You can use a picture to help understand the problem.

The picture shows the whole liter divided into 6 equal parts.



Five shaded parts show how much water is in the bottle. Alberto drinks 4 sixths of a liter, so take away 4 shaded parts. The 1 shaded part that is left shows the fraction of a liter that is left.



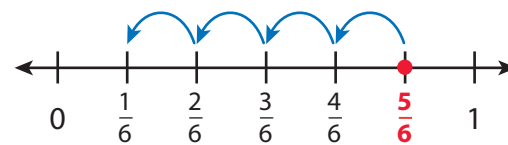
MODEL IT

You can also use a number line to help understand the problem.

The number line at the right is divided into sixths, with a point at $\frac{5}{6}$.



Start at $\frac{5}{6}$ and count back 4 sixths to subtract $\frac{4}{6}$.



CONNECT IT

Now you will use the problem from the previous page to help you understand how to subtract any two fractions that have the same denominator.

1 In **Picture It**, why does $\frac{1}{6}$ represent 1 of the equal parts of the liter?

2 What do the numerators, 5 and 4, tell you?

3 How many sixths of a liter are left in the bottle after Alberto drinks 4 sixths?

4 Complete the equations to show what fraction of a liter is left in the bottle.

Use words: **5 sixths** − **4 sixths** = sixth

Use fractions: $\frac{5}{6}$ − $\frac{4}{6}$ = $\frac{\square}{6}$

5 Explain how you subtract fractions with the same denominator.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for subtracting fractions? Explain.

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APPLY IT

Use what you just learned to solve these problems.

- 7 Carmen has $\frac{8}{10}$ of the lawn left to mow. She mows $\frac{5}{10}$ of the lawn. Now what fraction of the lawn is left to mow? Show your work.



Solution

- 8 Mrs. Kirk has $\frac{3}{4}$ of a carton of eggs. She uses some for baking and has $\frac{2}{4}$ of the carton left. What fraction of the carton does she use? Show your work.

Solution

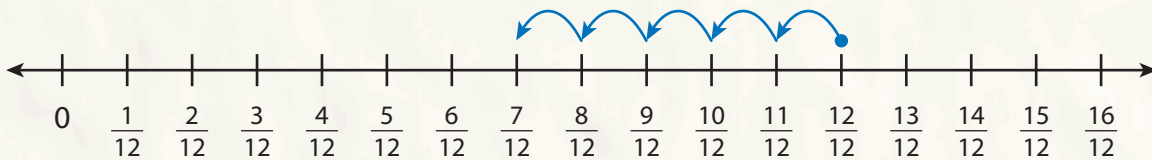
- 9 Badru reads $\frac{4}{8}$ of a book. How much of the book does he have left to read?
- (A) $\frac{1}{8}$
 - (B) $\frac{2}{8}$
 - (C) $\frac{4}{8}$
 - (D) $\frac{6}{8}$

Practice Subtracting Fractions

Study the Example showing one way to subtract fractions.
Then solve problems 1–7.

EXAMPLE

Ali buys a carton of eggs. He uses $\frac{3}{12}$ of the eggs to cook breakfast. He uses another $\frac{2}{12}$ to make a dessert for dinner. What fraction of the carton is left?



$$\frac{12}{12} - \frac{3}{12} = \frac{9}{12}$$

$$\frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

So, $\frac{7}{12}$ of the carton is left.



Keisha is at her friend's house. Her friend's house is $\frac{8}{10}$ of a mile from Keisha's home. Keisha walks $\frac{3}{10}$ of a mile toward home. Then her mother drives her the rest of the way home.

- 1 Divide the number line below to show tenths. Then label each tick mark.



- 2 Use arrows to show the problem on the number line you labeled in problem 1.
- 3 How far does Keisha's mother drive her?
- 4 Write an equation for this problem that includes your answer.

- 5 Anna makes a quilt by sewing together green, white, and yellow fabric. When she finishes, $\frac{2}{6}$ of the quilt is green, and $\frac{3}{6}$ is yellow. The rest is white. What fraction of the quilt is white? Show your work.

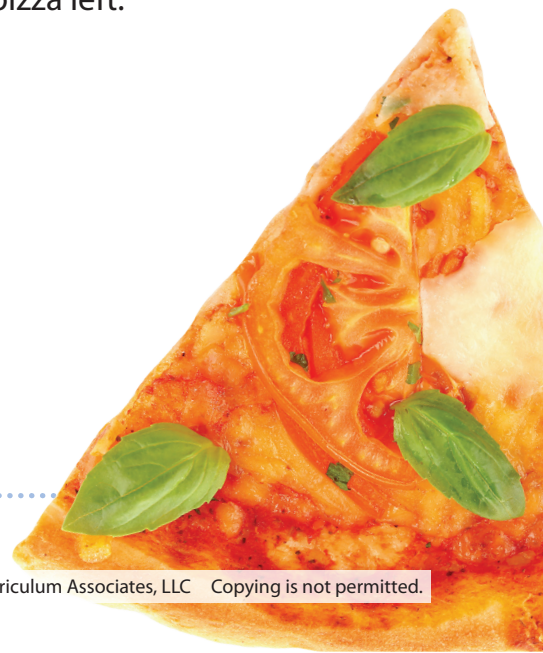
Solution

- 6 Find $\frac{9}{8} - \frac{8}{8}$.
Use a number line or an area model to show your thinking.

Solution

- 7 Shanice has 1 whole pizza. She eats some of it and has $\frac{4}{6}$ of the pizza left. What fraction of the pizza does she eat? Show your work.

Solution



Develop Decomposing Fractions

Read and try to solve the problem below.

Dan has $\frac{5}{6}$ of his reading left to complete for the week. He plans to complete his reading on two or more days of the week from Monday to Friday. What are two different ways he could plan to complete his reading? Use a fraction to describe the part of his reading he does each day.



TRY IT



Math Toolkit

- counters
- fraction circles
- fraction tiles
- fraction bars
- number lines



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I do not understand how ...

Explore different ways to understand decomposing fractions.

Dan has $\frac{5}{6}$ of his reading left to complete for the week. He plans to complete his reading on two or more days of the week from Monday to Friday. What are two different ways he could plan to complete his reading? Use a fraction to describe the part of his reading he does each day.

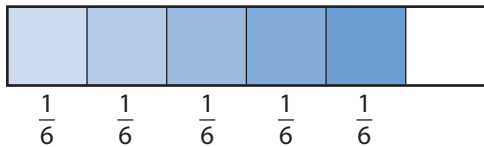
MODEL IT

You can use models to show how to decompose a fraction in different ways.

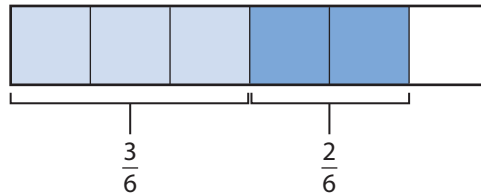
When you decompose a fraction, you break it into parts.

The models show two ways to decompose $\frac{5}{6}$.

One way:



Another way:



MODEL IT

You can also use equations to decompose a fraction in different ways.

You can list different ways to add fractions to make $\frac{5}{6}$.

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{2}{6}$$

$$\frac{5}{6} = \frac{1}{6} + \frac{2}{6} + \frac{2}{6}$$

$$\frac{5}{6} = \frac{3}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\frac{5}{6} = \frac{3}{6} + \frac{2}{6}$$

$$\frac{5}{6} = \frac{4}{6} + \frac{1}{6}$$

CONNECT IT

Now you will use the problem from the previous page to help you understand how to decompose a fraction in different ways.

- 1 Look at the first **Model It**. How many equal parts are in each model?
How many shaded parts are in each model?
- 2 Look at the equations in the second **Model It**. How can you tell if two or more fractions add to make $\frac{5}{6}$?
- 3 What is the greatest amount of his reading that Dan could do in one day?
- 4 What are two different ways that Dan could do his reading?
- 5 Explain how to find all the different ways to decompose a fraction.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for decomposing a fraction? Explain.

.....

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APPLY IT

Use what you just learned to solve these problems.

- 7 Find three ways to decompose $\frac{7}{8}$ into a sum of other fractions. Draw a model for each way to show how you know the way is correct. Show your work.

Solution

- 8 Complete the equations to show a way to decompose each fraction.

a. $+$ $\frac{1}{4}$ $+$ $\frac{3}{4}$ $=$ $\frac{5}{4}$

b. $\frac{3}{4} = \frac{1}{4} +$

c. $\frac{9}{12} = \frac{3}{12} + \frac{3}{12} +$

- 9 Draw a diagram to justify your answer to problem 8b.

Practice Decomposing Fractions

Study the Example showing how to decompose a fraction in different ways. Then solve problems 1–5.

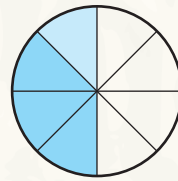
EXAMPLE

Sarah's family has $\frac{4}{8}$ of a cherry pie left over. Sarah and her sister share the leftover pie. What are two different ways that Sarah and her sister can each get some of the pie?



$$\frac{2}{8} + \frac{2}{8} = \frac{4}{8}$$

Sarah and her sister each get $\frac{2}{8}$ of the pie.



$$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$$

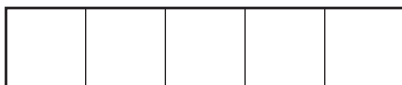
Sarah gets $\frac{1}{8}$ of the pie, and her sister gets $\frac{3}{8}$ of the pie.

- 1 Complete the equations to show how to decompose $\frac{3}{5}$ in two different ways.

a. $\frac{3}{5} = \frac{1}{5} + \dots$

b. $\frac{3}{5} = \frac{1}{5} + \dots + \frac{1}{5}$

- 2 Shade the area model below to show the equation in problem 1a.



3 Select all the equations that show a correct way to represent $\frac{7}{10}$.

Ⓐ $\frac{1}{10} + \frac{5}{10} = \frac{7}{10}$

Ⓑ $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

Ⓒ $\frac{1}{10} + \frac{2}{10} + \frac{4}{10} = \frac{7}{10}$

Ⓓ $\frac{1}{10} + \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

Ⓔ $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{7}{10}$

4 Vijay has $\frac{6}{10}$ of a cup of raisins. He wants to put the raisins into three snack bags. What are two different ways he could put raisins into three snack bags? Use a model to show each way. Show your work.

Solution

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.....

5 Is $\frac{7}{12} + \frac{1}{12}$ equivalent to $\frac{4}{12} + \frac{4}{12}$? Explain your answer.

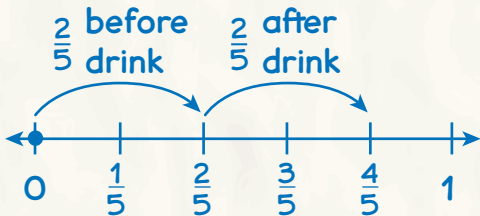
Refine Adding and Subtracting Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Jessica hikes $\frac{2}{5}$ of a mile on a trail before she stops to get a drink of water. After her drink, Jessica hikes another $\frac{2}{5}$ of a mile. How far does Jessica hike in all?

Look at how you could show your work using a number line.



Solution

The student used labels and “jump” arrows to show each part of the hike on a number line. It is just like adding whole numbers!



PAIR/SHARE

How else could you solve this problem?

APPLY IT

- 1 Ruth makes 1 fruit smoothie. She drinks $\frac{1}{3}$ of it. What fraction of the fruit smoothie is left? Show your work.

Solution

What fraction represents the whole fruit smoothie?

PAIR/SHARE

How did you and your partner decide what fraction to start with?

- 2 Mr. Chang has a bunch of balloons. $\frac{3}{10}$ of the bunch is red. $\frac{2}{10}$ of the bunch is blue. What fraction of the bunch is not red or blue? Show your work.

Solution

- 3 Emily eats $\frac{1}{6}$ of a bag of carrots. Nick eats $\frac{2}{6}$ of the same bag of carrots. What fraction of the bag of carrots do Emily and Nick eat altogether?

- (A) $\frac{1}{6}$
- (B) $\frac{1}{3}$
- (C) $\frac{3}{6}$
- (D) $\frac{3}{12}$

Rob chose (D) as the correct answer. How did he get that answer?

I think that there are at least two different steps to solve this problem.



PAIR/SHARE

What other problem in this lesson is similar to this one?

To find the fraction of the bag Emily and Nick ate together, should you add or subtract?

PAIR/SHARE

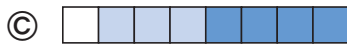
Does Rob's answer make sense?

- 4 Lin buys some cloth. He uses $\frac{5}{8}$ of a yard for a school project. He has $\frac{2}{8}$ of a yard left. How much cloth does Lin buy?
- (A) $\frac{3}{8}$ of a yard
- (B) $\frac{7}{16}$ of a yard
- (C) $\frac{7}{8}$ of a yard
- (D) $\frac{8}{8}$ of a yard
- 5 Carmela cuts a cake into 12 equal-sized pieces. She eats $\frac{2}{12}$ of the cake, and her brother eats $\frac{3}{12}$ of the cake. What fraction of the cake is left?
- (A) $\frac{1}{12}$
- (B) $\frac{5}{12}$
- (C) $\frac{7}{12}$
- (D) $\frac{12}{12}$
- 6 Lee makes muffins. She uses $\frac{2}{3}$ of a cup of milk and $\frac{1}{3}$ of a cup of oil. How many more cups of milk than oil does she use?



	/	/	/	/	/	
.
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9

- 7 Lucy and Melody work together to paint $\frac{6}{8}$ of a room. Which models could be used to show how much of the room each girl paints?



(D) $\frac{6}{8} = \frac{3}{8} + \frac{3}{8}$

(E) $\frac{6}{8} = \frac{5}{8} + \frac{1}{8}$

- 8 Cole and Max pick $\frac{9}{10}$ of a bucket of blueberries in all. Cole picks $\frac{3}{10}$ of a bucket of blueberries. What fraction of a bucket of blueberries does Max pick? Show your work.

Solution

9 MATH JOURNAL

Ms. Jones cuts an apple into eighths. She eats $\frac{3}{8}$ of the apple and gives the rest to her son and daughter. Describe two different ways her son and daughter can share the rest of the apple if they each have some of the apple.



SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

Teacher Sample, Lesson 20

The *Ready Classroom Mathematics* Teacher's Guide includes support for planning, differentiation, and facilitating meaningful mathematical discourse.

The following pages, 413a–440b, represent a complete teacher lesson that corresponds to the student lesson included in this sample.

CCSS Focus



Domain

Number and Operations—Fractions

Cluster

- B.** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards

- 4.NF.B.3** Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.
- b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.
- d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Additional Standard

- 4.NF.B.3a** (See Standards Correlations at the end of the book for full text.)

Standards for Mathematical Practice (SMP)

- 1 Make sense of problems and persevere in solving them.
- 2 Reason abstractly and quantitatively.
- 3 Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.
- 5 Use appropriate tools strategically.
- 6 Attend to precision.
- 7 Look for and make use of structure.

Lesson Objectives

Content Objectives

- Add fractions with like denominators.
- Subtract fractions with like denominators.
- Decompose fractions as a sum of fractions with the same denominators in more than one way.
- Use fraction models, number lines, and equations to represent word problems.

Language Objectives

- Draw pictures or diagrams to represent word problems involving fraction addition and subtraction.
- Use fraction vocabulary, including *numerator* and *denominator*, to explain how to add and subtract fractions with like denominators.
- Orally define and use the key mathematical terms *add*, *subtract*, *equal parts*, *fraction*, *unit fraction*, *numerator*, and *denominator* when reasoning and arguing about fraction addition and subtraction and about decomposing fractions.
- Draw models and write equations to represent ways to decompose a fraction.
- Write and solve equations to represent word problems involving fraction addition or subtraction.

Prerequisite Skills

- Understand addition as joining parts.
- Understand subtraction as separating parts.
- Know addition and subtraction basic facts.
- Understand the meaning of fractions.
- Identify numerators and denominators.
- Write whole numbers as fractions.
- Compose and decompose fractions.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In the previous lesson students began developing an understanding of adding and subtracting fractions with like denominators. They developed an understanding of adding fractions as combining parts referring to the same whole.

This lesson extends students' understanding of fraction addition and subtraction. Here students begin to deal with addition and subtraction in the abstract. They learn to decompose fractions as a sum of fractions with the

same denominators in more than one way. Students use visual models to represent word problems involving the addition and subtraction of fractions with the same whole. Students also use equations to solve word problems.

In the next lesson students will add and subtract mixed numbers with like denominators. The focus in Grade 4 is on adding and subtracting fractions with like denominators. In Grade 5, students begin to add and subtract fractions with unlike denominators.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Lesson Pacing Guide

Teacher Toolbox

Whole Class Instruction

SESSION 1

Explore

45–60 min

Interactive Tutorial* (Optional)

Prerequisite Review: Understand Adding and Subtracting Fractions

Adding and Subtracting Fractions

- Start 5 min
- Try It 10 min
- Discuss It 10 min
- Connect It 15 min
- Close: Exit Ticket 5 min

Additional Practice

Lesson pages 417–418

SESSION 2

Develop

45–60 min

Adding Fractions

- Start 5 min
- Try It & Discuss It 15 min
- Picture It & Model It 5 min
- Connect It & Apply It 15 min
- Close: Exit Ticket 5 min

Additional Practice

Lesson pages 423–424

Fluency

Adding Fractions

SESSION 3

Develop

45–60 min

Subtracting Fractions

- Start 5 min
- Try It & Discuss It 15 min
- Picture It & Model It 5 min
- Connect It & Apply It 15 min
- Close: Exit Ticket 5 min

Additional Practice

Lesson pages 429–430

Fluency

Subtracting Fractions

SESSION 4

Develop

45–60 min

Decomposing Fractions

- Start 5 min
- Try It & Discuss It 10 min
- Model Its 5 min
- Connect It & Apply It 15 min
- Close: Exit Ticket 5 min

Additional Practice

Lesson pages 435–436

Fluency

Decomposing Fractions

SESSION 5

Refine

45–60 min

Adding and Subtracting Fractions

- Start 5 min
- Example & Problems 1–3 15 min
- Practice & Small Group Differentiation 20 min
- Close: Exit Ticket 5 min

Lesson Quiz

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 3

- Lesson 20 Understand What a Fraction Is
- Lesson 20 Understand Fractions on a Number Line

RETEACH

Tools for Instruction

Grade 3

- Lesson 20 Modeling Fractions
- Lesson 20 Fractions on a Number Line

Grade 4

- Lesson 20 Add and Subtract Fractions

REINFORCE

Math Center Activities

Grade 4

- Lesson 20 Make a Whole!
- Lesson 20 Different Ways to Show Sums

EXTEND

Enrichment Activity

Grade 4

- Lesson 20 Addition Grids



Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 4

- Add and Subtract Fractions

Learning Game

- Prerequisite: Bounce

Lesson Materials

Lesson *Per student:* 1 set of fraction tiles or fraction circles
(Required)

Activities *Per student:* scissors, ruler, heavy paper or card stock, paper plates, markers
Per pair: 1 set of fraction tiles or fraction circles
Activity Sheet: Fraction Bars

Math Toolkit counters, fraction circles, fraction tiles, fraction bars, number lines, index cards

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Connect to Family, Community, and Language Development

The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox

Add and Subtract Fractions

Dear Family,
This week your child is learning how to add and subtract fractions with like denominators.

Fractions with the same number below the line have like denominators.
like denominators: $\frac{1}{4}$ and $\frac{3}{4}$ unlike denominators: $\frac{1}{2}$ and $\frac{3}{4}$

To find the sum of fractions with like denominators, understand that you are adding like units. Just as 3 apples plus 2 apples is 5 apples, 3 eighths plus 2 eighths is 5 eighths. Similarly, when you take away, or subtract, 2 eighths from 5 eighths, you have 3 eighths left.

$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

You can also use a number line to understand adding and subtracting like fractions.

Remember that the denominator names units the same way that "apples" names units.
So, when you add two fractions with like denominators, the sum of the numerators tells how many of those units you have.
When you subtract two fractions with like denominators, the difference of the numerators tells how many of those units you have.
Invite your child to share what he or she knows about adding and subtracting fractions by doing the following activity together.

LESSON 20

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ACTIVITY ADDING AND SUBTRACTING FRACTIONS

Do this activity with your child to add and subtract fractions.

Materials bowl, measuring cup, ingredients shown in the recipe

Follow the recipe below to make a creamy cracker spread or veggie dip.

Creamy Spread

Ingredients

- $\frac{5}{8}$ cup cream cheese
- $\frac{2}{8}$ cup sour cream
- herbs
- crackers or veggies

Directions
Mix the cream cheese, sour cream, and herbs together in a medium bowl. Serve immediately with crackers or sliced fresh veggies. Enjoy!

After you have made the spread, ask your child questions such as these:

- What fraction of a cup is the total amount of spread?
- If you spread $\frac{1}{8}$ of a cup on crackers or veggies, how much spread is left?

Make up a simple recipe using fractions for someone else in the family to make!

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Goal

The goal of the Family Letter is to reinforce the concept that adding and subtracting fractions with the same denominators is like adding and subtracting parts of the same whole. The sum or difference of the numerators tells how many of those parts you have. Understanding fractions is a critical life skill necessary for hobbies and jobs such as cooking, sewing, architecture, and construction.

Activity

Students and family members work with fractional measurements of ingredients to prepare a creamy cracker spread. Look at the *Adding and Subtracting Fractions* activity and adjust it if necessary to connect with your students.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Math Talk at Home

Encourage students to discuss with their family any foods they eat at home that can be separated into equal parts, such as pizza, pie, or another favorite food their family enjoys.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- Do you use a measuring cup when you cook? Can you show it to me so that I can see the markings?
- What favorite recipe can we make together?
- What do you eat that can be cut into parts?

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 Use with *Try It*.

- You may want to ask students what type of cards or other items they collect. Substitute the collectible items students name to help them connect to the problem.

Session 2 Use with *Try It*.

- Ask students if they have helped build a fence or any other structure or have used any tools around their home. Share with students that before a fence or a structure is built, an architect or planner draws plans with very accurate measurements that include fractions.
- Show an image of a plan if possible. Point out that adding fractions will allow them to know how to build something so that it fits in the space available.
- Bring a measuring tape to class and point out how each inch—which is an example of a whole—can be separated into equal parts. Measure the door frame and the door of your classroom and ask students what would happen if the door was made wider than the frame.

Session 3 Use with *Additional Practice* problem 5.

- Many cultures record important events through weaving, tapestries, or quilting. To help build cultural connections, ask students to draw a quilt or tapestry that represents their cultural background.

Session 5 Use anytime during the session.

- Share with students that adding and subtracting fractions is not only used in construction or to build structures. Explain that clothing has to be carefully measured to determine how much material is needed. If time permits, have students use a tape measure to measure the length between their shoulders. Have students work in small groups to determine how much material they would need to make the shirts they are wearing.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1 Use with *Try It*.

Levels 1–3

Listening/Speaking Read the *Try It* problem. As students draw representations, ask them to identify the whole by pointing to their drawings. Provide a sentence frame: *These (circles, squares) represent the whole.* Ask students to explain what *whole* means. Say: *Whole means complete.* Ask students to identify the number of cards Lynn and Paco get and the number of cards Todd gets. Help students write a fraction for the part of the pack of cards that Todd gets: $\frac{5}{12}$. Then have them identify the numerator and denominator and explain what they mean: *The 5 means _____. The 12 means _____.*

Levels 2–4

Listening/Speaking Read the *Try It* problem and have students form pairs. Write the following sentence frames:

- *First, I identified the whole by _____.*
- *Next, I found out how many total cards Lynn and Paco had by _____.*
- *Then I decided the fraction for the part of the pack of cards that Todd got is _____ because _____.*
- *My answer is reasonable because _____.*

Read the sentences to the students and then have them take turns reading the sentences with their partners. Ask students to explain to their partners how they determined the fraction of the pack of cards that Todd got. Call on students to share their explanations.

Levels 3–5

Writing/Reading Read the *Try It* problem and have students form pairs. Have students describe in writing the strategies they used to determine the fraction of the pack of cards that Todd got. Ask them to read what they have written to their partners. Provide the following vocabulary for students to use in their responses: *denominator, numerator, whole, fraction, and reasonable.*

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session, students draw on the similarities between adding or subtracting whole numbers and adding or subtracting fractions. They share models to explore how various solution methods are based on unit fractions. They will look ahead to think about problem situations that involve subtracting from a whole.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' facility with composing and decomposing fractions, foreshadowing the twelfths they will work with to solve the problem.

How Have students use fraction tiles or fraction circles to show one whole built from twelfths, in any way they choose.

Use $\frac{1}{12}$ s to build 1 whole. Complete the equation to show how you made 1 whole.

$$\frac{\square}{12} + \frac{\square}{12} = 1 \text{ whole}$$

Possible Solutions

$$\begin{array}{ll} \frac{1}{12} + \frac{11}{12} & \frac{4}{12} + \frac{8}{12} \\ \frac{2}{12} + \frac{10}{12} & \frac{5}{12} + \frac{7}{12} \\ \frac{3}{12} + \frac{9}{12} & \frac{6}{12} + \frac{6}{12} \end{array}$$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify how many cards are in the pack they are sharing.

DISCUSS IT

Support Partner Discussion

To reinforce the units of twelfths, encourage students to use *twelfths* as they talk to each other.

Look for, and prompt as necessary, for understanding of:

- 12 as the number of parts in the whole
- 3 and 4 as parts of the total
- a part of the total that is unknown

Explore Adding and Subtracting Fractions

Previously, you learned that adding fractions is similar to adding whole numbers. Use what you know to try to solve the problem below.

Lynn, Paco, and Todd share a pack of 12 cards. Lynn gets 4 cards, Paco gets 3 cards, and Todd gets the rest of the cards. What fraction of the pack does Todd get?



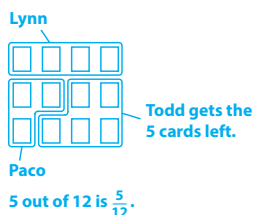
Learning Targets

- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

TRY IT

Possible student work:

Sample A



Sample B

$3 + 4 = 7$. Lynn and Paco get 7 cards.
 $12 - 7 = 5$. Todd gets the 5 cards left.
 5 out of 12 is $\frac{5}{12}$.

Math Toolkit

- counters
- fraction circles
- fraction tiles
- fraction bars
- number lines

DISCUSS IT

Ask your partner: Why did you choose that strategy?
Tell your partner: At first, I thought ...

Common Misconception Look for students who are not comfortable with twelfths and who try to use 3 or 4 as the denominator. As students present solutions, be sure to have them specify why they used 12 as the denominator.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing twelfths
- drawings representing twelfths
- number lines marked in twelfths
- whole-number equations to find 5 out of 12 cards are left ($\frac{5}{12}$)

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask How do [student name]'s and [student name]'s models show the whole and the parts?

Listen for 12 is the denominator of all the fractions. $\frac{12}{12}$ shows the whole. The parts are $\frac{4}{12}$, $\frac{3}{12}$, $\frac{7}{12}$, and $\frac{5}{12}$.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

1 LOOK BACK

Look for understanding that the whole set of cards is 12 twelfths and that only some of those twelfths, $\frac{5}{12}$, are left for Todd.



Hands-On Activity

Use models to add fractions.

If . . . students are unsure about the concept of adding fractions,

Then . . . use this activity to have them model similar problems.

Materials For each student: scissors, ruler, heavy paper or card stock

- Distribute heavy paper or card stock to each student. Tell students to use scissors to cut out 12 equal-sized cards. Explain to students that the 12 cards represent one pack of cards, or one whole, and that there are 12 parts in the whole.
- Tell students to hold up 2 cards. Have students write the name of the fraction of the whole pack of cards that is represented by the 2 cards on a sheet of paper. $\left[\frac{2}{12}\right]$ Review the meaning of the fraction. [2 cards out of 12] Then repeat with 7 cards.
- Tell students to add (join) the fractions and write the sum on their sheets of paper. $\left[\frac{9}{12}\right]$ Have volunteers explain how they determined their answers.
- Repeat the activity for additional fractions, such as eighths and sixths.

2 LOOK AHEAD

Point out that sometimes students will encounter a whole that does not look like it is composed of fractional parts. Students may have to imagine the whole being cut into equal parts.

Students should be able to use fraction language to describe the whole pizza in terms of eighths, to discuss taking eighths away from the whole pizza, and to determine how many eighths are left.

CONNECT IT

1 LOOK BACK

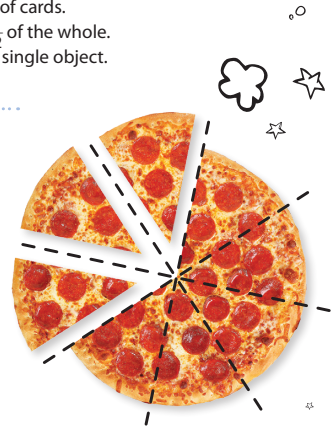
Explain how you can find the fraction of the pack that Todd gets.

Possible answer: Todd gets 5 cards. There are 12 cards in the pack. The numerator tells the number of cards Todd gets, and the denominator tells the number of cards in the whole pack, so Todd gets $\frac{5}{12}$ of the pack.

2 LOOK AHEAD

In the problem on the previous page, the whole is the pack of cards. Since there are 12 cards in the pack, each card represents $\frac{1}{12}$ of the whole. Look at the whole shown here. The whole is the pizza. It is a single object.

- How many equal parts are shown in the pizza? $\dots 8 \dots$
- What fraction can you use to describe each piece of pizza? $\dots \frac{1}{8} \dots$
- What fraction can you use to describe the whole? $\dots \frac{8}{8} \dots$
- What fraction can you use to describe the 3 pieces being taken away? $\dots \frac{3}{8} \dots$



3 REFLECT

How does knowing about equal parts help you add and subtract fractions?

Possible answer: Adding fractions means joining equal parts of the whole.

Subtracting fractions means taking away equal parts of the whole.

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Close: Exit Ticket

3 REFLECT

Look for understanding of adding or subtracting fractions as joining together or separating parts referring to the same whole. Student responses should include references to the whole, joining equal parts of the whole to add, and taking away equal parts of the whole to subtract. Some students may use the terms *numerator* and *denominator* in their explanations.

Common Misconception If students do not reference the whole or are unclear in their explanations that they are adding or subtracting equal parts of the whole, then provide fraction tiles and have students “join” tiles to add and “take away” tiles to subtract. Discuss what students notice about the numerators and denominators.



Real-World Connection

Encourage students to think about everyday places or situations in which people might need to add or subtract fractions with like denominators. Have volunteers share their ideas. Examples include cooking, construction sites, and distances on a map.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Solutions

Support Vocabulary Development

1 Ask students to circle *frac* in the word *fraction*. Share with students that *frac* is a Latin root that means “break.” Point out that in math, a *fraction* is a part of a whole. Ask students if they know of other words that have the root word *frac* (or *frag*). Provide examples, such as *fracture* and *fragment*.

If students struggle completing any part of the graphic organizer, have them use manipulatives, such as fraction tiles or fraction bars, to break a whole into parts.

2 Have students count the parts and ask: *How many parts does the model have? How would you describe the size of the parts?* Have students discuss the answers with partners. Then have them answer the questions in the problem. Provide a sentence frame:

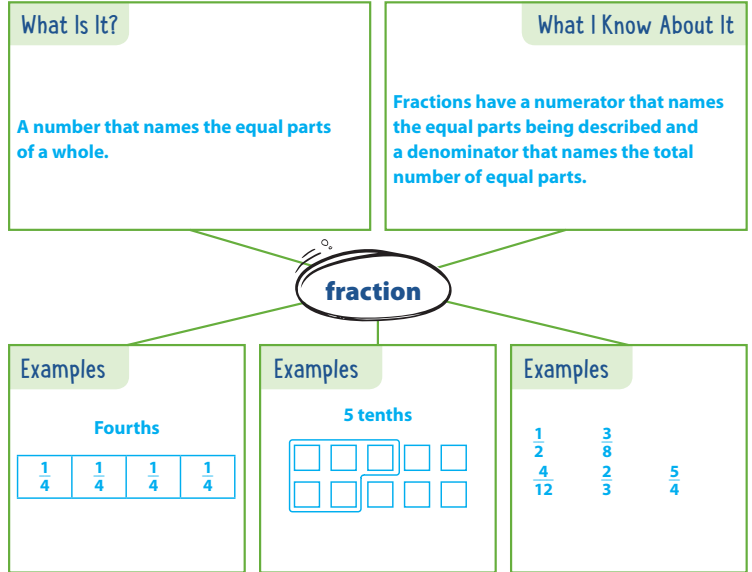
The model is/is not _____ because _____.

Supplemental Math Vocabulary

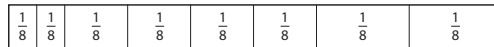
- numerator
- unit fraction
- whole

Prepare for Adding and Subtracting Fractions

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Does the model below show eighths? Why or why not?



Possible answer: The model does not show equal parts, so it cannot be an example of eighths.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

3 Assign problem 3 to provide another look at solving a problem with fractional parts of a set.

This problem is very similar to the problem about the set of space exploration cards. In both problems, the whole is a set, and the set is shared among three friends. The question asks what fraction of the whole is left for the third friend.

Students may want to use fraction tiles, sticky notes, or counters.

Suggest that students read the problem three times, asking themselves one of the following questions each time.

- What is the problem about?
- What is the question I am trying to answer?
- What information is important?

Solution:

Kara gets $\frac{4}{10}$ of the stickers. After Maria takes 2 and Jon takes 4, there are 4 stickers left. 4 out of 10 equal parts is $\frac{4}{10}$.

Medium

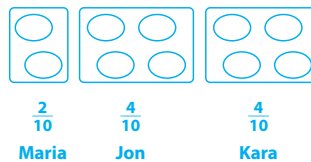
4 Have students solve the problem a different way to check their answer.

3 Solve the problem. Show your work.

Maria, Jon, and Kara share a set of 10 animal stickers. Maria gets 2 stickers, Jon gets 4 stickers, and Kara gets the rest of the stickers. What fraction of the stickers does Kara get?



Possible student work using a picture:



Solution $\frac{4}{10}$

4 Check your answer. Show your work.

Possible student work:

There are 10 stickers in all. $2 + 4 + 4 = 10$, so $\frac{4}{10}$ is correct.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Speaking/Writing Read *Connect It* problems 1–2. Ask students to draw a fence divided into 10 equal sections. Point to a section and ask: *How do I know this is $\frac{1}{10}$ of the fence?* Provide sentence frames: *1 is the numerator. 10 is the denominator. The fraction $\frac{1}{10}$ tells this is 1 out of 10 sections.* Point to and count 3 sections. Say: *Josie paints 3 sections of the fence. Write $\frac{3}{10}$. What does the numerator 3 in this fraction mean?* Have students answer in writing: *The numerator 3 tells me that Josie paints 3 sections of the fence.* Repeat for the numerator 4.

Levels 2–4

Speaking/Writing Read *Connect It* problems 1–2. Have students form pairs and explain to their partners how they know each section is $\frac{1}{10}$ of the whole. Ask: *When you see the numerator 1, what does that tell you? When you see the denominator 10, what does that tell you?* After students have discussed their thinking, have them write their responses. Write $\frac{3}{10}$ and $\frac{4}{10}$. Ask partners to discuss what is the same and different about the fractions. Then call on pairs to say how they determined the total number of sections each girl paints.

Levels 3–5

Speaking/Writing Have students form pairs and read *Connect It* problems 1–6 and write their responses. Ask them to discuss their thinking with partners. Extend the discussions by asking: *How would the answer be different if Josie paints 5 sections and Margo paints 1 section? What fractions would you need to solve the problem?* After students have discussed, have them work with partners to make new problems using fractions that assume Josie and Margo paint different numbers of sections of the fence.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session, students solve a problem that requires finding the sum of $\frac{3}{10}$ and $\frac{4}{10}$. Students model the fractions in the word problem either on paper or with manipulatives to represent the sum. The purpose of this problem is to have students develop strategies to add fractions.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' understanding of adding fractions.

How Have students use fraction tiles or fraction circles to compare two different decompositions of $\frac{3}{5}$.



Look at the two expressions.

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} \quad \frac{1}{5} + \frac{2}{5}$$

How are they the same?

How are they different?

Possible Solution
The expressions have the same value of $\frac{3}{5}$, but they have different addends.

Develop Language

Why Clarify the meaning of the multiple-meaning word *end*.

How Have students circle the word *end*. Explain that in this case, the *end* is the first or last part of something that is long, such as the fence in the problem or a rope. If necessary, illustrate the meaning with four students in a line. Show that there are two ends of the line with the first person being at the first end and the last person being at the last end.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify 10 as the total number of equal parts.

Ask How many equal parts of the fence are there if Josie paints $\frac{3}{10}$ of it?

Develop Adding Fractions

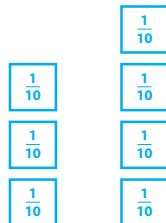
Read and try to solve the problem below.

Josie and Margo are painting a fence green. Josie starts at one end and paints $\frac{3}{10}$ of the fence. Margo starts at the other end and paints $\frac{4}{10}$ of it. What fraction of the fence do they paint altogether?

TRY IT

Possible student work:

Sample A

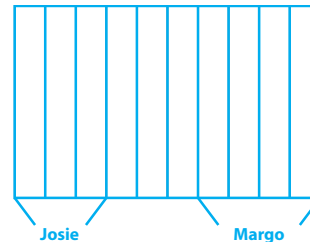


The fence is the whole. It has 10 equal parts.

Josie paints $\frac{3}{10}$. Margo paints $\frac{4}{10}$.

They paint $\frac{7}{10}$ altogether.

Sample B



Josie and Margo paint 7 out of 10 parts of the fence.

They paint $\frac{7}{10}$ altogether.



Math Toolkit

- fraction circles
- fraction tiles
- fraction bars
- number lines
- index cards

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner:
A model I used was ...
It helped me ...

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *tenths* as they discuss their solutions.

Support as needed with questions such as:

- How did you get started?
- How would you describe your model?

Common Misconception Look for students who write a fraction comparing the painted parts to the unpainted parts and write $\frac{7}{3}$, instead of comparing to the whole.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing tenths
- drawings representing tenths
- whole-number solutions showing that 7 out of 10 parts are painted ($\frac{7}{10}$)
- number lines marked in tenths

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the total number of equal parts in the fence? the part Josie paints? the part Margo paints? the total number of tenths the two girls paint?*

Listen for Students should recognize that accurate responses include fractions with a denominator of 10 and representations that show equal parts. Responses may include 10 as the total number of equal parts, $\frac{3}{10}$ as the part Josie paints, $\frac{4}{10}$ as the part Margo paints, and $\frac{7}{10}$ as the total number of parts both girls paint.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the number of equal parts
- the number of parts each girl paints
- the total amount of fence painted

Ask *What number tells the number of equal parts in the whole in the picture? in the number line? Is it the same or different?*

Listen for 10 is the denominator. It tells the total number of equal parts in both the picture and the number line. Both show 10 equal parts because they represent the same whole.

For a sketch of the fence, prompt students to identify how the fence is labeled to represent the problem.

- *Is there any way that this picture is more or less helpful than the one drawn by [student name]?*
- *How is it helpful that the fence has 10 boards?*

For a number line model, prompt students to identify the greatest number on the number line as well as the number of divisions.

- *How is the number line divided?*
- *Why is it done that way?*

Explore different ways to understand adding fractions.

Josie and Margo are painting a fence green. Josie starts at one end and paints $\frac{3}{10}$ of the fence. Margo starts at the other end and paints $\frac{4}{10}$ of it. What fraction of the fence do they paint altogether?

PICTURE IT

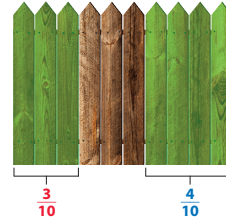
You can use a picture to help understand the problem.

Think what the fence might look like if it has 10 equal-sized parts.



Each part is $\frac{1}{10}$ of the whole.

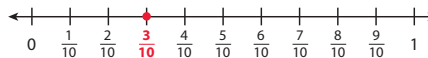
The girls paint 3 tenths and 4 tenths of the fence.



MODEL IT

You can also use a number line to help understand the problem.

The number line below is divided into tenths with a point at $\frac{3}{10}$.



Start at $\frac{3}{10}$ and count 4 tenths to the right to add $\frac{4}{10}$.



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Deepen Understanding Number Line Model

SMP 7 Look for structure.

When discussing the number line model, prompt students to consider how it could be used to demonstrate the commutative property.

Ask *What if you drew the starting point at $\frac{4}{10}$ instead of at $\frac{3}{10}$? Could you still model the problem?* To emphasize the point, draw a tenths number line on the board with a point at $\frac{4}{10}$.

Listen for Yes, you could count on $\frac{3}{10}$ from $\frac{4}{10}$ to find the answer.

Encourage a volunteer to come to the board and demonstrate how to find the sum.

Generalize *Do you think this is true no matter what numbers you are adding? If you were using a number line to add 3 and 4, would it be true?* Have students explain their reasoning. Listen for understanding that when adding whole numbers or fractions, the order of the addends does not matter; the sum stays the same.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

Remind students that one thing that is alike about all the representations is the numbers.

Explain that on this page, students will use those numbers to write one equation that matches all the representations, including concrete, visual, and symbolic.

Monitor and Confirm

1–3 Check for understanding that:

- 10 is the number of equal parts
- 3 and 4 tell how many parts each girl paints
- 7 is the total number of parts painted

Support Whole Class Discussion

4–5 Tell students that problem 4 will prepare them to provide the explanation required in problem 6.

Be sure students understand that the problem is asking them to represent the same equation twice: once with words and once with fractions.

Ask *What part of the problem do each of the fractions in the equations show?*

Listen for $\frac{3}{10}$ is the fraction of the fence Josie paints. $\frac{4}{10}$ is the fraction Margo paints. $\frac{7}{10}$ is the fraction they paint altogether.

Ask *What is the same about the two equations?*

Listen for The numerators, 3 and 4, are numbers in each equation; the denominators are words in one equation and numbers in the other.

Ask *Is the sum of $\frac{3}{10}$ and $\frac{5}{10}$ greater than or less than the sum of $\frac{3}{10}$ and $\frac{4}{10}$?*

Listen for It is greater because $\frac{3}{10}$ is the same in both problems, but $\frac{5}{10}$ is more than $\frac{3}{10}$.

Explain that problem 5 is asking about adding two different fractions not shown in the fence problem.

6 Look for the idea that you add the numerators and keep the same denominator because the size of the parts does not change when you add them.

7 **REFLECT** Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to add any two fractions that have the same denominator.

1 Look at **Picture It**. How do you know that each section of fence is $\frac{1}{10}$ of the whole fence?

Possible answer: The denominator tells the total number of fence sections.

There are 10 equal parts in all, so one part equals $\frac{1}{10}$.

2 What do the numerators, 3 and 4, tell you?

3 tells the number of fence sections that Josie paints. 4 tells the number of fence sections that Margo paints.

3 How many tenths of the fence do Josie and Margo paint altogether? $\dots 7 \dots$

4 Complete the equations to show what fraction of the fence Josie and Margo painted altogether.

Use words: **3 tenths** + **4 tenths** = **7** tenths

Use fractions: $\frac{3}{10}$ + $\frac{4}{10}$ = $\frac{7}{10}$

5 What would be the sum if the fractions were $\frac{3}{10}$ and $\frac{5}{10}$? $\dots \frac{8}{10} \dots$

6 Explain how you add fractions that have the same denominator.

Add the numerators of the addends to get the numerator of the sum to find how many parts you added. The denominators of the sum and the addends are the same because that tells what kind of parts you added.

7 **REFLECT**

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for adding fractions? Explain.

Some students may respond that they like the strategy of drawing a picture because it helps them see the parts of the whole, while other students may like using a number line because it helps them see how fraction addition is like whole number addition.

**Hands-On Activity**

Connect fraction words with fraction symbols using familiar fractions, such as fourths.

If . . . students are unsure about what the numerator and denominator name,

Then . . . use the activity below to connect symbolic fractions with verbal descriptions.

Materials For each student: Activity Sheet *Fraction Bars* (3 bars for fourths, 3 bars for tenths)

- Have students shade and label 1 part of a fourths fraction bar with the words *one fourth* and the symbol $\frac{1}{4}$. Discuss the connections—for each, ask: *Where do you see the 1? Where do you see the 4?*
- Repeat with other fourths fraction bars, shading 2 parts and labeling *two fourths* and $\frac{2}{4}$, and then shading 3 parts, labeling *three fourths* and $\frac{3}{4}$. Ask: *Where is the 2? the 3? Why does 4 show up so many times?*
- Extend to tenths, using tenths fraction bars, and ask students to name, with words and numbers, $\frac{1}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$.
- Prompt students to recognize that they can use drawings and words as well as numbers to keep track of the math and the meaning of fractions.



APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing thirds or fifths accurately is difficult and here precise measurements are not necessary.

- 8 $\frac{2}{3}$ of the rooms; Students may also show $\frac{1}{3}$ on a number line divided into thirds and count 1 tick mark to the right. They also may write the equation $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.
- 9 $\frac{4}{5}$ of a meter; Students may show $\frac{1}{5}$ on a number line divided into fifths and count 3 marks to the right. They also may write the equation $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$.

Close: Exit Ticket

- 10 $\frac{6}{8}$ of a pound of fruit; Students may write the equation $\frac{2}{8} + \frac{4}{8} = \frac{6}{8}$. They may also show $\frac{2}{8}$ on a number line divided into eighths and count 4 marks to the right.

Students' solutions should indicate understanding of:

- adding (joining) parts that refer to the same whole
- accurate use of visual fraction models and equations to represent the problem

Error Alert If students' solutions are $\frac{6}{16}$, then review the structure of fractions to help them see that only numerators should be combined. Explain that denominators tell the kind of parts that are being added. Have them write *2 apples + 4 apples = 6 apples* on an index card and then write below it *2 eighths + 4 eighths = 6 eighths*.

APPLY IT

Use what you just learned to solve these problems.

- 8 Lita and Otis help their mom clean the house. Lita cleans $\frac{1}{3}$ of the house. Otis cleans $\frac{1}{3}$ of the house. What fraction of the house do Lita and Otis clean altogether? Show your work.

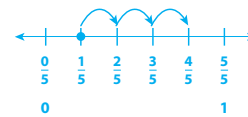
Possible student work using a model:



Solution $\frac{2}{3}$ of the house

- 9 Mark and Imani use string for a project. Mark's string is $\frac{1}{5}$ of a meter long. Imani's string is $\frac{3}{5}$ of a meter long. How long are the two strings combined? Show your work.

Possible student work using a number line:



$\frac{4}{5}$ of a meter

- 10 Paola makes a fruit smoothie. She uses $\frac{2}{8}$ of a pound of strawberries and $\frac{4}{8}$ of a pound of blueberries. How many pounds of fruit does she use? Show your work.

Possible student work using an equation:

$$\frac{2}{8} + \frac{4}{8} = \frac{6}{8}$$

Solution $\frac{6}{8}$ of a pound of fruit



Solutions

- 1 Students should shade any 2 of the 12 muffin cups.

Basic

- 2 Students should shade any 6 of the 12 muffin cups.

Medium

- 3 $\frac{8}{12}$ of the tray is filled; $\frac{2}{12} + \frac{6}{12} = \frac{8}{12}$

Challenge

Name: _____

Practice Adding Fractions

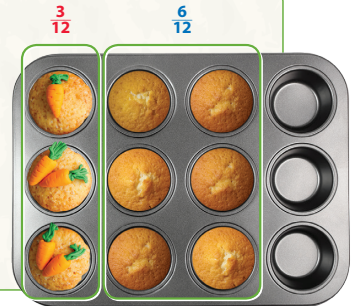
Study the Example showing one way to add fractions. Then solve problems 1–9.

EXAMPLE

Shrina has a muffin pan that holds 12 muffins. She fills $\frac{3}{12}$ of the pan with carrot muffin batter. Then she fills $\frac{6}{12}$ with pumpkin muffin batter. What fraction of the pan does she fill?

$$\frac{3}{12} + \frac{6}{12} = \frac{9}{12}$$

So, she fills $\frac{9}{12}$ of the muffin pan.

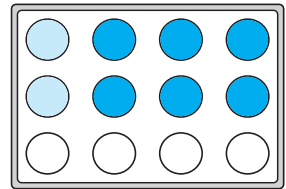


- 1 Sam fills $\frac{2}{12}$ of another pan with banana muffin batter. Shade $\frac{2}{12}$ of the muffin pan diagram at the right.

Any 2 muffins may be shaded.

- 2 Then Sam fills $\frac{6}{12}$ with lemon muffin batter. Shade $\frac{6}{12}$ of the diagram to show this.

Any 6 muffins may be shaded.



- 3 In problem 2, what fraction of the pan in all is filled now? $\frac{8}{12}$
Write an equation for this problem that includes your answer.

$$\frac{2}{12} + \frac{6}{12} = \frac{8}{12}$$

Fluency & Skills Practice **Teacher Toolbox**

Assign Adding Fractions

Fluency and Skills Practice
Adding Fractions

Name: _____

Write the missing digits in the boxes to make each addition problem true.

1 $\frac{1}{6} + \frac{4}{6} = \frac{\square}{6}$ 2 $\frac{2}{8} + \frac{4}{8} = \frac{\square}{8}$ 3 $\frac{1}{10} + \frac{4}{10} = \frac{\square}{10}$

4 $\frac{1}{12} + \frac{\square}{12} = \frac{2}{12}$ 5 $\frac{2}{6} + \frac{\square}{6} = \frac{2}{6}$ 6 $\frac{6}{7} + \frac{\square}{7} = \frac{7}{7}$

7 $\frac{\square}{4} + 2 = \frac{5}{4}$ 8 $\frac{\square}{10} + \frac{2}{10} = \frac{5}{10}$ 9 $\frac{\square}{8} + \frac{2}{8} = \frac{5}{8}$

10 $\frac{\square}{6} + 2 = \frac{\square}{6}$ 11 $\frac{\square}{5} + \frac{1}{5} = \frac{\square}{5}$ 12 $\frac{4}{10} + \frac{\square}{10} = \frac{\square}{10}$

Write a digit from 1–12 in each box so that the addition problem is true.

$\frac{\square}{12} + \frac{\square}{12} = \frac{\square}{12}$

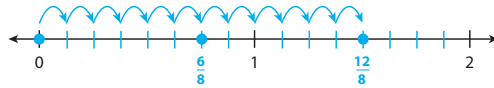
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Kay runs $\frac{6}{8}$ of a mile and rests. Then she runs another $\frac{6}{8}$ of a mile.

- 4 Divide the number line below to show eighths.



- 5 Label $\frac{6}{8}$ on the number line above.

- 6 Use arrows to show $\frac{6}{8} + \frac{6}{8}$ on the number line.

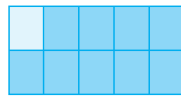
- 7 What is the total distance Kay runs? $\frac{12}{8}$ miles

- 8 Write an equation for this problem that includes your answer.

$$\frac{6}{8} + \frac{6}{8} = \frac{12}{8}$$

- 9 Jin cleans $\frac{1}{10}$ of the patio before lunch and $\frac{9}{10}$ of the patio after lunch. What fraction of the patio does Jin clean altogether? Show your work.

Possible student work:



$$\frac{1}{10} + \frac{9}{10} = \frac{10}{10}$$

Solution $\frac{10}{10}$ (or 1 whole) of the patio

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- 4 Tick marks should divide each whole into 8 (approximately) equal parts.

Basic

- 5 dot on sixth tick mark after 0

Basic

- 6 12 arrows on the number line, six from 0 to $\frac{6}{8}$, and then six from $\frac{6}{8}$ to $\frac{12}{8}$. Some students may start at $\frac{6}{8}$ and count 6 more eighths, showing only 6 arrows from $\frac{6}{8}$ to $\frac{12}{8}$.

Medium

- 7 $\frac{12}{8}$ miles

Medium

- 8 $\frac{6}{8} + \frac{6}{8} = \frac{12}{8}$

Challenge

- 9 $\frac{10}{10}$ (or 1 whole) of the patio; Students may draw a rectangular area model divided into tenths, shading $\frac{1}{10}$ one way and $\frac{9}{10}$ another way, or they may write an equation $\frac{1}{10} + \frac{9}{10} = \frac{10}{10}$.

Challenge

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3 Use with Try It.

Levels 1–3

Writing Read the *Try It* problem and then write *numerator* and *denominator*. Point to each word as you read it aloud. Ask students to form pairs and explain the meaning of each word to partners. Point to the fraction $\frac{5}{6}$. Ask students to identify the numerator and denominator and tell what each refers to. If students respond, “It is the top/bottom number,” encourage them to provide additional information using the sentence frames: _____ is the _____. It tells me _____. Follow the same procedure for $\frac{4}{6}$. Ask students to work with their partners to write the operation needed to solve the problem and the equation.

Levels 2–4

Speaking/Writing Read the *Try It* problem with students. Divide students into groups of three to discuss what each numerator and denominator mean. Ask: *What operation will you use to solve the problem? Why?* Have students discuss their answers to the questions in their small groups. Ask them to write the procedure they followed to solve the problem. Provide sentence frames for groups to use if additional support is needed.

Levels 3–5

Writing/Speaking Have students form pairs and read the *Try It* problem. Ask them to discuss with their partners how they will solve it using what they know about numerators and denominators. Then have students write the steps they use to solve the problem. Ask: *How much water would Alberto have now if he put $\frac{3}{6}$ of a liter of water in the bottle from the water fountain?* After students have solved the problem, have them work with their partners to write additional scenarios to increase or decrease the amount of water in the bottle. Provide support as needed. Have pairs exchange scenarios and try to solve the problems.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session, students solve a word problem that requires finding the difference between $\frac{5}{6}$ and $\frac{4}{6}$. Students model the fractions either on paper or with manipulatives to represent the difference. The purpose is to have students develop strategies to subtract fractions.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' understanding of subtracting fractions.

How Have students use fraction tiles or fraction circles to find $\frac{1}{10}$ more than and $\frac{1}{10}$ less than $\frac{5}{10}$.

1 $\frac{1}{10}$ more than $\frac{5}{10}$ is

2 $\frac{1}{10}$ less than $\frac{5}{10}$ is

Solutions

1. $\frac{6}{10}$
2. $\frac{4}{10}$

Develop Language

Why Review the names of some units in the customary and the metric systems of measurement.

How Remind students that two systems of measurement are used in the United States: the customary system and the metric system. Have students try to recall some of the units for each system. Then ask them to identify the unit in the *Try It* problem and say to which system it belongs. Explain that the *liter* is a metric unit of liquid volume.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify 6 as the total number of equal parts.

Ask *How many equal parts is the liter divided into if Alberto drinks $\frac{4}{6}$ of a liter of water? How do you know?*

Develop Subtracting Fractions

Read and try to solve the problem below.

Alberto's water bottle has $\frac{5}{6}$ of a liter of water in it. He drinks $\frac{4}{6}$ of a liter. What fraction of a liter of water is left in the bottle?

TRY IT

Possible student work:

Sample A

Alberto drinks water, so you subtract to find what's left.

$$\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

The water bottle has $\frac{1}{6}$ of a liter of water left.

Sample B



The water bottle has $\frac{1}{6}$ of a liter of water left.



Math Toolkit

- fraction circles
- fraction tiles
- fraction bars
- number lines
- index cards

DISCUSS IT

- Ask your partner:** Can you explain that again?
- Tell your partner:** I disagree with this part because ...

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *sixths* as they discuss their solutions.

Support as needed with questions such as:

- *Did you draw a picture or make a model to show the problem?*
- *How can you explain what the problem is asking?*

Common Misconception Students may believe that because the whole bottle is 1 liter that they must subtract either $\frac{5}{6}$ or $\frac{4}{6}$ from $\frac{6}{6}$. Have students restate the problem in their own words, using a diagram to support their explanation.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing sixths
- drawings representing sixths
- number lines marked in sixths

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the total number of equal parts in the bottle? the part Alberto drinks? the part that is left?*

Listen for Students should recognize that accurate representations show 6 as the total number of equal parts, 4 out of 6 parts, or $\frac{4}{6}$, as the part Alberto drinks, and 1 out of 6 parts, or $\frac{1}{6}$, as the part that is left.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the number of equal parts
- the number of parts Alberto drinks

Ask *What number tells the whole in the picture? in the number line? Is it the same or different?*

Listen for 6 is the denominator; it tells the total number of equal parts in both the picture and the number line. Both show 6 equal parts because they represent the same whole.

For a sketch of the water bottle, prompt students to identify how the bottle is labeled to represent the problem.

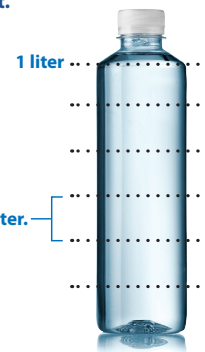
- *Is there any way that this picture is more or less helpful than the one drawn by [student name]?*
- *How is it helpful that the bottle shows 1 liter divided into sixths?*
- *Why are some parts of the bottle blue and some clear?*

For a number line model, prompt students to identify the greatest number on the number line and the number of divisions.

- *How is the number line divided?*
- *Why is the point $\frac{5}{6}$ marked?*

Explore different ways to understand subtracting fractions.

Alberto's water bottle has $\frac{5}{6}$ of a liter of water in it. He drinks $\frac{4}{6}$ of a liter. What fraction of a liter of water is left in the bottle?



PICTURE IT

You can use a picture to help understand the problem.

The picture shows the whole liter divided into 6 equal parts.

Each part is $\frac{1}{6}$ of a liter.

Five shaded parts show how much water is in the bottle. Alberto drinks 4 sixths of a liter, so take away 4 shaded parts. The 1 shaded part that is left shows the fraction of a liter that is left.

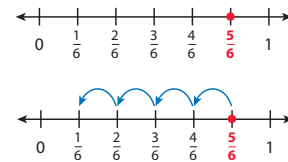


MODEL IT

You can also use a number line to help understand the problem.

The number line at the right is divided into sixths, with a point at $\frac{5}{6}$.

Start at $\frac{5}{6}$ and count back 4 sixths to subtract $\frac{4}{6}$.



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Deepen Understanding

Connect Visual Representations to Models

SMP 4 Model with mathematics.

When discussing the number line model, prompt students to consider how it and the visual representation of the water bottle are connected.

- Draw the number line on the board. Then draw the $\frac{5}{6}$ -full water bottle on its side above the number line, making sure the bottom of the bottle is aligned with 0 and each part of the bottle is aligned with a sixths tick mark.
- Have students identify that $\frac{5}{6}$ on the number line lines up with the amount of water in the bottle.
- Then cross out (or erase) 4 parts of the bottle, one part at a time, moving from right to left along the number line to show the water Alberto drinks.

Generalize To help students identify important quantities and map their relationships, ask: *What do you notice about the amount of water remaining in the water bottle?* Have students share their observations. Listen for understanding that it lines up with the $\frac{1}{6}$ mark on the number line and that both representations show that there is $\frac{1}{6}$ of a liter remaining.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

Remind students that one thing that is alike about all the representations is the numbers.

Explain that on this page, students will use those numbers to write one equation that matches all the representations.

Monitor and Confirm

1 – 3 Check for understanding that:

- 6 is the number of equal parts
- 5 tells how many parts are in the bottle to start
- 4 tells how many parts Alberto drinks

Support Whole Class Discussion

4 Tell students that this problem will prepare them to provide the explanation required in problem 5.

Be sure students understand that the problem is asking them to represent the same equation twice: once with words and once with fractions.

Ask *What part of the problem do each of the fractions in the equations show?*

Listen for $\frac{5}{6}$ is the amount of water in the bottle. After Alberto drinks $\frac{4}{6}$, there is $\frac{1}{6}$ left.

Ask *What is the same about the two equations?*

Listen for The numerators, 5 and 4, are numbers in each equation; the denominators are words in one equation and numbers in the other.

5 Look for the idea that you subtract the numerators and keep the same denominator because the size of the parts does not change when you subtract them.

6 **REFLECT** Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to subtract any two fractions that have the same denominator.

1 In **Picture It**, why does $\frac{1}{6}$ represent 1 of the equal parts of the liter?

Possible answer: The denominator tells the number of equal parts the bottle is divided into. The water bottle is divided into 6 equal parts, so one part equals $\frac{1}{6}$.

2 What do the numerators, 5 and 4, tell you?

5 tells the number of parts of the bottle that had water to begin with. 4 tells the number of parts that Alberto drank.

3 How many sixths of a liter are left in the bottle after Alberto drinks 4 sixths? 1 sixth

4 Complete the equations to show what fraction of a liter is left in the bottle.

Use words: **5 sixths** – **4 sixths** = 1 sixth

Use fractions: $\frac{5}{6}$ – $\frac{4}{6}$ = $\frac{1}{6}$

5 Explain how you subtract fractions with the same denominator.

Possible answer: Subtract the numerator of the amount you take away from the numerator of the starting amount to get the numerator of the answer because that tells you how many parts you have left. The denominator of the answer is the same as the denominator of the other amounts because that just tells you what kind of parts you have.

6 **REFLECT**

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for subtracting fractions? Explain.

Some students may respond that they like the strategy of using a number line because it helps them see how subtracting fractions is like subtracting whole numbers, while other students may like writing equations because it helps them solve the problem using just numbers.

**Hands-On Activity**

Use paper plates to subtract fractions.

If . . . students are unsure about subtracting fractions,

Then . . . use the activity below to provide a concrete model to connect to the visual and symbolic representations.

Materials For each student: paper plates, markers, scissors

- Distribute a paper plate, markers, and scissors to each student. Model how to divide the plate into 8 equal sections by folding the plate on top of itself three times.
- Direct students to color $\frac{5}{8}$ of the plate and then cut out that fraction of the plate. Ask students to name the fraction of the plate they have. $\left[\frac{5}{8}\right]$
- Tell students to subtract 2 eighths from the 5 eighths. Guide students to cut 2 sections from the colored portion of the plate they are holding.
- Ask students to name the fraction of the plate they are left with. $\left[\frac{3}{8}\right]$
- Write $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$ on the board.
- If time allows, repeat for other subtraction problems, such as $\frac{7}{8} - \frac{3}{8}$ and $\frac{3}{4} - \frac{1}{4}$.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing fractional parts accurately is difficult, and here precise measurements are not necessary.

7 $\frac{3}{10}$ of the lawn; $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$

8 $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$, $\frac{1}{4}$ of the carton.

Close: Exit Ticket

9 C; $\frac{8}{8} - \frac{4}{8} = \frac{4}{8}$

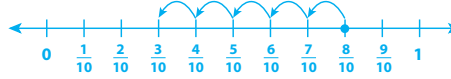
Error Alert If students choose A, B, or D, then have them identify what information is missing from the problem (the whole). Review the meaning of *denominator* (the number of equal parts in a whole) and have students explain how that relates to the missing information by writing the whole as a fraction with a denominator of 8.

APPLY IT

Use what you just learned to solve these problems.

- 7 Carmen has $\frac{8}{10}$ of the lawn left to mow. She mows $\frac{5}{10}$ of the lawn. Now what fraction of the lawn is left to mow? Show your work.

Possible student work using a number line:



Solution $\frac{3}{10}$ of the lawn

- 8 Mrs. Kirk has $\frac{3}{4}$ of a carton of eggs. She uses some for baking and has $\frac{2}{4}$ of the carton left. What fraction of the carton does she use? Show your work.

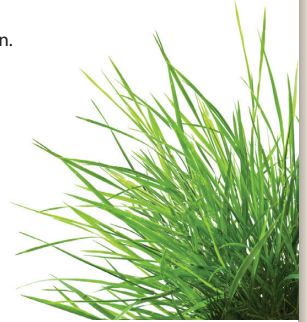
Possible student work using an equation:

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

Solution $\frac{1}{4}$ of the carton

- 9 Badru reads $\frac{4}{8}$ of a book. How much of the book does he have left to read?

- (A) $\frac{1}{8}$
 (B) $\frac{2}{8}$
 (C) $\frac{4}{8}$
 (D) $\frac{6}{8}$



Solutions

1 The number line should be divided into 10 equal sections and each tick mark labeled as tenths, as shown on the Student Worktext page.
Basic

2 Arrows should start at $\frac{8}{10}$ and jump left 3 times to $\frac{5}{10}$ and then 5 times to 0, as shown on the Student Worktext page.
Medium

3 $\frac{5}{10}$ of a mile
Medium

4 $\frac{8}{10} - \frac{3}{10} = \frac{5}{10}$; Students may use an addition equation, $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$.
Challenge

Name: _____

Practice Subtracting Fractions

Study the Example showing one way to subtract fractions. Then solve problems 1–7.

EXAMPLE

Ali buys a carton of eggs. He uses $\frac{3}{12}$ of the eggs to cook breakfast. He uses another $\frac{2}{12}$ to make a dessert for dinner. What fraction of the carton is left?



$$\frac{12}{12} - \frac{3}{12} = \frac{9}{12}$$

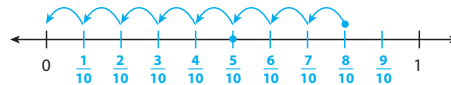
$$\frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

So, $\frac{7}{12}$ of the carton is left.



Keisha is at her friend's house. Her friend's house is $\frac{8}{10}$ of a mile from Keisha's home. Keisha walks $\frac{3}{10}$ of a mile toward home. Then her mother drives her the rest of the way home.

1 Divide the number line below to show tenths. Then label each tick mark.



2 Use arrows to show the problem on the number line you labeled in problem 1.

3 How far does Keisha's mother drive her? $\frac{5}{10}$ of a mile

4 Write an equation for this problem that includes your answer.

$$\frac{8}{10} + \frac{3}{10} = \frac{11}{10} \text{ or } \frac{8}{10} - \frac{3}{10} = \frac{5}{10}$$

Fluency & Skills Practice **Teacher Toolbox**

Assign Subtracting Fractions

Fluency and Skills Practice
Subtracting Fractions

Name: _____

Solve each problem.

1 Sammy has $\frac{4}{5}$ of his art project left to paint. He paints $\frac{2}{5}$ of the project. What fraction of the project is left to paint?

2 Marianne has $\frac{8}{9}$ of a yard of green ribbon. She uses $\frac{3}{9}$ of a yard for a craft project. How much green ribbon is left?

3 Yuna plans to run 1 mile. She has run $\frac{7}{10}$ of a mile so far. What fraction of a mile does she have left to run?

4 Alex and Brady are helping to pack books into a box. Together they pack $\frac{7}{8}$ of the books. Alex packs $\frac{3}{8}$ of the books. What fraction of the books does Brady pack?

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Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

5 $\frac{1}{6}$ of the quilt is white; $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$; $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$

Medium

6 $\frac{1}{8}$; models should show $\frac{9}{8} - \frac{8}{8} = \frac{1}{8}$; See Student Worktext page for a number line model. Area models should be divided into eighths and have 9 parts shaded, and 8 parts crossed out.

Basic

7 She eats $\frac{2}{6}$ of the pizza; $\frac{6}{6} - \frac{4}{6} = \frac{2}{6}$

Challenge

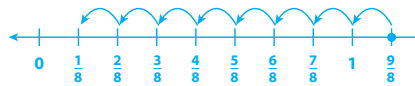
5 Anna makes a quilt by sewing together green, white, and yellow fabric. When she finishes, $\frac{2}{6}$ of the quilt is green, and $\frac{3}{6}$ is yellow. The rest is white. What fraction of the quilt is white? Show your work.

Student work may include an area model, number line, or equations, and should show that the whole quilt is $\frac{6}{6}$.

Solution $\frac{1}{6}$ of the quilt is white.

6 Find $\frac{9}{8} - \frac{8}{8}$. Use a number line or an area model to show your thinking.

Possible student work:

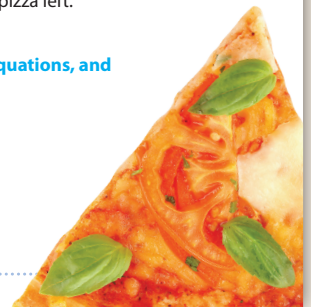


Solution $\frac{1}{8}$

7 Shanice has 1 whole pizza. She eats some of it and has $\frac{4}{6}$ of the pizza left. What fraction of the pizza does she eat? Show your work.

Student work may include an area model, number line, or equations, and should show that the whole pizza is $\frac{6}{6}$.

Solution She eats $\frac{2}{6}$ of the pizza.



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ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Connect It*.

Levels 1–3

Listening/Speaking Use with *Connect It* problem 2. Point to the first equation for the second **Model It**. Ask: *What do you notice about the denominators?* Provide a sentence frame for student responses: *The fractions have the same denominator.* Point to the numerators. Say: $1 + 1 + 1 + 1 + 1 = 5$. *What do you notice when the numerators are added?* Provide a sentence frame for student responses: *The numerators have a sum of 5.* Ask students to look at the remaining equations and tell what they notice about the denominators and numerators.

Levels 2–4

Speaking/Writing Use with *Connect It* problem 2. Ask students to look at the equations for the second **Model It** and then answer the following discussion questions:

- *What do you notice about the denominators?*
- *What do you notice about the numerators when you add them?*

If students respond with incomplete sentences, restate their responses in complete sentences. For example, *Yes, the denominators are the same, 6.* Have students reread *Connect It* problem 2. Have them verbally respond to the question before writing their responses.

Levels 3–5

Speaking/Writing Have partners read *Connect It* problem 2. Provide the following discussion questions for partners to use as they review the equations for the second **Model It**:

- *What do you observe about the equations?*
- *What do the equations have in common?*
- *How can you decompose the fraction $\frac{5}{6}$ into sums of different fractions?*

Have students reread *Connect It* problem 2. Encourage them to verbally respond before writing their responses. Provide the following terms for them to use in their responses: *fractions, numerator, denominator, add, and sum.*

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session, students solve a problem that requires them to decompose a fraction into a sum of fractions in more than one way. Students model the fractions either on paper or with manipulatives to show different ways to break apart a fraction. The purpose of this problem is to have students develop strategies for decomposing fractions.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' facility with adding fractions.

How Have students use fraction tiles or fraction circles to find sums of two eighths fractions.

1. $\frac{1}{8} + \frac{1}{8} = \dots\dots\dots$

2. $\frac{1}{8} + \frac{3}{8} = \dots\dots\dots$

3. $\frac{1}{8} + \frac{5}{8} = \dots\dots\dots$

Solutions

1. $\frac{2}{8}$
2. $\frac{4}{8}$
3. $\frac{6}{8}$

Develop Language

Why Develop understanding of the term *justify*.

How Explain that to justify an answer to a math problem, students need to prove that it is correct. Say: *When you decompose fractions, you can use a diagram to justify your answer.* Ask them to review the *Try It*, *Model It*, and *Connect It* activities to find examples of when they have justified an answer using a diagram or model.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify what the problem is asking them to do.

Ask *What fraction describes the amount of reading Dan has left to complete? What are the days of the week on which Dan could complete his reading? On how many of those days does he plan to complete his reading?*

Develop Decomposing Fractions

Read and try to solve the problem below.

Dan has $\frac{5}{6}$ of his reading left to complete for the week. He plans to complete his reading on two or more days of the week from Monday to Friday. What are two different ways he could plan to complete his reading? Use a fraction to describe the part of his reading he does each day.



TRY IT

Possible student work:

Sample A:

M	T	W	Th	F
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	0	0

$\frac{1}{6}$ on each day Monday to Friday; $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}$

$\frac{1}{6}$ on Monday, $\frac{2}{6}$ on Tuesday, and $\frac{2}{6}$ on Wednesday; $\frac{1}{6} + \frac{2}{6} + \frac{2}{6} = \frac{5}{6}$

Sample B:

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\frac{5}{6} = \frac{2}{6} + \frac{3}{6}$$

Dan could do $\frac{1}{6}$ of his reading on each day Monday to Friday.

Dan could do $\frac{2}{6}$ of his reading on Monday and $\frac{3}{6}$ of his reading on Tuesday.



Math Toolkit

- counters
- fraction circles
- fraction tiles
- fraction bars
- number lines



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I do not understand how...

DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did as they talk to each other.

Support as needed with questions such as:

- *How did you get started?*
- *How did you decide what strategy to use?*

Common Misconception Look for students who list fractions that do not have a sum of $\frac{5}{6}$. Have students use fraction tiles to show different ways to make a sum of $\frac{5}{6}$.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical models showing two ways to decompose $\frac{5}{6}$
- drawings showing two ways to decompose $\frac{5}{6}$
- equations with two or more addends that make a sum of $\frac{5}{6}$

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the part of his reading that Dan does each day? Where does your model show the part of his reading that Dan completes for the week?*

Listen for Students should recognize that accurate responses include fractions with a denominator of 6 and representations with 6 equal parts. Students' responses should also include two or more addends that have a sum of $\frac{5}{6}$.

MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the $\frac{5}{6}$ of his reading that Dan has left to complete
- different ways to break apart $\frac{5}{6}$

Ask *What does it mean to decompose a fraction? How do the models show the parts that $\frac{5}{6}$ is decomposed into?*

Listen for Decompose means to break a fraction into parts. The models show fractions with denominators of 6 that have a sum of $\frac{5}{6}$.

For area models, prompt students to identify how the models are labeled to represent the problem.

- *How are the two models alike? How are they different?*
- *How is $\frac{5}{6}$ shown in each model?*
- *How are the parts shown in each model?*

For equations, prompt students to recognize the strategy used to generate the list of equations.

- *What is the same about each of the equations? What is different?*
- *What do you notice about the numerators of the fractions? the denominators?*
- *What pattern do you notice in how the equations are listed?*

Explore different ways to understand decomposing fractions.

Dan has $\frac{5}{6}$ of his reading left to complete for the week. He plans to complete his reading on two or more days of the week from Monday to Friday. What are two different ways he could plan to complete his reading? Use a fraction to describe the part of his reading he does each day.

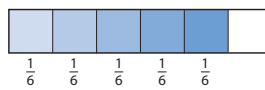
MODEL IT

You can use models to show how to decompose a fraction in different ways.

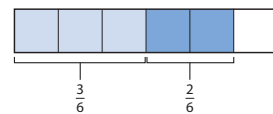
When you decompose a fraction, you break it into parts.

The models show two ways to decompose $\frac{5}{6}$.

One way:



Another way:



MODEL IT

You can also use equations to decompose a fraction in different ways.

You can list different ways to add fractions to make $\frac{5}{6}$.

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{2}{6}$$

$$\frac{5}{6} = \frac{1}{6} + \frac{2}{6} + \frac{2}{6}$$

$$\frac{5}{6} = \frac{3}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\frac{5}{6} = \frac{3}{6} + \frac{2}{6}$$

$$\frac{5}{6} = \frac{4}{6} + \frac{1}{6}$$

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Deepen Understanding

Equation Model

SMP 7 Use structure.

When discussing the second *Model It*, prompt students to consider how the first equation is the sum of unit fractions.

Ask *How is the first equation different from the other equations?*

Listen for It has the most addends, and all the addends are $\frac{1}{6}$.

Ask *How would you describe the addends in the first equation?*

Listen for All the addends are unit fractions.

Ask *If you want to add fractions to make $\frac{4}{6}$ instead of $\frac{5}{6}$, how could the first equation help you?*

Listen for $\frac{4}{6}$ could also be written as a sum of unit fractions.

Generalize *For any fraction with a numerator greater than 1, what is one way you can always decompose the fraction? Have students explain their reasoning. Listen for understanding that a fraction with a numerator greater than 1 can always be decomposed into a sum of unit fractions with the same denominator.*

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use those numbers to consider general methods for decomposing a fraction into two or more parts.

Monitor and Confirm

- ① – ④ Check for understanding that:
- 6 is the number of equal parts and 5 is the number of parts left
 - the numerators of the addends have a sum of 5
 - $\frac{4}{6}$ is the greatest amount in one day because reading is done on two or more days
 - $\frac{5}{6}$ can be decomposed in different ways

Support Whole Class Discussion

- ⑤ Look for the idea that finding all the ways to make the numerator helps find all the ways to decompose a fraction.

Deepen Understanding**Decompose a Fraction**

SMP 2 Reason abstractly.

To support discussion of problem 5, prompt students to consider how decomposing whole numbers can help them decompose fractions.

Ask Look at the second Model It. How does the first equation show a way to make 5? How does the second equation show a way to make 5?

Listen for In the first equation, the sum of the numerators is $1 + 1 + 1 + 1 + 1 = 5$ and, in the second, it is $1 + 1 + 1 + 2 = 5$.

Generalize How does finding all the ways to make the numerator of a fraction help you find all the ways to decompose a fraction? Have students explain their reasoning. Listen for understanding that when the denominators are all the same, the number of ways to make the numerator is the same as the number of ways to make the fraction.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to decompose a fraction in different ways.

- ① Look at the first **Model It**. How many equal parts are in each model?6.....
How many shaded parts are in each model?5.....
- ② Look at the equations in the second **Model It**. How can you tell if two or more fractions add to make $\frac{5}{6}$?

Possible answer: All the fractions are sixths. If the numerators of the fractions have a sum of 5, then the fractions add to make $\frac{5}{6}$.

- ③ What is the greatest amount of his reading that Dan could do in one day?
Dan could do $\frac{4}{6}$ of his reading.

- ④ What are two different ways that Dan could do his reading?

Possible answer: Dan could do $\frac{1}{6}$ of his reading on each of five days. Dan could also do $\frac{3}{6}$ of his reading on one day and $\frac{2}{6}$ on another day.

- ⑤ Explain how to find all the different ways to decompose a fraction.

Possible answer: Find all the combinations of numbers as numerators that added together equal the numerator of the fraction.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for decomposing a fraction? Explain.

Students may respond that they like drawing an area model to help them visualize ways to decompose a fraction. Other students may respond that they like making a list of equations to keep track of ways to decompose a fraction.

**Hands-On Activity**

Use fraction tiles to decompose fractions.

If . . . students are unsure about breaking a fraction into parts,

Then . . . use the activity below to provide a more concrete experience.

Materials For each pair: 1 set of fraction tiles or fraction circles

- Distribute fraction tiles or fraction circles to each pair.
- Have one student build $\frac{4}{5}$ using 4 one-fifth fraction tiles or circles. Then have the student record the relationship shown as an equation:
 $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$.
- Have the partner break apart the fraction tiles in a different way and record the relationship (e.g., $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$).
- Challenge pairs to find other ways to break apart $\frac{4}{5}$.
- Have students switch roles and repeat the activity for another fraction, such as $\frac{7}{10}$. Make sure students start by building the fraction with unit fraction tiles.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in the precision of students' models as drawing equal parts accurately is difficult and here precise drawings are not necessary.

- 7 Answers will vary. Check that the numerators of the addends have a sum of 7. Possible answer: $\frac{7}{8} = \frac{6}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$
- 8 a. $\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4}$
 b. $\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$
 c. $\frac{9}{12} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12}$

Close: Exit Ticket

- 9 See possible diagram on the Student Worktext page. Check that students' diagrams show $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$.

Students' solutions should indicate understanding of:

- how to divide a whole into equal parts
- using a model to show the addition of fractions that refer to the same whole

Error Alert If students cannot make a visual model to represent $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$, then have students use fraction tiles or fraction circles to model the equation and then make a sketch of the physical model, labeling each part of the model.

APPLY IT

Use what you just learned to solve these problems.

- 7 Find three ways to decompose $\frac{7}{8}$ into a sum of other fractions. Draw a model for each way to show how you know the way is correct. Show your work.

Possible student work:



Possible answer: $\frac{7}{8} = \frac{6}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

Solution

- 8 Complete the equations to show a way to decompose each fraction.

a. $\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4}$

b. $\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$

c. $\frac{9}{12} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12}$

- 9 Draw a diagram to justify your answer to problem 8b.

Possible student work:



$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

Solutions

- 1 a. $\frac{3}{5} = \frac{1}{5} + \frac{2}{5}$
 b. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$
Basic

- 2 See Student Worktext page for completed model. The model should show $\frac{1}{5}$ shaded one way and $\frac{2}{5}$ shaded in another way.
Basic

Practice Decomposing Fractions

Study the Example showing how to decompose a fraction in different ways. Then solve problems 1–5.

EXAMPLE

Sarah's family has $\frac{4}{8}$ of a cherry pie left over. Sarah and her sister share the leftover pie. What are two different ways that Sarah and her sister can each get some of the pie?



$\frac{2}{8} + \frac{2}{8} = \frac{4}{8}$

Sarah and her sister each get $\frac{2}{8}$ of the pie.



$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

Sarah gets $\frac{1}{8}$ of the pie, and her sister gets $\frac{3}{8}$ of the pie.

- 1 Complete the equations to show how to decompose $\frac{3}{5}$ in two different ways.

a. $\frac{3}{5} = \frac{1}{5} + \frac{2}{5}$

b. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

- 2 Shade the area model below to show the equation in problem 1a.



Fluency & Skills Practice **Teacher Toolbox**

Assign Decomposing Fractions

Fluency and Skills Practice
 Decomposing Fractions
 Name: _____

Find three ways to decompose each fraction into a sum of other fractions with the same denominator.

1 $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
 $\frac{3}{4} = \frac{2}{4} + \frac{1}{4}$
 $\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$

2 $\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$
 $\frac{5}{6} = \frac{2}{6} + \frac{1}{6} + \frac{2}{6}$
 $\frac{5}{6} = \frac{1}{6} + \frac{2}{6} + \frac{2}{6}$

3 $\frac{7}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
 $\frac{7}{8} = \frac{3}{8} + \frac{2}{8} + \frac{2}{8}$
 $\frac{7}{8} = \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$

4 $\frac{9}{10} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$
 $\frac{9}{10} = \frac{4}{10} + \frac{3}{10} + \frac{2}{10}$
 $\frac{9}{10} = \frac{1}{10} + \frac{4}{10} + \frac{4}{10}$

5 Describe your strategy for finding the missing numbers.

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3 B; The equation is true.

C; The equation is true.

E; The equation is true.

Medium

4 Answers will vary. Possible answers: $\frac{1}{6} + \frac{1}{6} + \frac{4}{6}$, $\frac{1}{6} + \frac{2}{6} + \frac{3}{6}$, or $\frac{2}{6} + \frac{2}{6} + \frac{2}{6}$. See possible models on the Student Worktext page.

Medium

5 Yes; Possible explanation: $\frac{7}{12} + \frac{1}{12} = \frac{8}{12}$ and $\frac{4}{12} + \frac{4}{12} = \frac{8}{12}$. Since both expressions have a value of $\frac{8}{12}$, $\frac{7}{12} + \frac{1}{12} = \frac{4}{12} + \frac{4}{12}$. Students may also draw models to represent each expression and show that the models represent the same part of the whole.

Challenge

3 Select all the equations that show a correct way to represent $\frac{7}{10}$.

(A) $\frac{1}{10} + \frac{5}{10} = \frac{7}{10}$

(B) $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

(C) $\frac{1}{10} + \frac{2}{10} + \frac{4}{10} = \frac{7}{10}$

(D) $\frac{1}{10} + \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

(E) $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{7}{10}$

4 Vijay has $\frac{6}{6}$ of a cup of raisins. He wants to put the raisins into three snack bags.

What are two different ways he could put raisins into three snack bags?

Use a model to show each way. Show your work.

Possible student work:



Solution Possible answer: One way: He could put $\frac{4}{6}$ of a cup of raisins in one bag, $\frac{1}{6}$ of a cup in the second bag, and $\frac{1}{6}$ of a cup in the third bag. Another way: He could put $\frac{2}{6}$ of a cup of raisins in each snack bag: $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{6}{6}$.

5 Is $\frac{7}{12} + \frac{1}{12}$ equivalent to $\frac{4}{12} + \frac{4}{12}$? Explain your answer.

Yes. Possible explanation: $\frac{7}{12} + \frac{1}{12} = \frac{8}{12}$ and $\frac{4}{12} + \frac{4}{12} = \frac{8}{12}$. Since both expressions have a value of $\frac{8}{12}$, $\frac{7}{12} + \frac{1}{12}$ is equivalent to $\frac{4}{12} + \frac{4}{12}$.

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ELL English Language Learners: Differentiated Instruction

Prepare for Session 5
Use with *Apply It*.

Levels 1–3

Reading/Speaking Mime the words that name actions as you read *Apply It* problem 1. Write: *Add the numerators* and *Subtract the numerators*. Read the problem with students. Ask them to find the whole number and circle it. Ask: *How can you subtract $\frac{1}{3}$ from 1?* Have students discuss possible solutions with partners. Ask students to say how else they could represent 1 using fractions. Write $1 = \frac{3}{3}$. Say: *This whole number and fraction are equal. They have the same value.* Have students say: *One whole is equal to three thirds.* Ask: *How much of the smoothie does Ruth drink?* Write $\frac{1}{3}$. Have students write an equation to subtract.

Levels 2–4

Speaking/Writing Have students read *Apply It* problem 1. Have students retell the problem with partners. As they retell, draw a number line divided into three equal parts and label points $\frac{1}{3}$, $\frac{2}{3}$, and 1. Ask students how else they could label the 1 on the number line and then cross out the 1 and write $\frac{3}{3}$ under it. Ask: *Why did I rewrite 1 as $\frac{3}{3}$?* Have pairs discuss why these numbers have the same value. Ask: *How can the number line be used to solve the problem? What equation can I write to represent the problem?* Ask students to work with partners to write an equation and solve the problem. Ask: *What steps did you use to solve the problem?* Write the steps with students and have them read the sentences aloud.

Levels 3–5

Reading/Writing Have students read *Apply It* problem 1 with their partners and discuss their ideas. Ask guiding questions as you listen to discussions: *Why do you think you need to write the whole number as a fraction? Why do the whole number 1 and the fraction $\frac{3}{3}$ have the same value? How will you write the whole number as a fraction to write an equation? Why do you use subtraction to solve the problem?* When students have solved the problem, ask them to write on strips of paper the steps they followed to solve it. Then have the partner groups shuffle the strips and exchange them with other groups. Ask students to read the strips with their partners and put them in order.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session, students solve word problems involving addition and subtraction of fractions and decomposing fractions and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of adding fractions with like denominators.

How Have students find $\frac{4}{10} + \frac{2}{10}$ using any strategy they want.

$$\frac{4}{10} + \frac{2}{10} = ?$$

Solution
 $\frac{6}{10}$

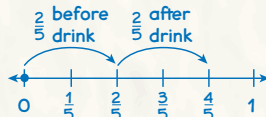
Refine Adding and Subtracting Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Jessica hikes $\frac{2}{5}$ of a mile on a trail before she stops to get a drink of water. After her drink, Jessica hikes another $\frac{2}{5}$ of a mile. How far does Jessica hike in all?

Look at how you could show your work using a number line.



Solution Jessica hikes $\frac{4}{5}$ of a mile.

The student used labels and “jump” arrows to show each part of the hike on a number line. It is just like adding whole numbers!



PAIR/SHARE

How else could you solve this problem?

APPLY IT

1 Ruth makes 1 fruit smoothie. She drinks $\frac{1}{3}$ of it. What fraction of the fruit smoothie is left? Show your work.

Possible student work using an equation:

$$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

What fraction represents the whole fruit smoothie?

PAIR/SHARE

How did you and your partner decide what fraction to start with?

Error Alert

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{6}{20}$	have added both the numerators and the denominators.	Remind students that the denominator tells the kind of parts you are adding. Explain that just as 4 apples + 2 apples = 6 apples, 4 tenths + 2 tenths = 6 tenths.
$\frac{3}{10}$	have added numerators, added denominators, and then written an equivalent fraction with a denominator of 10.	Remind students that the denominator tells the kind of parts you are adding. Explain that just as 4 apples + 2 apples = 6 apples, 4 tenths + 2 tenths = 6 tenths.
$\frac{2}{10}$	have subtracted the fractions.	Remind students to read the problem carefully to be sure they are using the correct operation.
$\frac{1}{5}$	have subtracted the fractions and written an equivalent fraction.	Remind students to read the problem carefully to be sure they are using the correct operation.

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EXAMPLE

Jessica hikes $\frac{4}{5}$ of a mile; the number line shown is one way to solve the problem. Students could also solve the problem by drawing a model that is divided into fifths and shading 4 parts (2 parts out of 5 and 2 parts out of 5).

Look for Add the numerators, $2 + 2$.

APPLY IT

- 1 $\frac{2}{3}$ of the smoothie; Students could solve the problem using the equation $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$.

DOK 2

Look for $\frac{3}{3}$ is the fraction representing 1 whole that is hidden in the problem.

- 2 $\frac{5}{10}$ of the bunch; Students could solve the problem by drawing a picture of 10 balloons and labeling 3 balloons as red and 2 balloons as blue.

DOK 2

Look for The solution requires two steps: addition $(\frac{3}{10} + \frac{2}{10})$ and subtraction $(\frac{10}{10} - \frac{5}{10})$.

- 3 C; Students could solve this problem using the equation $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$.

Explain why the other two answer choices are not correct:

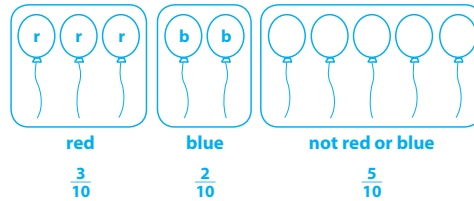
A is not correct because you are not subtracting $\frac{1}{6}$ from $\frac{2}{6}$; this is an addition problem.

B is not correct because $\frac{1}{3}$ is not equivalent to $\frac{3}{6}$.

DOK 3

- 2 Mr. Chang has a bunch of balloons. $\frac{3}{10}$ of the bunch is red. $\frac{2}{10}$ of the bunch is blue. What fraction of the bunch is not red or blue? Show your work.

Possible student work using a model:



Solution $\frac{5}{10}$ of the bunch

- 3 Emily eats $\frac{1}{6}$ of a bag of carrots. Nick eats $\frac{2}{6}$ of the same bag of carrots. What fraction of the bag of carrots do Emily and Nick eat altogether?

- Ⓐ $\frac{1}{6}$
 Ⓑ $\frac{1}{3}$
 Ⓒ $\frac{3}{6}$
 Ⓓ $\frac{3}{12}$

Rob chose Ⓒ as the correct answer. How did he get that answer?

Rob added both the numerators and the denominators.

I think that there are at least two different steps to solve this problem.

**PAIR/SHARE**

What other problem in this lesson is similar to this one?

To find the fraction of the bag Emily and Nick ate together, should you add or subtract?

PAIR/SHARE

Does Rob's answer make sense?

4 C; Add the number of yards Lin uses for the project and the number of yards left, $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$.
DOK 2

5 C; Find the combined amount of cake eaten, $\frac{2}{12} + \frac{3}{12} = \frac{5}{12}$. Subtract the sum from the whole, $\frac{12}{12} - \frac{5}{12} = \frac{7}{12}$.
DOK 2

Error Alert Students may not recognize this as a two-step problem and either fail to add $\frac{2}{12}$ and $\frac{3}{12}$ before subtracting, or subtract $\frac{2}{12}$ from $\frac{3}{12}$.

6 $\frac{1}{3}$ of a cup more milk than oil; Subtract the amount of oil Lee uses from the amount of milk she uses, $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$.
DOK 2

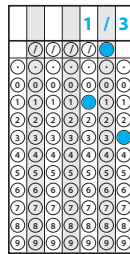
4 Lin buys some cloth. He uses $\frac{5}{8}$ of a yard for a school project. He has $\frac{2}{8}$ of a yard left. How much cloth does Lin buy?

- A $\frac{3}{8}$ of a yard
- B $\frac{7}{16}$ of a yard
- C $\frac{7}{8}$ of a yard
- D $\frac{8}{8}$ of a yard

5 Carmela cuts a cake into 12 equal-sized pieces. She eats $\frac{2}{12}$ of the cake, and her brother eats $\frac{3}{12}$ of the cake. What fraction of the cake is left?

- A $\frac{1}{12}$
- B $\frac{5}{12}$
- C $\frac{7}{12}$
- D $\frac{12}{12}$

6 Lee makes muffins. She uses $\frac{2}{3}$ of a cup of milk and $\frac{1}{3}$ of a cup of oil. How many more cups of milk than oil does she use?



Differentiated Instruction

RETEACH

Hands-On Activity
Use fraction bars to add.

Students struggling with concepts that fractions written as numbers or shown as visual models represent a part or multiple parts of a whole

Will benefit from additional work with concrete representations of fraction addition and subtraction

Materials For each student: markers, Activity Sheet *Fraction Bars* (2 bars for fourths, 2 bars for thirds, 2 bars for sixths, 2 bars for eighths)

- Distribute fourths fraction bars and markers. Tell students to color $\frac{1}{4}$ of the fraction bar. Then have them color another $\frac{1}{4}$ of the fraction bar.
- Write $\frac{1}{4} + \frac{1}{4}$ on the board. Have students use their fraction bars to show that the sum is $\frac{2}{4}$.
- Then have students color $\frac{3}{4}$ of another fourths fraction bar and cross out $\frac{2}{4}$. Write $\frac{3}{4} - \frac{2}{4}$ and have students show that the difference is $\frac{1}{4}$.
- Repeat for other fractions with denominators such as thirds, sixths, and eighths.

EXTEND

Challenge Activity
Write a problem for a given sum.

Students who achieved proficiency
Will benefit from deepening understanding of fraction addition and subtraction

- Tell students that the sum of two fractions is $\frac{2}{5}$. However, the original fractions did not have denominators of 5.
- Challenge students to write a fraction addition problem using denominators other than 5 that has a sum of $\frac{2}{5}$.
[Possible answer: $\frac{3}{10} + \frac{1}{10}$]

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- 7** **A**; The model shows $\frac{2}{8}$ shaded light blue for one girl's section and $\frac{4}{8}$ shaded dark blue for the other girl's section. The total shaded sections represent the total fraction of the room they paint.
- D**; The equation $\frac{6}{8} = \frac{3}{8} + \frac{3}{8}$ models the problem and shows that each girl could paint $\frac{3}{8}$ of the room.
- E**; The equation $\frac{6}{8} = \frac{5}{8} + \frac{1}{8}$ models the problem and shows that one girl could paint $\frac{5}{8}$ of the room and the other could paint $\frac{1}{8}$.

DOK 2

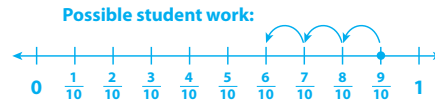
- 8** $\frac{6}{10}$ of a bucket; Possible student work using an equation: $\frac{9}{10} - \frac{3}{10} = \frac{6}{10}$

DOK 2

- 7** Lucy and Melody work together to paint $\frac{6}{8}$ of a room. Which models could be used to show how much of the room each girl paints?

- (A)** 
- (B)** 
- (C)** 
- (D)** $\frac{6}{8} = \frac{3}{8} + \frac{3}{8}$
- (E)** $\frac{6}{8} = \frac{5}{8} + \frac{1}{8}$

- 8** Cole and Max pick $\frac{9}{10}$ of a bucket of blueberries in all. Cole picks $\frac{3}{10}$ of a bucket of blueberries. What fraction of a bucket of blueberries does Max pick? Show your work.




Solution $\frac{6}{10}$ of a bucket

9 **MATH JOURNAL**

Ms. Jones cuts an apple into eighths. She eats $\frac{3}{8}$ of the apple and gives the rest to her son and daughter. Describe two different ways her son and daughter can share the rest of the apple if they each have some of the apple.

Possible answer: The whole apple is $\frac{8}{8}$. $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$, so Ms. Jones's son and daughter share $\frac{5}{8}$ of the apple. They could share $\frac{5}{8}$ as $\frac{2}{8}$ for one of them and $\frac{3}{8}$ for the other, or as $\frac{1}{8}$ for one of them and $\frac{4}{8}$ for the other.

 **SELF CHECK** Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–9

Add and subtract fractions.

All students will benefit from additional work with adding and subtracting fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE



Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:


- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 **MATH JOURNAL**

Student responses should indicate an understanding of fraction subtraction as well as an understanding that there is more than one way to decompose a fraction.

Error Alert If students decompose $\frac{3}{8}$ rather than $\frac{5}{8}$, then have students model the problem using 8 one-eighth fraction tiles or fraction circles to represent the whole apple and then model the amount Ms. Jones eats and different ways to show the amount her son and daughter can share.

 **SELF CHECK** Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Tested Skills

Assesses 4.NF.B.3b, 4.NF.B.3d

Problems on this assessment form require students to be able to use fraction models and number lines to add and subtract fractions with like denominators. Students will also need to be familiar with basic addition and subtraction facts, composing and decomposing fractions, and writing whole numbers as fractions.

Error Alert Students may:

- add instead of subtracting or vice versa.
- add or subtract denominators as well as numerators.
- interpret units on the number line incorrectly.
- add or subtract whole numbers incorrectly.

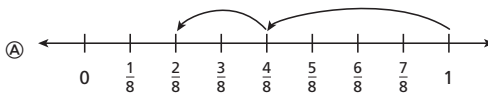
Solutions

1 B, D
2 points
DOK 2

2 D
1 point
DOK 2

Solve the problems.

- 1** Nobu is making a bracelet with 8 equal sections. He makes $\frac{4}{8}$ of the bracelet on Saturday and $\frac{2}{8}$ of the bracelet on Sunday. Which model can be used to find the total fraction of the bracelet that Nobu makes on Saturday and Sunday? Select all the correct answers.



- 2** In a science class, students spend $\frac{2}{10}$ of the time reading and $\frac{7}{10}$ of the time doing an experiment. They spend the rest of the time cleaning up. What fraction of science class time do students spend cleaning?

- A** $\frac{9}{10}$
B $\frac{5}{10}$
C $\frac{9}{20}$
D $\frac{1}{10}$

Short Response Scoring Rubric (2 points)

Points	Expectations
2	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of mathematical concepts and/or procedures.
1	Response contains mostly correct solution(s) and shows partial understanding of mathematical concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of mathematical concepts and/or procedures.

Multiple Select Scoring

2 points	1 point	0 points
All answers are correct	1 incorrect answer	2 or more incorrect answers

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Lesson 20 Quiz continued

- 3 Student models may have any 4 parts shaded.



$$\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$$

2 points

DOK 2

- 4 A (True);
D (False);
E (True);
H (False);
J (False)

5 points

DOK 2

- 5 $\frac{2}{6}$

1 point

DOK 2

- 3 Max eats $\frac{1}{6}$ of an orange. His sister eats $\frac{3}{6}$ of the orange. Shade the model and write an equation to show the fraction of the orange they eat in all.

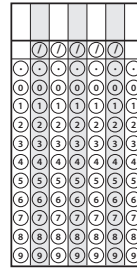


- 4 Lisa needs to measure $\frac{5}{8}$ of a cup of flour. Determine whether each equation shows a way Lisa can use smaller measuring cups to measure that amount.

Choose *True* or *False* for each statement.

	True	False
$\frac{5}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$	(A)	(B)
$\frac{5}{8} = \frac{2}{4} + \frac{3}{4}$	(C)	(D)
$\frac{5}{8} = \frac{3}{8} + \frac{2}{8}$	(E)	(F)
$\frac{5}{8} = \frac{1}{8} + \frac{3}{8}$	(G)	(H)
$\frac{5}{8} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$	(I)	(J)

- 5 What is $\frac{5}{6} - \frac{3}{6}$?



Differentiated Instruction

Teacher Toolbox

RETEACH

Tools for Instruction

Students who require additional support for prerequisite or on-level skills

Will benefit from activities that provide targeted skills instruction

- Grade 4, Lesson 20

REINFORCE

Math Center Activities

Students who require additional practice to reinforce concepts and skills and deepen understanding

Will benefit from small group collaborative games and activities (available in three versions—on-level, below-level, and above-level)

- Grade 4, Lesson 20

EXTEND

Enrichment Activities

Students who have achieved proficiency with concepts and skills and are ready for additional challenges

Will benefit from group collaborative games and activities that extend understanding

- Grade 4, Lesson 20

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