Ready® Classroom

Grade 3 | Volume 2

Mathematics

Sampler

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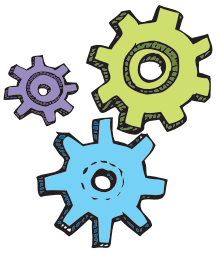

Student Handbook

UNIT 1

Three-Digit Numbers

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UNIT 2

Multiplication and Division
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UNIT 3

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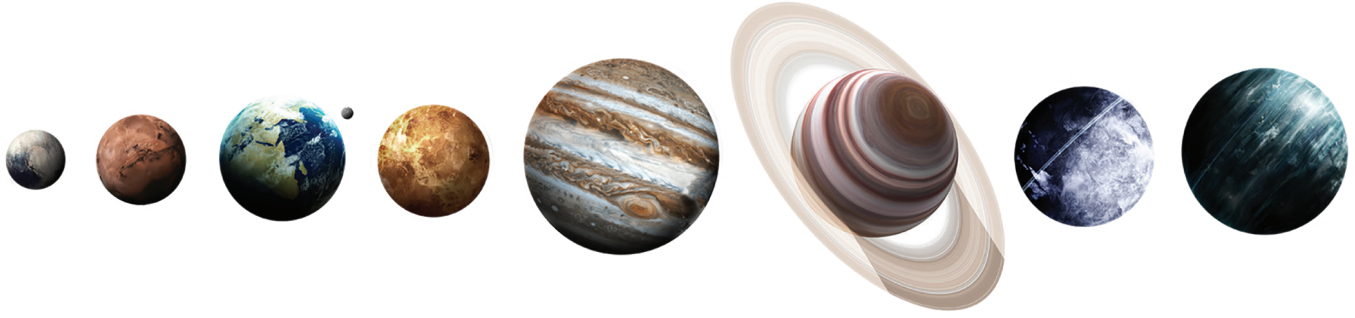
Finding Area, Solving Word Problems, and Using Scaled Graphs



UNIT 4

Fractions

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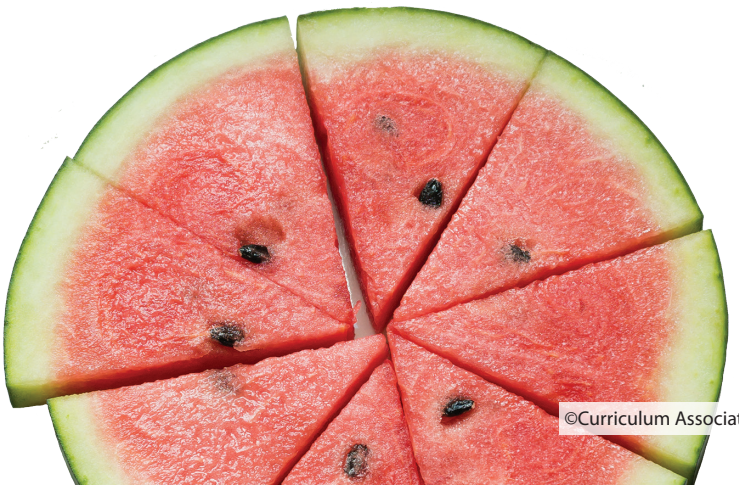

UNIT 5

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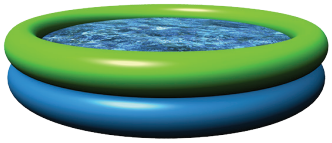


UNIT 6

Shapes

Attributes and Categories, Perimeter and Area, and Partitioning

Student Sample, Lesson 23

Ready Classroom Mathematics lessons begin with a Family Letter that provides activities and instructional supports to foster school, family, and community involvement and partnerships.

Each multiday lesson includes three types of sessions: Explore, Develop, and Refine.

The following pages, 493–520, represent a complete student lesson.

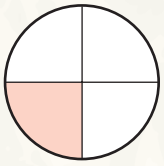
Find Equivalent Fractions

LESSON 23

Dear Family,

This week your child is learning to find equivalent fractions.

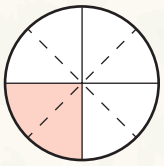
Using a model or diagram to represent equivalent fractions helps make it clear why they are equivalent.



$\frac{1}{4}$

The models to the right show that $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent because they cover the same amount of same-sized circles.

The circle showing $\frac{2}{8}$ has solid lines showing fourths and dotted lines showing how each fourth was cut to make eighths. It helps you see that since eighths are smaller than fourths, you need more of them to cover the same amount.



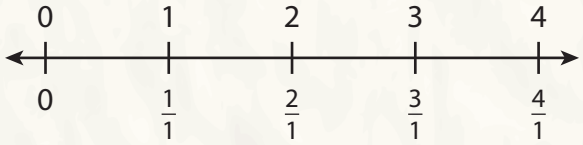
$\frac{2}{8}$

A number line is another model that can show equivalent fractions.

This number line shows both fourths and eighths. Since $\frac{1}{4}$ and $\frac{2}{8}$ are at the same point, $\frac{\mathbf{1}}{\mathbf{4}}$ and $\frac{\mathbf{2}}{\mathbf{8}}$ are equivalent.

A whole number can be written as a fraction too, with a denominator of 1.

A denominator of 1 means the whole has not been cut into parts. One whole can be written $\frac{1}{1}$, 2 wholes as $\frac{2}{1}$, and so forth.



Invite your child to share what he or she knows about finding equivalent fractions by doing the following activity together.

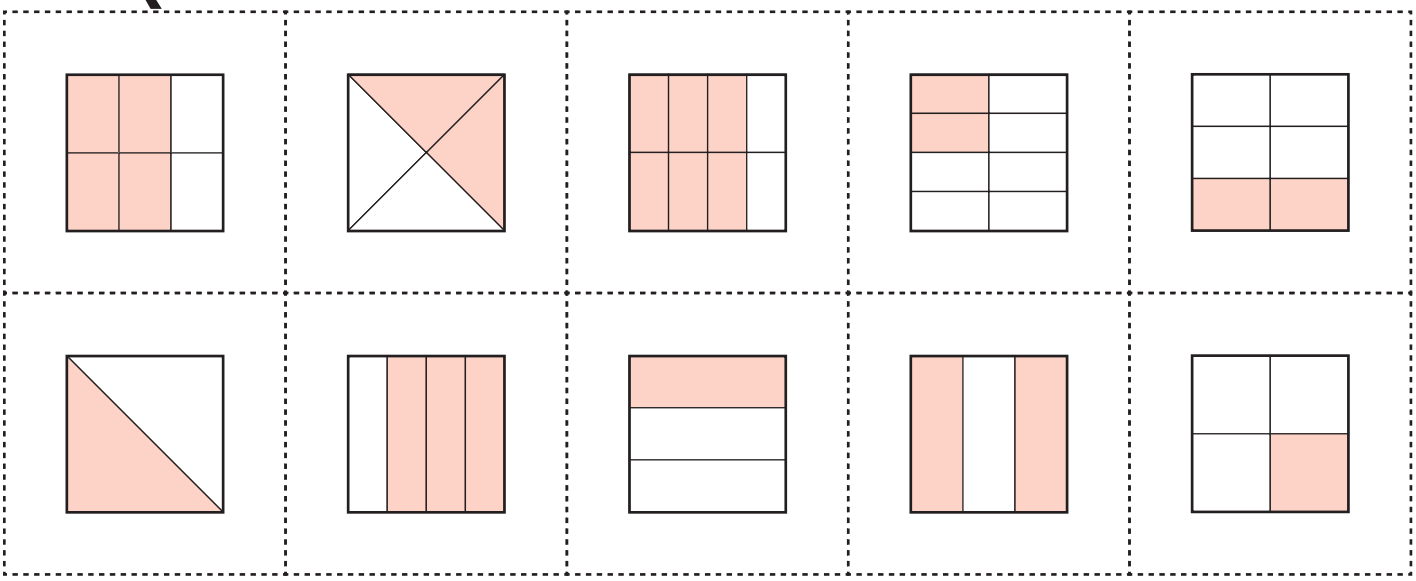
ACTIVITY EQUIVALENT FRACTIONS

Do this activity with your child to recognize equivalent fractions.

Materials cards below, scissors

Play this matching game to practice recognizing equivalent fractions.

- Cut out the cards below and color the backs.
- Mix the cards and place them facedown in two rows.
- Take turns. On your turn, flip two cards. Name the fractions.
- If the cards show equivalent fractions, keep them. If they are not equivalent, turn them back over in the same places as before.
- When all the equivalent fractions have been found, the player with the most cards is the winner.
- As you play, ask your child questions such as:
  - *If you keep the cards, how do you know the fractions are equivalent?*
  - *If you turn the cards back over, how do you know the fractions are not equivalent?*



Explore Equivalent Fractions

Learning Targets

- Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Previously you learned that equivalent fractions name the same amount of the whole. In this lesson you will learn more about finding equivalent fractions. Use what you know to try to solve the problem below.

Izzy's mom bakes a cake. She puts chocolate frosting on half of the cake and vanilla frosting on half of the cake. Then Izzy's mom cuts the cake into fourths so that each fourth has either all chocolate or all vanilla frosting.

What fraction other than $\frac{1}{2}$ names the part of the cake that has chocolate frosting?

TRY IT

Math Toolkit

- fraction circles
- 1-inch grid paper
- index cards
- crayons

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

CONNECT IT

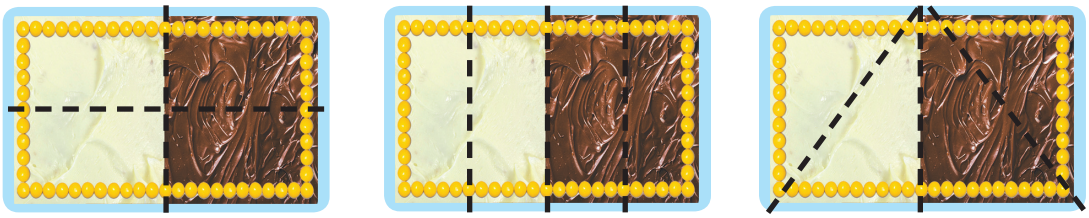
1 LOOK BACK

What fraction other than $\frac{1}{2}$ names the part of the cake that has chocolate frosting? ..... How did you get your answer?

2 LOOK AHEAD

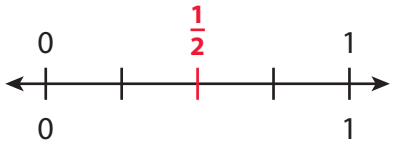
You have seen many different types of fraction models, such as area models, number lines, and fractions bars. You can find equivalent fractions by dividing the same model in different ways.

a. Each cake below shows fourths. Draw lines on one of the cakes to show eighths.



b. How many pieces of the cake have chocolate frosting now? .....

c. You can also look at different equal-sized parts on a number line to find equivalent fractions. Fill in the fraction for fourths that is equivalent to $\frac{1}{2}$.



3 REFLECT

Why does it make sense that $\frac{1}{2}$ and $\frac{2}{4}$ can name the same amount?

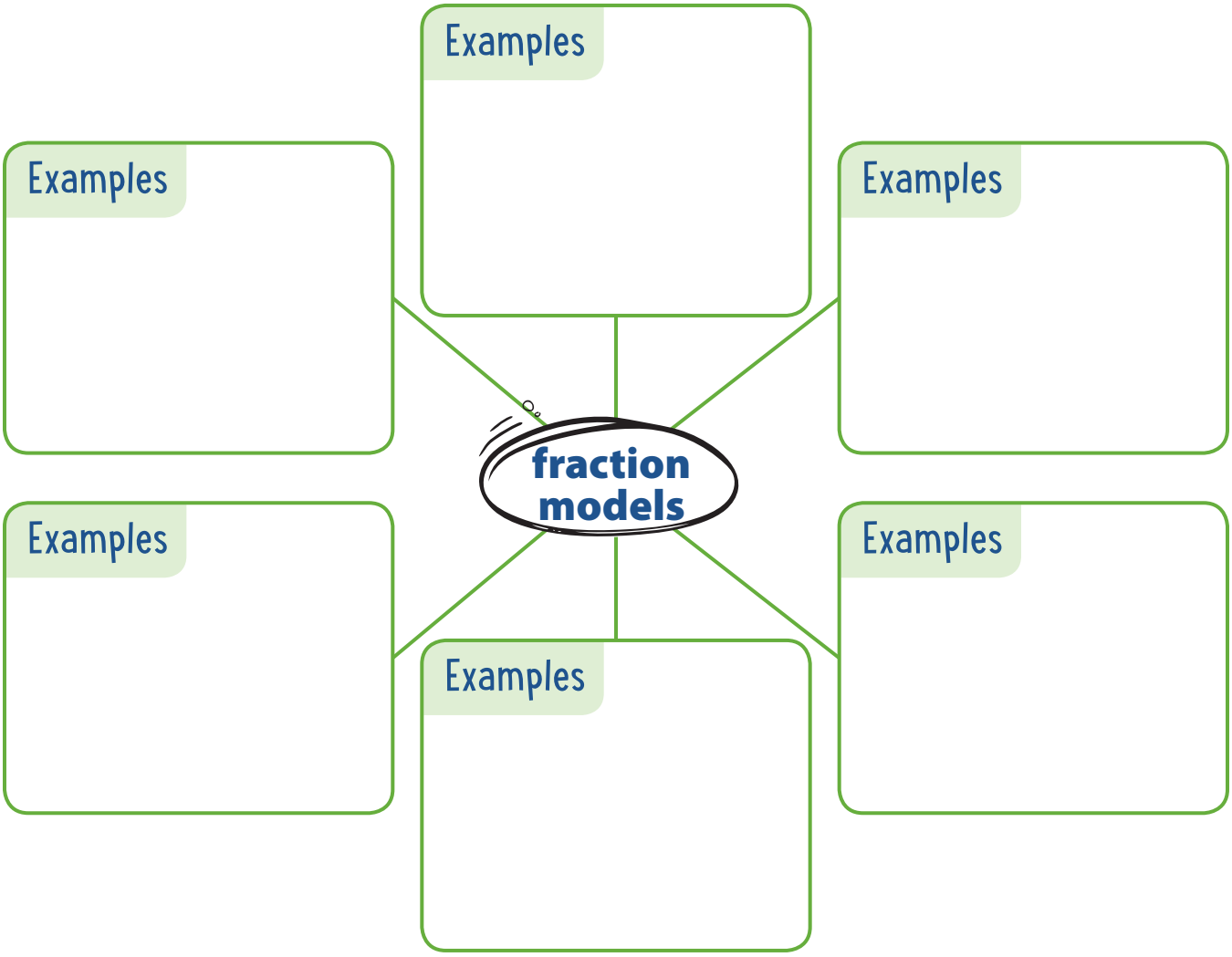
.....

.....

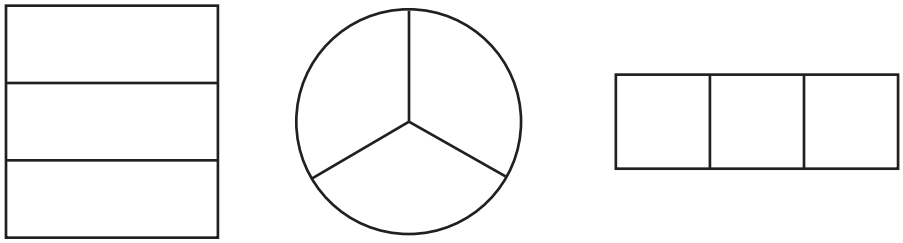
.....

Prepare for Finding Equivalent Fractions

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

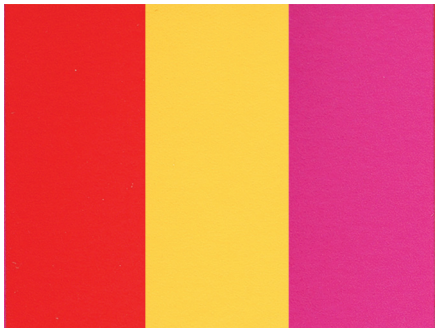


2 Each fraction model below shows thirds. Draw lines on each model to show sixths.



3 Solve the problem. Show your work.

Len has 3 strips of construction paper. Each strip is the same size and a different color—red, yellow, and pink. He tapes the strips together to make a rectangle.



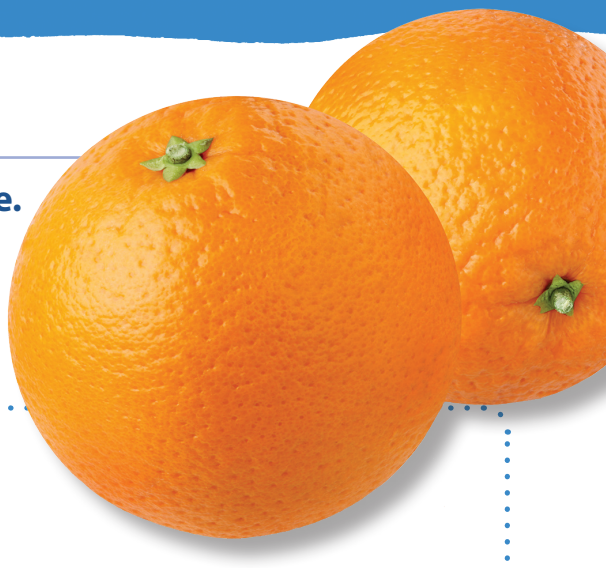
Then Len divides the rectangle into sixths so that each sixth is one color. What fraction other than $\frac{1}{3}$ names the part of the rectangle that is red?

4 Check your answer. Show your work.

Develop Finding Equivalent Fractions

Read and try to solve the problem below.

Carl eats $\frac{2}{8}$ of an orange. Trey's orange is the same size. He eats $\frac{1}{4}$ of it. Show that the two boys eat the same amount of an orange.



TRY IT

Math Toolkit

- fraction tiles
- fraction circles
- number lines
- grid paper

DISCUSS IT

Ask your partner: How did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

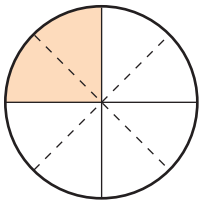
Explore different ways to understand finding equivalent fractions.

Carl eats $\frac{2}{8}$ of an orange. Trey's orange is the same size. He eats $\frac{1}{4}$ of it. Show that the two boys eat the same amount of an orange.

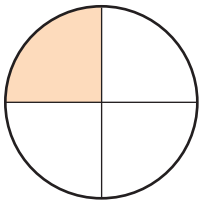
PICTURE IT

You can use models to help find equivalent fractions.

This model shows $\frac{2}{8}$.



This model shows $\frac{1}{4}$.

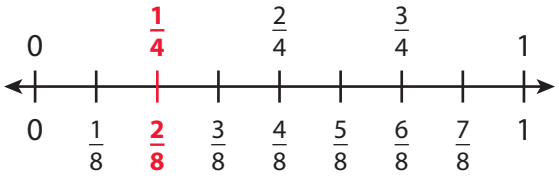


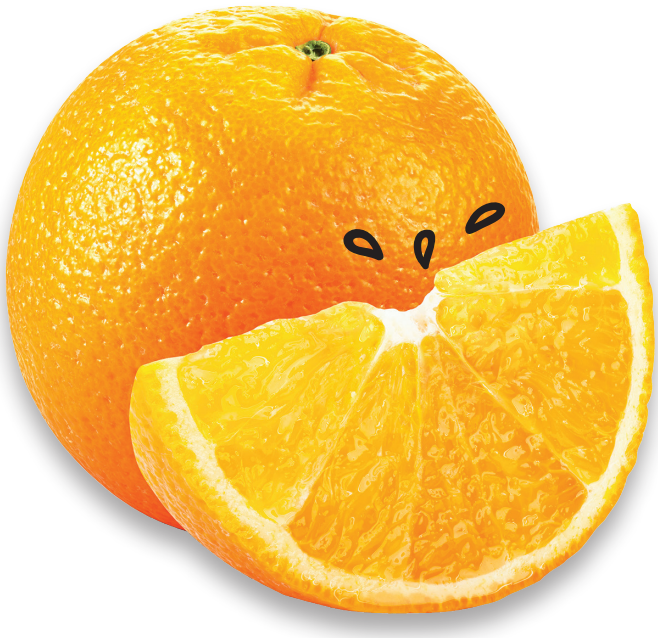
Look at the model of $\frac{2}{8}$. The solid lines divide the circle into fourths. The dashed lines divide each fourth in half to make eighths.

MODEL IT

You can also use a number line to help find equivalent fractions.

This number line shows both fourths and eighths.





CONNECT IT

Now you will use the problem from the previous page to help you understand how to find equivalent fractions.

1. Look at the models in **Picture It**. How do you know that $\frac{2}{8}$ of the first model is shaded?

2. How do you know that $\frac{1}{4}$ of the second model is shaded?

3. Explain how the models show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent.

4. How does the number line in **Model It** show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent?

5. Complete the sentences to show that the fractions of the two oranges name the same amount.

   Use words: Two eighths is equal to _____.

   Use fractions: $\frac{2}{8} =$ _____.

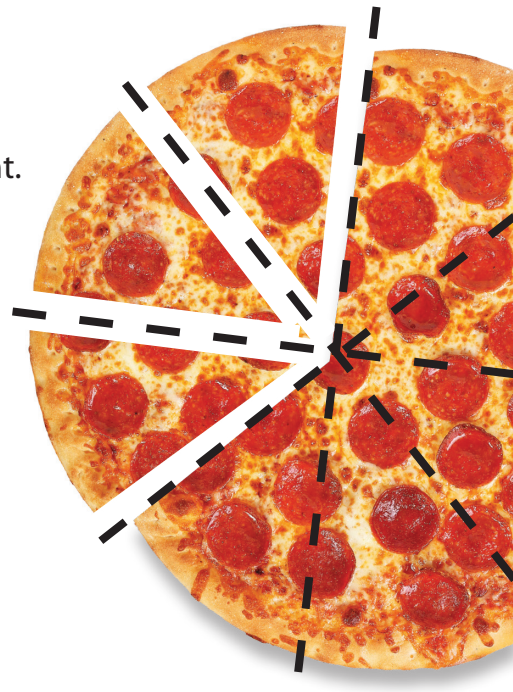
6. Describe two different ways to show two fractions are equivalent.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding equivalent fractions? Explain.

APPLY IT

Use what you just learned to solve these problems.



8 Lina and Adam each order a small pizza. They eat the same amount. Lina eats $\frac{3}{4}$ of her pizza. Adam's pizza is divided into 8 slices. How many slices of pizza did Adam eat? Show your work.

Solution

9 Draw a model to show $\frac{2}{3} = \frac{4}{6}$.

10 Use the number line to find a fraction equivalent to $\frac{1}{3}$. Show your work.

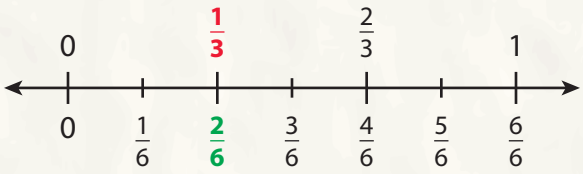
Solution

Practice Finding Equivalent Fractions

Study the Example showing how to find equivalent fractions. Then solve problems 1–8.

EXAMPLE

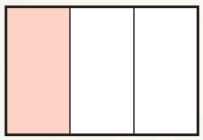
Maria colors $\frac{1}{3}$ of her art paper red. Erica colors $\frac{2}{6}$ of her art paper green. The papers are the same size. Do the two girls color the same amount of their art papers?



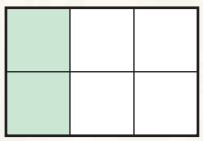
One third is equal to two sixths.

$$\frac{1}{3} = \frac{2}{6}$$

The girls color the same amount of their art papers.

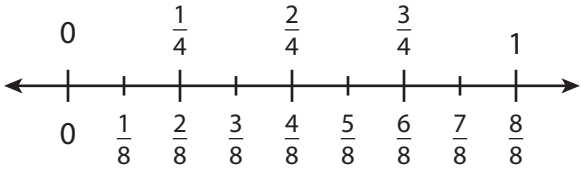


Maria colors $\frac{1}{3}$.



Erica colors $\frac{2}{6}$.

Use the number line to complete the equivalent fractions in problems 1–3.



1 $\frac{1}{4} = \frac{\square}{8}$

2 $\frac{6}{8} = \frac{\square}{4}$

3 $\frac{2}{4} = \frac{\square}{\square}$

Vocabulary

equivalent fractions fractions that name the same point on a number line. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.

Shade the models to show equivalent fractions in problems 4 and 5. Then fill in the blanks to write equivalent fractions.

4

$\frac{1}{2} = \frac{\square}{8}$

5

$\frac{2}{3} = \frac{\square}{6}$

Draw lines and shade to show equivalent fractions in problems 6 and 7. Then fill in the blanks to write equivalent fractions.

6

$\frac{1}{2} = \frac{\square}{4}$

7

$\frac{4}{6} = \frac{\square}{3}$

8 What is a fraction equivalent to $\frac{4}{4}$? Explain how you know.

Develop Writing a Whole Number as a Fraction

Read and try to solve the problem below.

Kacey uses 2 boards of the same size to build a birdhouse. He cuts each board into fourths. How can you write the number 2 as a fraction to find how many fourths Kacey cuts the boards into?

TRY IT

Math Toolkit

- fraction tiles
- fraction circles
- fraction bars
- number lines
- grid paper

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: A model I used was . . . It helped me . . .

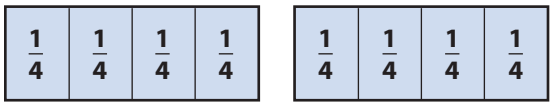
Explore different ways to understand writing a whole number as a fraction.

Kacey uses 2 boards of the same size to build a birdhouse. He cuts each board into fourths. How can you write the number 2 as a fraction to find how many fourths Kacey cuts the boards into?

PICTURE IT

You can use models to help you write a whole number as a fraction.

The fraction bars below show 2 wholes, each divided into fourths.

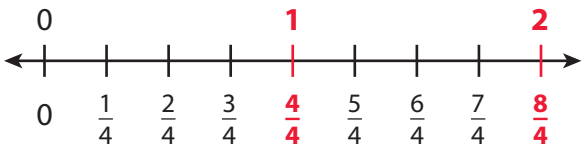


Each part is $\frac{1}{4}$ of a whole. There are eight $\frac{1}{4}$s in all.

MODEL IT

You can use a number line to help you write a whole number as a fraction.

This number line shows whole numbers on the top and fourths on the bottom.



Notice that each whole number has an equivalent fraction with a denominator of 4.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to write a whole number as a fraction.

1. Look at the models in **Picture It**. How many equal parts are shown in 1 whole? Explain how you know.

2. How many equal parts are shown in 2 wholes? Explain how you know.

3. Complete the sentences to show the fraction that is equivalent to 2.

   Use words: Two wholes equals _____.

   Use a fraction: 2 = _____.

   How many fourths does Kacey cut the boards into?

4. Explain how to find a fraction equivalent to a whole number.

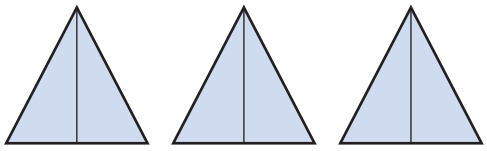
5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for writing a whole number as a fraction? Explain.

APPLY IT

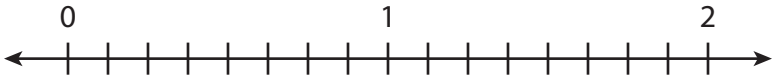
Use what you just learned to solve these problems.

6 Use the model below to write a fraction equivalent to 3.



Solution

7 Louisa has 2 ribbons that are the same length. She cuts each one into eighths. Use the number line below to help you write the number 2 as a fraction to show how many eighths she cuts the ribbons into.



Solution

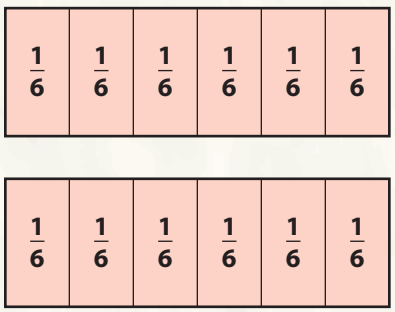
8 Draw a model to show $3 = \frac{18}{6}$. Show your work.

Practice Writing a Whole Number as a Fraction

Study the Example showing different ways to write whole numbers as fractions. Then solve problems 1–13.

EXAMPLE

Mrs. Clark cuts 2 same-sized pieces of colored paper into sixths to make strips for paper chains. How many strips does she make?



1 whole = six $\frac{1}{6}$s

$1 = \frac{6}{6}$

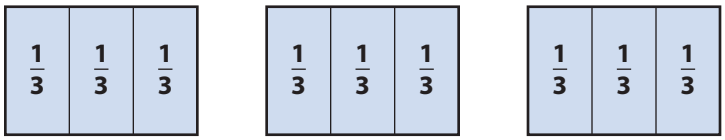
2 wholes = twelve $\frac{1}{6}$s

$2 = \frac{12}{6}$

Each strip is $\frac{1}{6}$ of a whole piece of paper.

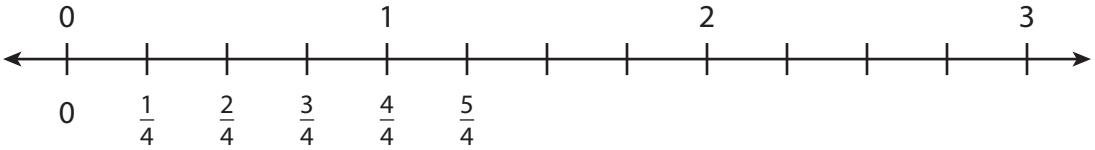
Mrs. Clark makes 12 strips.

Write the whole numbers as fractions in problems 1–4.



1. $1 = \frac{\square}{3}$
2. $2 = \frac{\square}{3}$
3. $3 = \frac{\square}{3}$
4. $4 = \frac{\square}{3}$

Use this number line to solve problems 5–8.



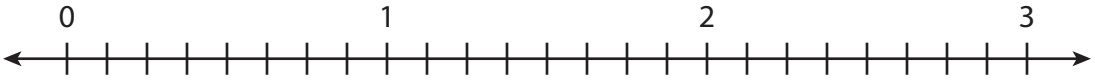
5 $1 = \frac{\square}{4}$

6 $2 = \frac{\square}{4}$

7 $3 = \frac{\square}{4}$

8 $0 = \frac{\square}{4}$

Use this number line to solve problems 9–11.



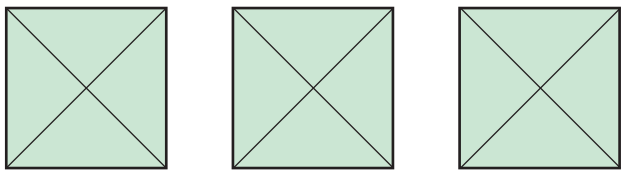
9 One whole is equal to _____ eighths.

10 16 eighths is equal to _____ wholes.

11 $3 = \frac{\square}{8}$

12 Use the model below to write a fraction equivalent to 3.

$3 =$ _____



13 Draw a model to show $2 = \frac{8}{4}$.

Develop Writing a Whole Number as a Fraction with a Denominator of 1

Read and try to solve the problem below.

Justin picks 4 green peppers from his garden. He does not cut them into pieces. How can you write the number of peppers Justin picks, 4, as a fraction?

TRY IT

Math Toolkit

- fraction circles
- fraction tiles
- fraction bars
- number lines
- grid paper

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

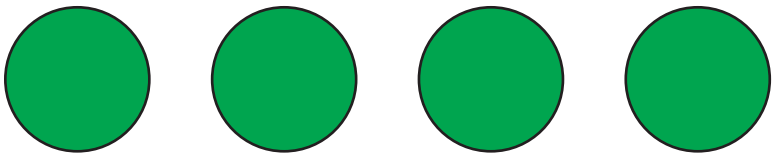
Explore different ways to understand writing a whole number as a fraction with a denominator of 1.

Justin picks 4 green peppers from his garden. He does not cut them into pieces. How can you write the number of peppers Justin picks, 4, as a fraction?

PICTURE IT

You can use models to help you write a whole number as a fraction with a denominator of 1.

Each circle stands for 1 green pepper.

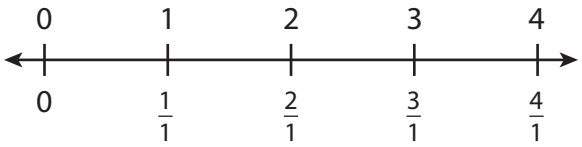


They are not divided into pieces, so each whole has one part.

MODEL IT

You can use a number line to help you write a whole number as a fraction with a denominator of 1.

This number line shows whole numbers on the top and fractions on the bottom.



Notice that each whole number has an equivalent fraction. The spaces between whole numbers are not divided into parts. Each whole number has one part, so the denominator of each equivalent fraction is 1.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to write a whole number as a fraction with a denominator of 1.

1. Look at the models in **Picture It**. Explain how you know each whole has only 1 part.

2. How many parts do the 4 green peppers make? .....

3. What does the numerator of a fraction show?

4. What does the denominator of a fraction show?

5. Write a fraction equivalent to 4. Use the fraction below to help you.

$$\frac{\text{number of parts described}}{\text{number of equal parts in the whole}}$$ .....

6. Explain how to write a whole number as a fraction with a denominator of 1.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for writing a whole number as a fraction with a denominator of 1? Explain.

.....

.....

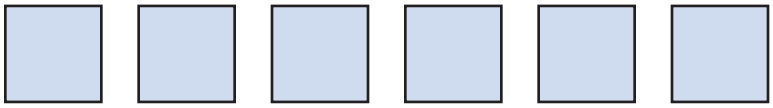
.....

.....

APPLY IT

Use what you just learned to solve these problems.

8 Use the model below to write a fraction equivalent to 6.



Solution

9 Draw a model to show $\frac{5}{1} = 5$.

10 Oscar has 3 loaves of bread that he has not sliced yet. Use a number line to write the pieces of bread Oscar has as a fraction. Show your work.

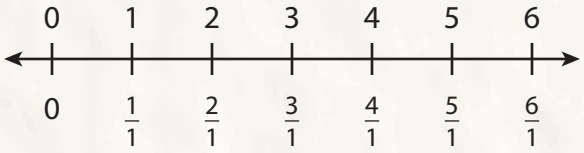
Solution

Practice Writing a Whole Number as a Fraction with a Denominator of 1

Study the Example showing how to write a whole number as a fraction with a denominator of 1. Then solve problems 1–14.

EXAMPLE

The spaces between whole numbers on this number line are not divided into smaller parts. So, each whole has only 1 part.



The number line shows that $\frac{3}{1}$ is equal to 3.

$\frac{3}{1}$ is a fraction name for 3.

Write the whole number for each fraction in problems 1–4.

1. $\frac{4}{1} =$ _____
2. $\frac{2}{1} =$ _____
3. $\frac{5}{1} =$ _____
4. $\frac{8}{1} =$ _____

Write a fraction with a denominator of 1 for each whole number in problems 5–8.

5. $2 =$ _____
6. $5 =$ _____
7. $1 =$ _____
8. $7 =$ _____

Write the whole number for each fraction in problems 9 and 10.

9 $\frac{9}{1} =$ .....

10 $\frac{10}{1} =$ .....

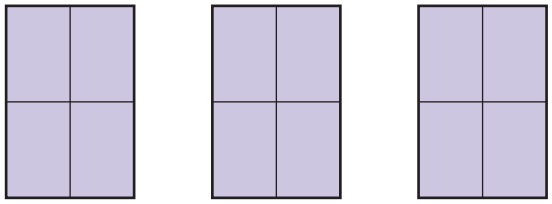
Write a fraction with a denominator of 1 for each whole number in problems 11 and 12.

11 $12 =$ .....

12 $18 =$ .....

13 Explain how to write a whole number as a fraction with a denominator of 1.

14 Bella says this model shows 3 wholes. She says it shows that if you write the whole number 3 as a fraction, you have to write $3 = \frac{12}{4}$.
How can you explain to Bella that there are other ways to write 3 as a fraction?



Vocabulary

numerator the number above the line in a fraction; it tells how many equal parts are being described.

denominator the number below the line in a fraction; it tells how many equal parts are in the whole.

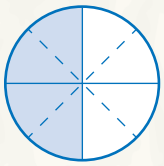
Refine Finding Equivalent Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Caleb and Hannah buy two melons that are the same size. Caleb cuts his melon into fourths. Hannah cuts her melon into eighths. Hannah eats $\frac{4}{8}$ of her melon. Caleb eats an equal amount of his melon. What fraction of his melon does Caleb eat?

Look at how you could show your work using a model.



Solution

The student used solid lines to show fourths. She used dashed lines to show how to divide fourths to make eighths.

PAIR/SHARE

How could you solve this problem using a number line?

APPLY IT

1 Matt says $\frac{3}{3}$ is equivalent to 1. Elisa says $\frac{8}{8}$ is equivalent to 1. Who is correct? Show your work.

How many thirds are in 1 whole? How many eighths are in 1 whole?

PAIR/SHARE

What is another fraction that is equivalent to 1?

Solution

2 Write two fractions that are equivalent to 5. Show your work.

There will be 5 wholes in all. Think about how many parts will be in each whole.

PAIR/SHARE

How did you decide what denominators to use in your fractions?

Solution

3 Kaia ate $\frac{3}{6}$ of a banana. Zoie ate an equivalent amount. Which fraction shows how much of a banana Zoie ate?

Ⓐ $\frac{1}{3}$

Ⓑ $\frac{2}{3}$

Ⓒ $\frac{5}{8}$

Ⓓ $\frac{1}{2}$

Find $\frac{3}{6}$ on a number line. What is another fraction that names the same location?

Landon chose Ⓐ as the correct answer. How did he get that answer?

PAIR/SHARE

Does Landon's answer make sense?

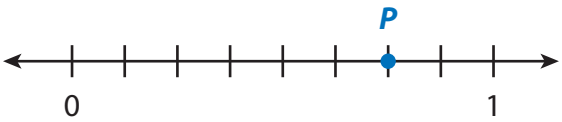
4 Which model shows a fraction equivalent to $\frac{2}{6}$?

Ⓐ Ⓑ Ⓒ Ⓓ

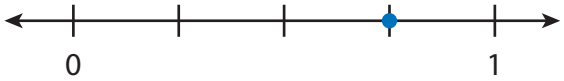
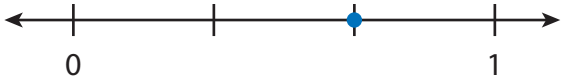
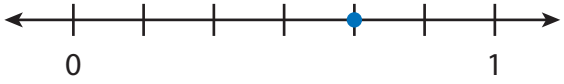
5 Draw a model to find a fraction equivalent to $\frac{1}{4}$. Show your work.

$\frac{1}{4}$ is equivalent to _____.

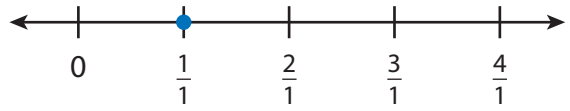
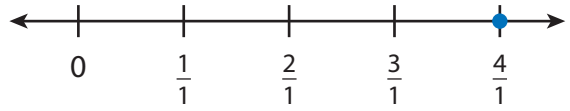
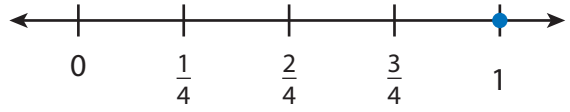
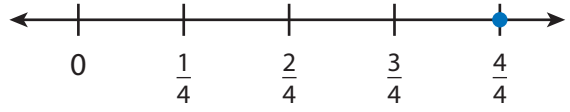
6 Look at point *P* on the number line.



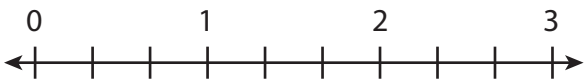
Does the point on each number line below represent a fraction equivalent to the fraction shown by point *P*?

	Yes	No
	Ⓐ	Ⓑ
	Ⓒ	Ⓓ
	Ⓔ	Ⓕ

7 Does the point on each number line represent one whole?

	Yes	No
	Ⓐ	Ⓑ
	Ⓒ	Ⓓ
	Ⓔ	Ⓕ
	Ⓖ	Ⓗ

8 Use the number line to find a fraction equivalent to 3. Show your work.



3 is equivalent to .....

9 MATH JOURNAL

Write two fractions equivalent to 4 using the denominators 1 and 3. Use a number line to show how you found your answers.

SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

Teacher Sample, Lesson 23

The *Ready Classroom Mathematics* Teacher's Guide includes support for planning, differentiation, and facilitating meaningful mathematical discourse.

The following pages, 493a–520b, represent a complete teacher lesson that corresponds to the student lesson included in this sample.

Lesson Overview

LESSON 23

Find Equivalent Fractions

CCSS Focus

Domain
Number and Operations—Fractions

Cluster
A. Develop understanding of fractions as numbers.

Standards
3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$; *recognize that* $\frac{6}{1} = 6$; *locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.*

Additional Standards
3.NF.A.2a, 3.NF.A.2b, 3.NF.A.3a
(See Standards Correlations at the end of the book for full text.)

Standards for Mathematical Practice (SMP)
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Lesson Objectives

Content Objectives
- Use fraction models and number lines to identify and create equivalent fractions, including those that are greater than or equal to one whole.
- Identify, model, and write equivalent fractions for whole numbers.

Language Objectives
- Write equivalent fractions for numbers greater than 1.
- Write whole numbers as fractions and justify, using area models or number lines.
- Write a fraction that represents a whole number.
- Tell why a fraction with a denominator of 1 is equivalent to a whole number.

Prerequisite Skills
- Understand the meaning of fractions.
- Identify fractions represented by models.
- Understand that the size of a fractional part is relative to the size of the whole.
- Understand how to use number lines to count and identify fractional parts.

Lesson Vocabulary
There is no new vocabulary. Review the following key terms.
- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **equivalent fractions** two or more different fractions that name the same part of a whole or the same point on a number line.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.

Learning Progression
In the previous lesson students developed a conceptual understanding of equivalent fractions by using fraction models and number lines.

In this lesson students extend their understanding to include identifying and generating equivalent fractions, including equivalent fractions for whole numbers that are equal to or greater than 1. Students continue to use fraction models and number lines to reason about fraction equivalency and to find equivalent fractions. Students learn to write a whole number as an equivalent number of fraction parts and as a fraction with a denominator of 1.

Students' work with equivalent fractions in this lesson provides a foundation for learning to compare fractions in the next two Grade 3 lessons.

In Grade 4 students will learn to find equivalent fractions by multiplying or dividing the numerator and denominator of a fraction by the same whole number.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 Explore *45–60 min*	**Equivalent Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 497–498
SESSION 2 Develop *45–60 min*	**Finding Equivalent Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 503–504 **Fluency** Finding Equivalent Fractions
SESSION 3 Develop *45–60 min*	**Writing a Whole Number as a Fraction** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 509–510 **Fluency** Writing a Whole Number as a Fraction
SESSION 4 Develop *45–60 min*	**Writing a Whole Number as a Fraction with a Denominator of 1** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 515–516 **Fluency** Writing a Whole Number as a Fraction with a Denominator of 1
SESSION 5 Refine *45–60 min*	**Finding Equivalent Fractions** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz**

Lesson Materials

Lesson *(Required)*	*Per student:* fraction tiles *Activity Sheet:* Number Lines**
Activities	*Per student:* scissors, tape, colored pencils; *Per pair:* fraction circles *Activity Sheets:* 1-Inch Grid Paper**, Multiplication Table
Math Toolkit	fraction circles, fraction tiles, fraction bars, number lines, grid paper, index cards, crayons

**Used for more than one activity.

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 2
- Lesson 29 Understand Partitioning Shapes Into Halves, Thirds, and Fourths

RETEACH

Tools for Instruction

Grade 2
- Lesson 30 Make Equal Shares

Grade 3
- Lesson 23 Find Equivalent Fractions

REINFORCE

Math Center Activities

Grade 3
- Lesson 23 Building Equivalent Fractions

EXTEND

Enrichment Activity

Grade 3
- Lesson 23 Colorful Quilts

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 3
- Find Equivalent Fractions

Learning Game
- *Bounce*

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Connect to Family, Community, and Language Development

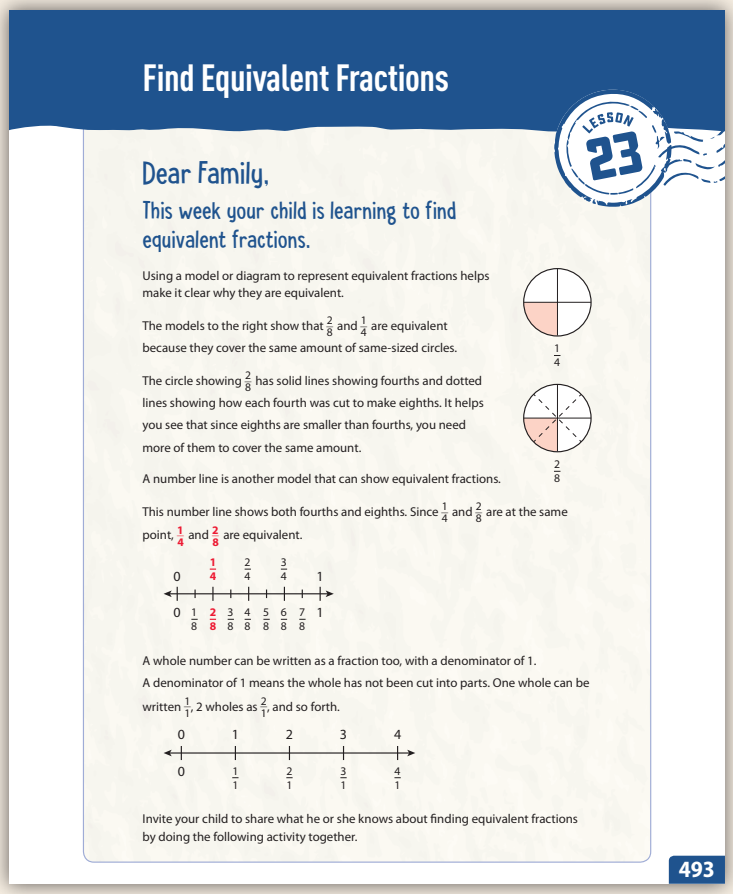
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Find Equivalent Fractions

LESSON 23

Dear Family,

This week your child is learning to find equivalent fractions.

Using a model or diagram to represent equivalent fractions helps make it clear why they are equivalent.

The models to the right show that $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent because they cover the same amount of same-sized circles.

$\frac{1}{4}$

The circle showing $\frac{2}{8}$ has solid lines showing fourths and dotted lines showing how each fourth was cut to make eighths. It helps you see that since eighths are smaller than fourths, you need more of them to cover the same amount.

$\frac{2}{8}$

A number line is another model that can show equivalent fractions.

This number line shows both fourths and eighths. Since $\frac{1}{4}$ and $\frac{2}{8}$ are at the same point, $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent.

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1

0 $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ 1

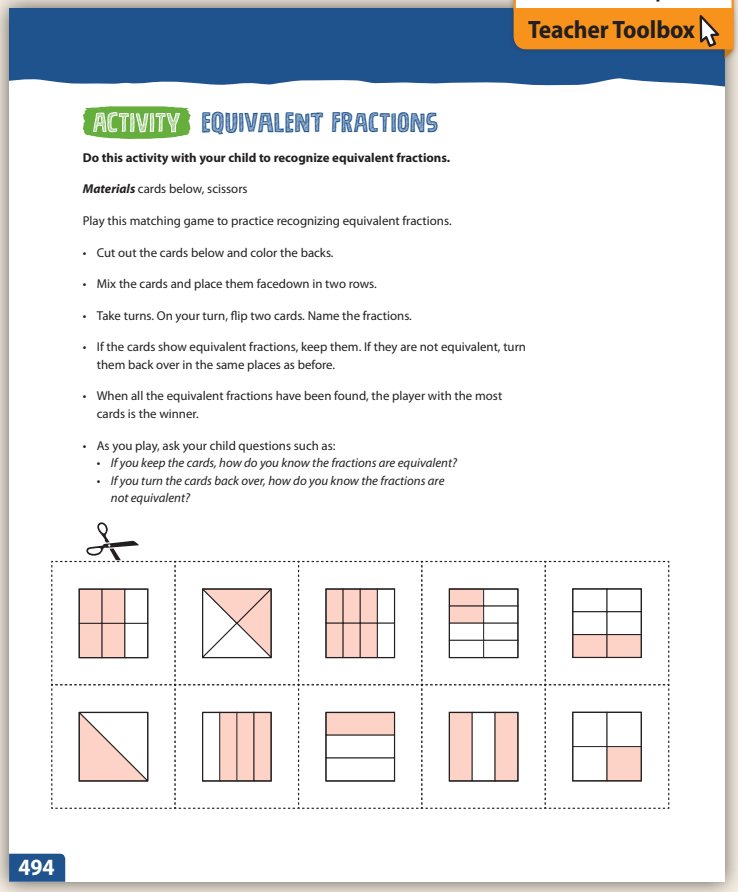
A whole number can be written as a fraction too, with a denominator of 1. A denominator of 1 means the whole has not been cut into parts. One whole can be written $\frac{1}{1}$, 2 wholes as $\frac{2}{1}$, and so forth.

0 1 2 3 4

0 $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$

Invite your child to share what he or she knows about finding equivalent fractions by doing the following activity together.

493



ACTIVITY EQUIVALENT FRACTIONS

Do this activity with your child to recognize equivalent fractions.

Materials cards below, scissors

Play this matching game to practice recognizing equivalent fractions.

- Cut out the cards below and color the backs.
- Mix the cards and place them facedown in two rows.
- Take turns. On your turn, flip two cards. Name the fractions.
- If the cards show equivalent fractions, keep them. If they are not equivalent, turn them back over in the same places as before.
- When all the equivalent fractions have been found, the player with the most cards is the winner.
- As you play, ask your child questions such as:
  - *If you keep the cards, how do you know the fractions are equivalent?*
  - *If you turn the cards back over, how do you know the fractions are not equivalent?*

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Goal

The goal of the Family Letter is to provide additional models of equivalent fractions. Students are expected to recognize and find equivalent fractions using area models and number lines. Whole numbers are represented as fractions.

Activity

Look at the *Equivalent Fractions* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students and their family members to talk about fractions. Challenge them to find examples of fractions at home, and to use these fractions as a basis for creating equivalent fractions.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members.

- *How did you learn about fractions in school? Did you use models and number lines?*
- *How do you figure out whether or not two fractions are equivalent?*
- *What does $\frac{5}{1}$ mean?*

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 Use with *Try It*.

- Ask students to tell their favorite snack. Suggest that many people like to snack on granola bars. Draw and label a model as you say: *When you eat a whole granola bar, the whole bar is represented as* $\frac{1}{1}$. *If you give your bar to two friends to share, the unit fraction they each get is* $\frac{1}{2}$ (display). *If you give your bar to three friends to share equally, the unit fraction they each get is* $\frac{1}{3}$ (display). *Turn to a partner and decide what unit fraction four friends receive if they share equally.* Display $\frac{1}{4}$. Point to the models: *What happens to the pieces of the granola bar as more friends share?* [the pieces get smaller] Point to the fractions. Ask: *What happens to the denominators as more friends share?* [the number gets larger] Display and have students complete the sentence frame: *The pieces get* ___*smaller*___ *as the denominators get* ___*larger*___ .

Session 2 Use throughout the session.

- Say: *We have focused on food items that can be divided into equal parts.* Display a dollar bill and scissors. Ask: *Why is cutting this dollar into equal parts not a good idea?* [The dollar cannot be used.] Say: *Are there things that cannot or should not be divided into fractional pieces? Turn to a partner and discuss something that you think cannot or should not be divided into fractional pieces. Be prepared to explain why you cannot or should not divide your item.* Select pairs to share.

Session 3 Use with *Try It*.

- Ask students if they have or have seen a birdhouse, bird feeder, or bird bath. Ask students to explain the purpose of each. Point out that these objects are often made of wood. Ask: *What are some things that can be built with wood?* (for example, a fence, a bookcase, and a tree house) Display a list of items students suggest. Ask students to share any experiences they may have had building something out of wood.

Session 4 Use with *Apply It* problem 10.

- Explain that breads in different cultures can vary quite a bit by ingredients, size, and shape. Invite students to tell about different types of breads that they know or like. You may also ask: *What type of bread is most common in your home?*

Session 5 Use with *Try It* the Example.

- Display the word *melon*. Ask students to share different types of melon they have eaten. Display the words *watermelon*, *cantaloupe*, and *honey dew*.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read the first two sentences of the ***Try It*** problem aloud. Have students form pairs and give each pair a square piece of paper. Say: *This is the cake.* Model how to fold the paper in half. Ask: *What unit fraction describes each part of the cake?* $\left[\frac{1}{2}\right]$ Have students label each half with a flavor. Read the rest of the problem. Model how to fold the paper in fourths. Ask: *What unit fraction describes each part of the cake now?* $\left[\frac{1}{4}\right]$ Have students label each part. Say: *Fold your square so that you can only see the half with chocolate frosting.* Display: $\frac{1}{2} = \frac{2}{4}$. Say: *Discuss with your partner how to complete this equation.*	**Speaking/Writing** Read the first two sentences of the ***Try It*** problem. Have students form pairs and give each pair a square piece of paper. Say: *This is the cake. Fold the paper in half. What unit fraction describes each part of the cake?* $\left[\frac{1}{2}\right]$ Have students label each half with a flavor. Read the rest of the problem. Ask: *How can you fold the paper to show fourths?* Display students' squares. Ask: *What unit fraction describes each part of the cake now?* $\left[\frac{1}{4}\right]$ *Label each part. Fold your square so that you see the half with chocolate frosting. With your partner, write an equation to show how many fourths of the cake are equal to one half.* Call on pairs to share their equations.	**Speaking/Writing** Have pairs read the ***Try It*** problem. Give each pair a square piece of paper. Say: *This is the cake. Fold the paper in half. What unit fraction describes each part of the cake?* $\left[\frac{1}{2}\right]$ *Label each half with a flavor. Fold the paper to show fourths.* Display students' squares. Ask: *What unit fraction describes each part of the cake now?* $\left[\frac{1}{4}\right]$ *Label each part. Fold your square so that you see the half with chocolate frosting.* Have students complete the sentence frame with a partner: *One half of the cake is equal to* _____ *fourths because* _____. Call on pairs to share their explanations.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

LESSON 23

SESSION 1 Explore

Purpose In this session students draw on their knowledge of area models and equivalent fractions. They compare models to explore how to rename a fraction using a different denominator. They will look ahead to think about how to find equivalent fractions using the same area model or number line.

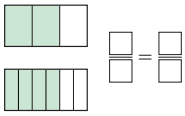
Start

Connect to Prior Knowledge

Why Support students' facility with recognizing and writing equivalent fractions.

How Have students write the equivalent fractions represented by two area models.

Write equivalent fractions for the shaded parts of the models.



$\square = \frac{\square}{\square}$

Solution

$\frac{2}{3} = \frac{4}{6}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that the cake is visually divided into halves by the kinds of frosting, but that the cake will be cut into fourths, and that each fourth must be all chocolate or all vanilla.

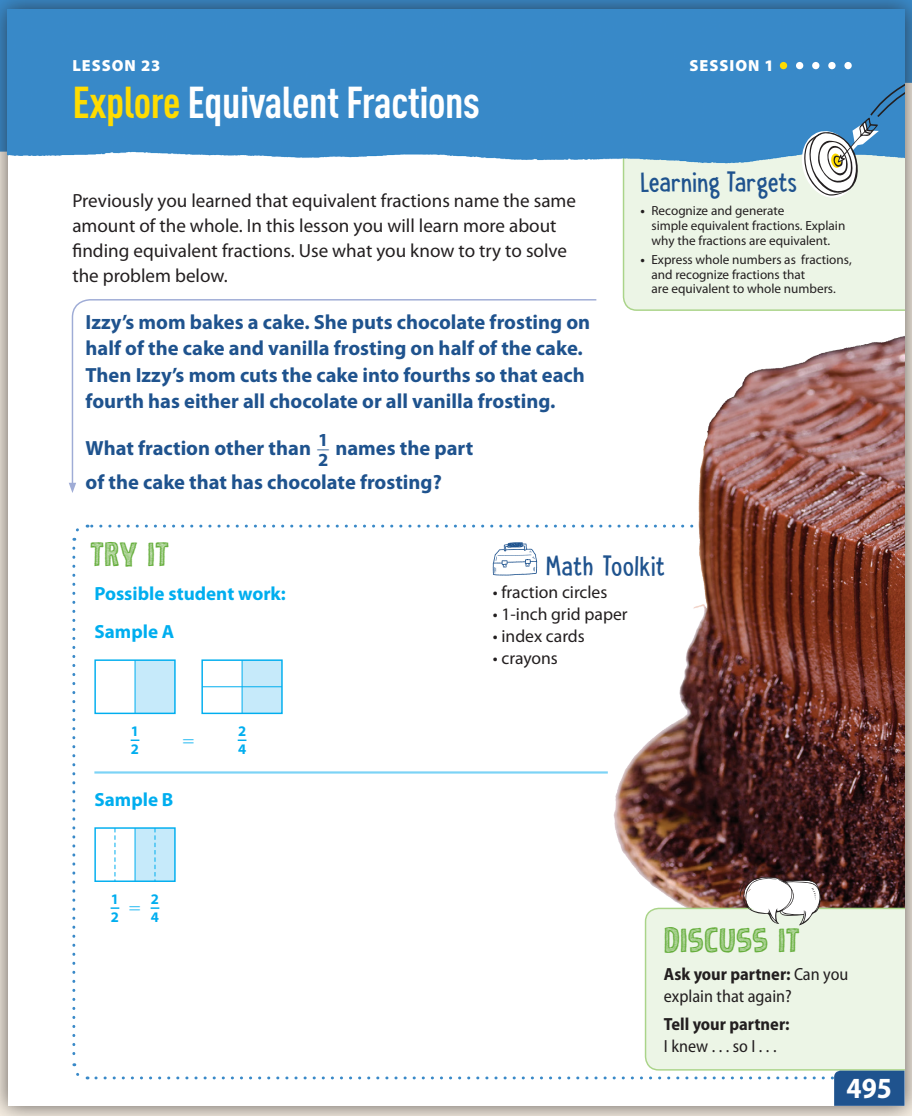
DISCUSS IT

Support Partner Discussion

To reinforce the fact that the problem is asking for another name for $\frac{1}{2}$, encourage students to use the terms *half* and *fourths* as they talk to each other.

Look for, and prompt as necessary for, understanding that:

- $\frac{1}{2}$ of the cake has chocolate frosting
- the cake is cut into fourths
- they are looking for another fraction that describes the part of the cake with chocolate frosting



LESSON 23

SESSION 1

Explore Equivalent Fractions

Previously you learned that equivalent fractions name the same amount of the whole. In this lesson you will learn more about finding equivalent fractions. Use what you know to try to solve the problem below.

Learning Targets

- Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Izzy's mom bakes a cake. She puts chocolate frosting on half of the cake and vanilla frosting on half of the cake. Then Izzy's mom cuts the cake into fourths so that each fourth has either all chocolate or all vanilla frosting.

What fraction other than $\frac{1}{2}$ names the part of the cake that has chocolate frosting?

TRY IT

Possible student work:

Sample A

$\frac{1}{2} = \frac{2}{4}$

Sample B

$\frac{1}{2} = \frac{2}{4}$

Math Toolkit

- fraction circles
- 1-inch grid paper
- index cards
- crayons

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

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Common Misconception Look for students who are not comfortable with visualizing how the cake is cut. As students present solutions, have them specify how they think the cake is cut.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- models that have been cut apart
- freehand drawings of the cake
- drawings on grid paper
- solutions with no drawing or visual model

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show which part(s) of the cake have chocolate frosting?*

Listen for One half, or two fourths, of the cake should be shaded or somehow marked to indicate chocolate frosting.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

1 LOOK BACK

Look for understanding that the amount of cake with chocolate frosting stays the same whether it is cut into halves or fourths.

Hands-On Activity

Use grid paper to model equivalent fractions.

If . . . *students have trouble understanding that each $\frac{1}{4}$ of the cake is the same amount regardless of its shape or how the cake is cut,*

Then . . . *use this activity to have them explore different ways to cut the cake into fourths.*

Materials For each student: scissors, tape, Activity Sheet *1-Inch Grid Paper*

- Have students draw at least two models of the cake, making each one a 2-inch by 4-inch rectangle. Then have them draw lines to show each model divided into fourths in a different way.
- Ask: *In each of the models you drew, how many grid squares are used to make one fourth?* [2] For models drawn so that the fourths are triangles, have students cut the triangle along a grid line and tape the pieces back together so that any partial grid squares are combined to form full squares.

2 LOOK AHEAD

Point out that no matter how the cake is cut into fourths, there are 2 pieces with chocolate frosting. No matter how the cake is cut into eighths, there are 4 pieces with chocolate frosting.

CONNECT IT

1 LOOK BACK

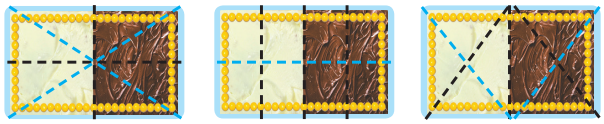
What fraction other than $\frac{1}{2}$ names the part of the cake that has chocolate frosting? $\frac{2}{4}$ How did you get your answer?

Possible answer: When you divide the cake into 4 equal parts, 2 of them have chocolate frosting.

2 LOOK AHEAD

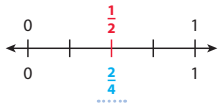
You have seen many different types of fraction models, such as area models, number lines, and fractions bars. You can find equivalent fractions by dividing the same model in different ways.

a. Each cake below shows fourths. Draw lines on one of the cakes to show eighths. **Possible answers:**



b. How many pieces of the cake have chocolate frosting now? 4

c. You can also look at different equal-sized parts on a number line to find equivalent fractions. Fill in the fraction for fourths that is equivalent to $\frac{1}{2}$.



3 REFLECT

Why does it make sense that $\frac{1}{2}$ and $\frac{2}{4}$ can name the same amount?

Possible answer: One half of a cake can be divided again to make two fourths, but the amount of cake does not change.

Close: Exit Ticket

3 REFLECT

Look for understanding that $\frac{1}{2}$ and $\frac{2}{4}$ represent the same quantity divided into different numbers of parts.

Common Misconception **If** students are confused by the single number line, **then** draw two number lines, one labeled with 0, $\frac{1}{2}$, and 1, and the other directly beneath it labeled with 0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and 1. Explain that combining the two number lines, as in problem 2c, makes identifying equivalent fractions easier.

Real-World Connection

Use two pieces of fruit that are the same size to help students reason that they are getting a fair share or equivalent amount if they cut the same-size fruits into halves two different ways. For example, if you have two lemons, cut one lemon in half the long way, and the other lemon in half the short way. Compare halves from each lemon. Emphasize that each half is the same amount even though the shapes are different. Then cut each half into two fourths. Hold the pieces together and then separate them again so that they can see that $\frac{2}{4}$ of the lemon is the same as $\frac{1}{2}$ of the lemon.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Solutions

Support Vocabulary Development

1 Have students point at and say the phrase *fraction models*. Point out that a model of a fraction can be a picture. Ask: *What models have you used in previous lessons to represent fractions?* [number lines, area models, fraction bars] *What shapes have you used to represent fractions?* [rectangles, squares, circles] *What fractions can you represent?* [possible responses include: unit fractions, $\frac{2}{3}$, and $\frac{3}{4}$] List students' responses to the questions as a scaffold.

2 Have students label each part of the first rectangle on the outside of the rectangle. Ask: *What unit fraction describes each piece of the first rectangle?* [$\frac{1}{3}$] *How many equal pieces must a model have to represent sixths?* [6] *With a partner, discuss how you can draw one or more lines on the rectangle to make six equal pieces.* Students may draw one vertical or three horizontal lines to create sixths. Validate both strategies. Repeat the process and questions for the next two models if students need additional support.

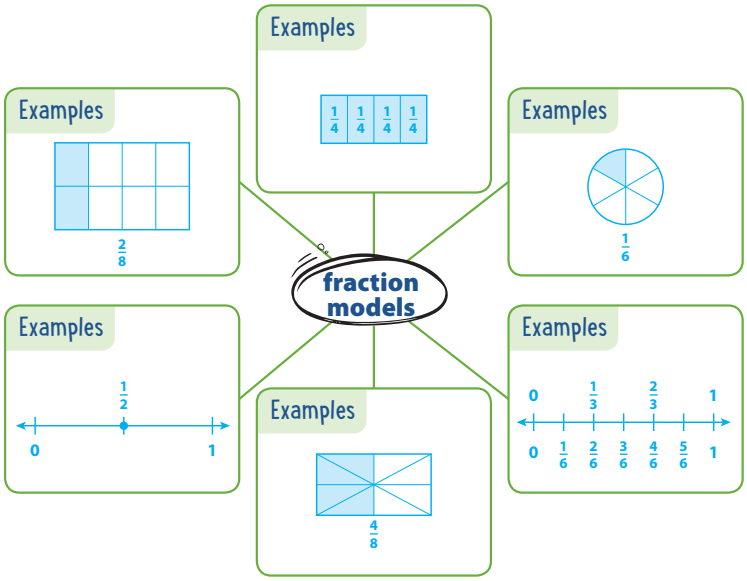
Supplemental Math Vocabulary

- *denominator*
- *equivalent fraction*
- *fraction*
- *numerator*

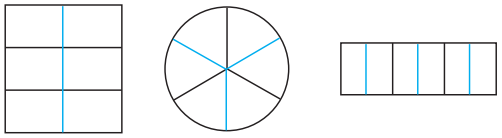
Name: _____ LESSON 23 SESSION 1

Prepare for Finding Equivalent Fractions

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Each fraction model below shows thirds. Draw lines on each model to show sixths. **Possible answers:**



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3 Assign problem 3 to provide another look at finding equivalent fractions.

This problem is very similar to the problem about Izzy's mom cutting a cake into equal pieces. In both problems, students are given a word problem where they must divide a shape into smaller parts to find an equivalent fraction. The question asks what fraction other than one third names the part of the rectangle that is red.

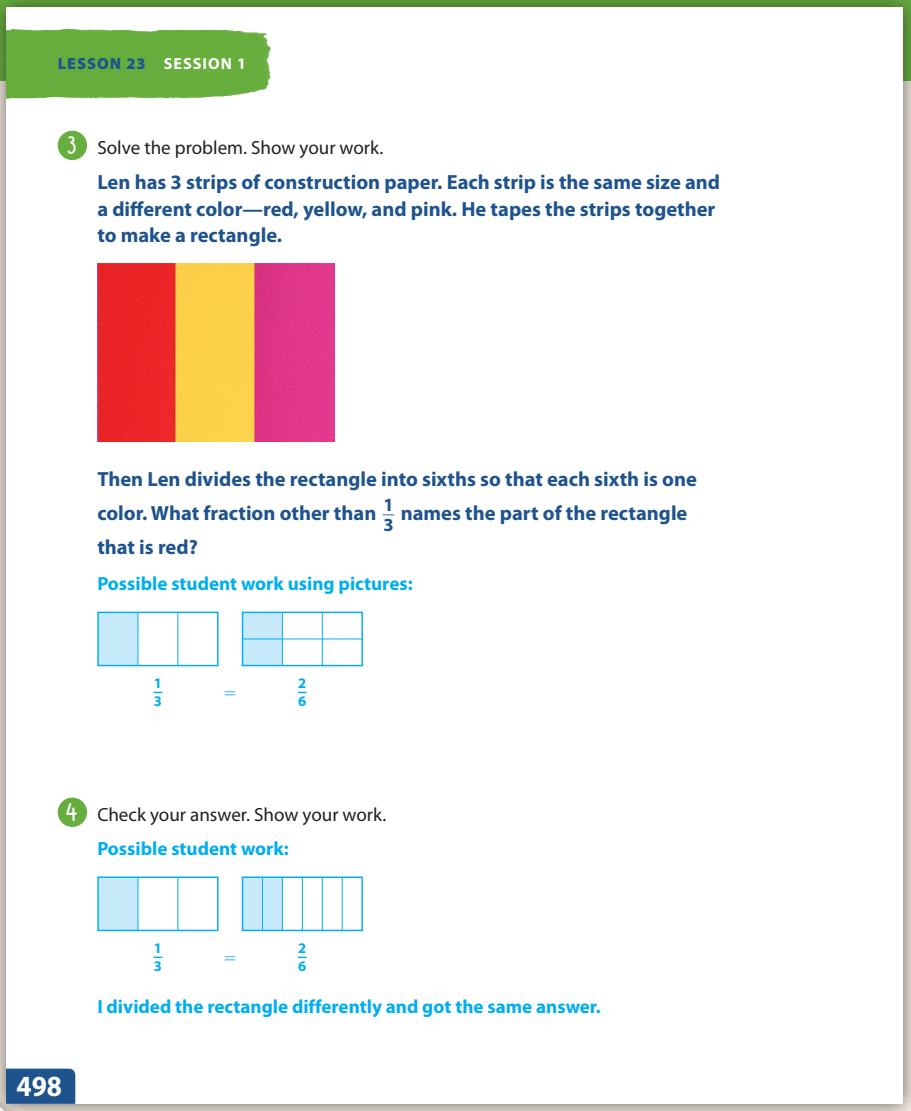
Students may want to use fraction tiles or construction paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: $\frac{2}{6}$ of the rectangle is red.
Medium

4 Have students solve the problem a different way to check their answer.



LESSON 23 SESSION 1

3 Solve the problem. Show your work.

Len has 3 strips of construction paper. Each strip is the same size and a different color—red, yellow, and pink. He tapes the strips together to make a rectangle.

Then Len divides the rectangle into sixths so that each sixth is one color. What fraction other than $\frac{1}{3}$ names the part of the rectangle that is red?

Possible student work using pictures:

$\frac{1}{3}$ = $\frac{2}{6}$

4 Check your answer. Show your work.

Possible student work:

$\frac{1}{3}$ = $\frac{2}{6}$

I divided the rectangle differently and got the same answer.

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ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Give pairs two congruent circles. Read aloud ***Apply It*** problem 8. Ask: *How many equal pieces are in Lina's pizza?* [4] *How do you know?* Provide the sentence frame: *The* ___*denominator*___ *is 4.* Model how to fold one circle to create fourths. Ask: *How many slices did Lina eat?* [3] *Shade the circle to represent* $\frac{3}{4}$. Repeat the process with the second circle. Say: *Both pizzas show* $\frac{3}{4}$. *Discuss how the four slices of Adam's pizza can be made into eight equal pieces.* Validate suggestions. Model how to fold or draw lines to create eighths. Display and have students complete the sentence frame: *Adam ate* ___*6*___ *slices.*	**Listening/Speaking** Give pairs two congruent circles. Read aloud ***Apply It*** problem 8. Ask: *How many equal pieces are in Lina's pizza?* [4] *How do you know?* [The denominator is 4.] *How many slices did Lina eat?* [3] *Discuss with your partner how you can use a circle to represent Lina's pizza.* Validate suggestions. Have them fold one circle to create fourths. Say: *Shade the circle to represent* $\frac{3}{4}$. Repeat the process with the second circle. Say: *With your partner, show eighths on the second circle to represent Adam's pizza. Decide how many eighths are equal to the three slices Lina ate.* Have students complete the sentence frame: *Adam ate* ___*6*___ *pizza slices.* Have students take turns reading the sentence to their partners.	**Listening/Speaking** Give pairs two congruent circles. Read aloud ***Apply It*** problem 8. Ask: *How many equal pieces are in Lina's pizza? How do you know? How many slices did Lina eat?* Encourage students to answer in complete sentences. Say: *Discuss with your partner how you can use a circle to represent Lina's pizza.* Validate suggestions. Have them fold one circle to create fourths. Say: *Shade the circle to represent* $\frac{3}{4}$. *With your partner, show eighths on the second circle to represent Adam's pizza. Decide how many eighths are equal to the three slices Lina ate. Write a sentence that tells how many slices Adam ate.* Select pairs to share their process.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session students solve a problem that requires showing that two fractions are equivalent. Students model and compare the fractions either on paper or with manipulatives. The purpose of this session is to have students understand how to use models such as area models and number lines to show two fractions are equivalent and represent the same quantity.

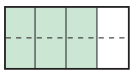
Start

Connect to Prior Knowledge

Why Reinforce the concept that two different fractions can name the same part of a whole.

How Have students write two different fractions for a rectangular area model.

Write two different fractions that describe the shaded part of the area model.



Solution

$\frac{3}{4}, \frac{6}{8}$

Develop Language

Why Clarify the meaning and use of *amount*.

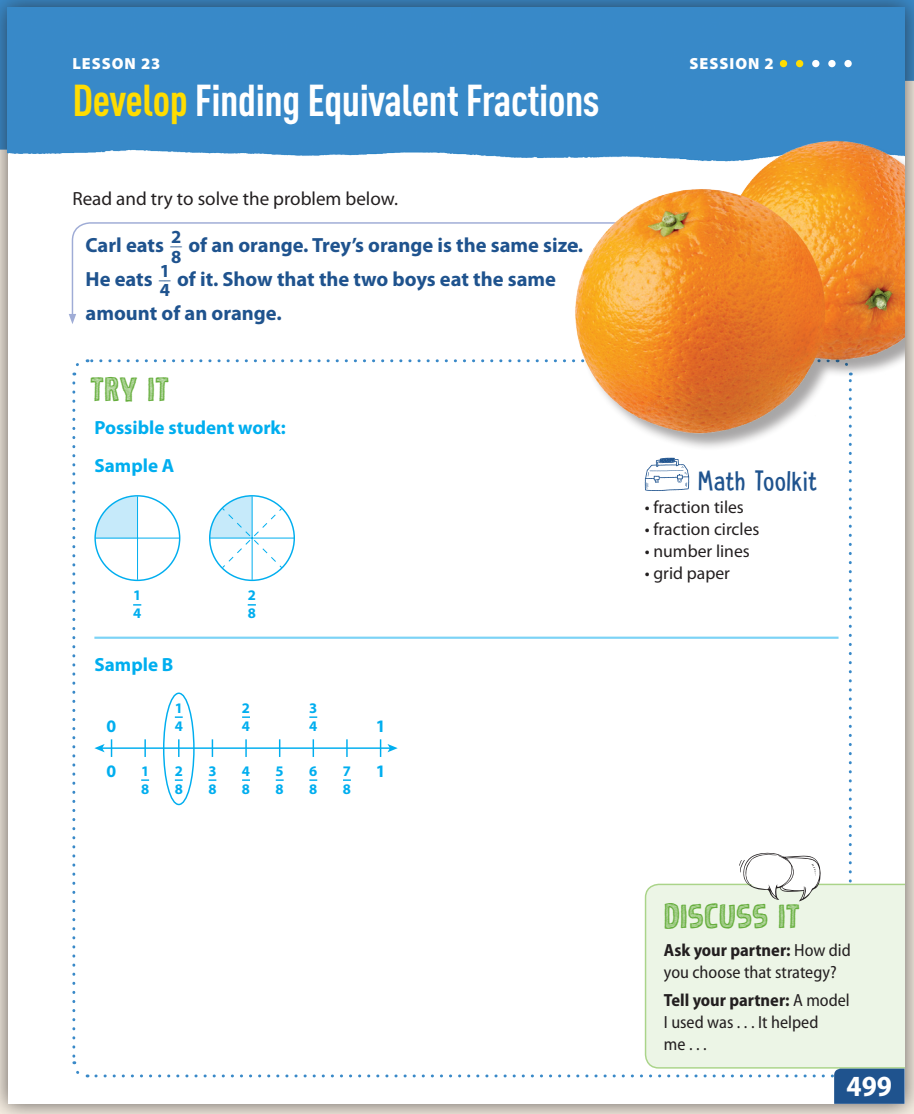
How Explain that the word *amount* refers to the quantity of something. Point out that *amount* is usually used with things that cannot be counted or that are difficult to count. Have students read the ***Try It*** problem. Explain that if the problem was about how many whole oranges Trey and Carl eat, a number could be used, for example: *Trey and Carl eat 3 oranges.* Explain that the problem refers to how much of an orange Trey eats. Since this quantity cannot be easily counted, we say: *Trey eats the same amount of orange as Carl.* Ask students whether they would use a number or the word *amount* with the following and to explain why: rice (amount), books (number), rain (amount), shoes (number)

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the fractions they are being asked to compare.

Ask *What fraction of an orange did each boy eat? Why is it important that Trey's orange is the same size as Carl's orange?*



LESSON 23 SESSION 2

Develop Finding Equivalent Fractions

Read and try to solve the problem below.

Carl eats $\frac{2}{8}$ of an orange. Trey's orange is the same size. He eats $\frac{1}{4}$ of it. Show that the two boys eat the same amount of an orange.

TRY IT

Possible student work:

Sample A

$\frac{1}{4}$ $\frac{2}{8}$

Sample B

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1

0 $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ 1

Math Toolkit
- fraction tiles
- fraction circles
- number lines
- grid paper

DISCUSS IT

Ask your partner: How did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *equivalent* as they discuss their solutions.

Support as needed with questions such as:

- *What model did you use?*
- *How did you show fourths? How did you show eighths?*

Common Misconception Look for students who use two different models that cannot be compared, such as an area model and a number line.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction tiles or fraction circles
- area models divided into fourths and eighths
- number lines showing fourths and eighths

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does each model show the two different denominators? Where does each model show the amount of orange each boy eats? How does each model show that the fractions are equivalent?*

Listen for Area models and number lines should be broken up into four parts and eight parts to show fourths and eighths, respectively, or fourths and eighths fraction tiles should be used. The amount eaten can be shown by shading on an area model, comparing the correct number of each tile, and by drawing points on a number line. The amounts shaded or tiles compared should be the same size and the points on the number line should coincide.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each show:

- the $\frac{2}{8}$ Carl eats
- the $\frac{1}{4}$ Trey eats
- $\frac{2}{8} = \frac{1}{4}$

Ask *How can you tell which area model shows each fraction? How do the area models show that the fractions are equivalent? How does the number line show that the fractions are equivalent?*

Listen for The number of parts the model is divided into shows the denominator and the number of shaded parts shows the numerator. The area models show that the same amount is shaded, even though they are broken into different numbers of parts. The number line shows that the two fractions name the same point.

For an area model, prompt students to identify the numerator and denominator of each fraction and how they are represented in the models.

- *How many equal parts is each orange divided into?*
- *How many parts did each boy eat?*

For a number line, prompt students to explain how to represent each fraction on a number line.

- *How can you label both fourths and eighths on the same number line?*
- *How can you show the amount each boy ate?*

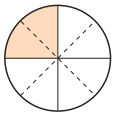
Explore different ways to understand finding equivalent fractions.

Carl eats $\frac{2}{8}$ of an orange. Trey's orange is the same size. He eats $\frac{1}{4}$ of it. Show that the two boys eat the same amount of an orange.

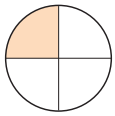
PICTURE IT

You can use models to help find equivalent fractions.

This model shows $\frac{2}{8}$.



This model shows $\frac{1}{4}$.

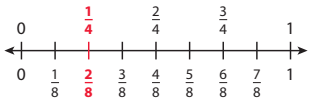


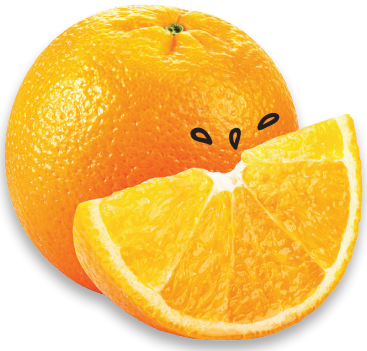
Look at the model of $\frac{2}{8}$. The solid lines divide the circle into fourths. The dashed lines divide each fourth in half to make eighths.

MODEL IT

You can also use a number line to help find equivalent fractions.

This number line shows both fourths and eighths.





Deepen Understanding

Area Models of Equivalent Fractions

SMP 4 Reason quantitatively.

When discussing the area models, prompt students to think about how they can divide each part into smaller equal parts to find other equivalent fractions.

Ask *How could you change the area model for $\frac{1}{4}$ to show $\frac{2}{8}$? Explain.*

Listen for Divide each fourth into 2 equal parts.

Ask *How do the numerator and denominator change when you divide each part into 2 parts?*

Listen for Both the numerator and denominator are doubled.

Ask *Suppose you have an area model showing halves. How can you use that model to show an equivalent fraction with a denominator of 6? How will the numerator and denominator change?*

Listen for Divide each half into 3 equal parts and count the number of shaded parts to find the numerator. Each number in the fraction will be multiplied by 3.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to explain why $\frac{2}{8} = \frac{1}{4}$.

Monitor and Confirm

1 – 2 Check for understanding that:

- The denominator tells how many parts the model is divided into.
- The numerator tells how many parts are shaded.

Support Whole Class Discussion

3 – 5 Tell students that these problems will prepare them to provide the explanation required in problem 6.

Be sure students understand that the problems are asking them to explain how the models on the previous page show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent, using both words and numbers.

Ask *Why is it important for the two circles to be the same size? Why is it helpful to use one number line for both fractions instead of two separate number lines?*

Listen for The wholes must be the same size to compare fractions. The circles should be the same size so that the amount shaded is the same for equivalent fractions. Using one number line ensures that the wholes are the same size, and it is easier to tell whether the fractions represent the same point.

6 Look for the understanding that equivalent fractions represent the same amount in same-sized models or the same point on a number line.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to find equivalent fractions.

1 Look at the models in **Picture It**. How do you know that $\frac{2}{8}$ of the first model is shaded? **There are 8 equal parts, and 2 are shaded.**

2 How do you know that $\frac{1}{4}$ of the second model is shaded?

There are 4 equal parts, and 1 is shaded.

3 Explain how the models show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent.

The wholes are equal in size, and the amount that is shaded is the same.

4 How does the number line in **Model It** show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent? **$\frac{1}{4}$ and $\frac{2}{8}$ are located at the same point on the number line.**

5 Complete the sentences to show that the fractions of the two oranges name the same amount.

Use words: Two eighths is equal to **one fourth**.

Use fractions: $\frac{2}{8} =$ **$\frac{1}{4}$**.

6 Describe two different ways to show two fractions are equivalent.

You can see if they show the same amount in equal-sized area models, or you can see if they are at the same point on a number line.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding equivalent fractions? Explain.

Some students may prefer drawing area models to represent the fractions because it just takes a simple visual check to see that the quantities are equal. Students with strong number sense may prefer using a number line.

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Hands-On Activity

Explore different area models showing fourths and eighths.

If . . . *students struggle with seeing how different models can model the same fractions,*

Then . . . *use this activity to let them explore different ways to divide a shape into equal parts.*

Materials For each student: colored pencils, Activity Sheet *1-Inch Grid Paper*

- Have students record on the board all the different models they drew to show $\frac{2}{8} = \frac{1}{4}$.
- Encourage them to think of additional ways they can show fourths and eighths on a single model. For example, students may have drawn a rectangle with three vertical lines to mark fourths and one dashed horizontal line to show eighths. Another way to show this is to used dashed vertical lines to show eighths, or to outline fourths with one color and outline eighths with another color.
- Have students draw a square on the grid paper and see how many ways they can divide it into fourths and then into eighths.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

8 6 slices; Area models should consist of two same-sized wholes, one divided into 4 parts with 3 shaded and the other divided into 8 parts with 6 shaded to show that $\frac{3}{4} = \frac{6}{8}$. Number lines should show fourths and eighths labeled, with the point that is labeled both $\frac{3}{4}$ and $\frac{6}{8}$ highlighted.

9 See Student Worktext page; Area models should be divided into three equal parts with two parts shaded, and each of the three parts should be further divided into two equal parts.

Close: Exit Ticket

10 Possible solution: $\frac{2}{6}$; The number line should show each third divided into two (or more) equal parts, and tick marks labeled appropriately.

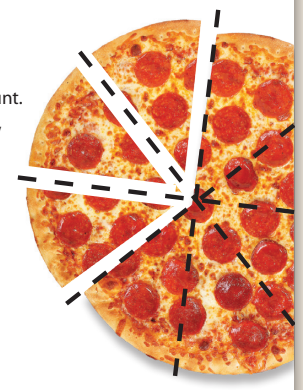
Students' solutions should indicate understanding that:

- equivalent fractions refer to the same point on a number line
- each third must be divided into equal parts in order to find a fraction equivalent to $\frac{1}{3}$

Error Alert **If** students' number lines show $\frac{1}{3}$ equal to $\frac{1}{4}$, $\frac{3}{8}$, or another incorrect fraction, **then** have the student redraw the number line on centimeter grid paper with either 12 or 24 grid squares between 0 and 1. Tell them how many grid squares make up each third and each of the other unit fractions so that they can label the number line appropriately and see their error.

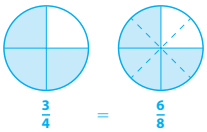
APPLY IT

Use what you just learned to solve these problems.



8 Lina and Adam each order a small pizza. They eat the same amount. Lina eats $\frac{3}{4}$ of her pizza. Adam's pizza is divided into 8 slices. How many slices of pizza did Adam eat? Show your work.

Possible student work:

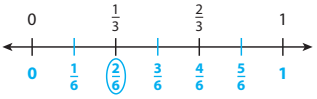


Solution **Adam ate 6 slices of pizza.**

9 Draw a model to show $\frac{2}{3} = \frac{4}{6}$.

Possible student model:

10 Use the number line to find a fraction equivalent to $\frac{1}{3}$. Show your work. **Possible student work:**



Solution **Possible answer: $\frac{2}{6}$**

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Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Solutions

1. $\frac{1}{4} = \frac{2}{8}$
Basic

2. $\frac{6}{8} = \frac{3}{4}$
Basic

3. $\frac{2}{4} = \frac{4}{8}$ or $\frac{2}{4} = \frac{1}{2}$
Basic

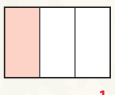
Name: _____ LESSON 23 SESSION 2

Practice Finding Equivalent Fractions

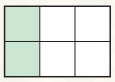
Study the Example showing how to find equivalent fractions.
Then solve problems 1–8.

EXAMPLE

Maria colors $\frac{1}{3}$ of her art paper red. Erica colors $\frac{2}{6}$ of her art paper green. The papers are the same size. Do the two girls color the same amount of their art papers?



Maria colors $\frac{1}{3}$.



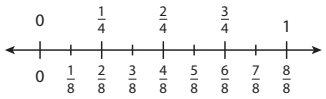
Erica colors $\frac{2}{6}$.

One third is equal to two sixths.

$$\frac{1}{3} = \frac{2}{6}$$

The girls color the same amount of their art papers.

Use the number line to complete the equivalent fractions in problems 1–3.



1. $\frac{1}{4} = \frac{2}{8}$

2. $\frac{6}{8} = \frac{3}{4}$

3. $\frac{2}{4} = \frac{4}{8}$ or $\frac{1}{2}$

Vocabulary

equivalent fractions
fractions that name the same point on a number line. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.

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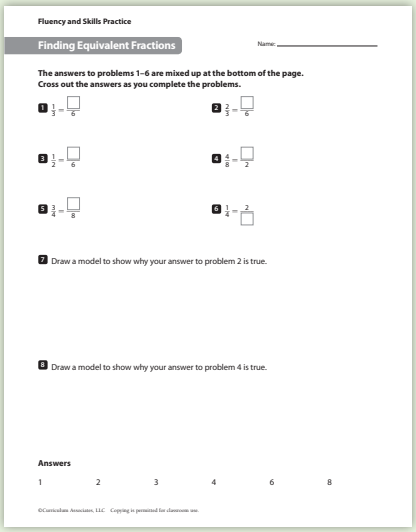
Fluency & Skills Practice

Teacher Toolbox

Assign Finding Equivalent Fractions



Fluency and Skills Practice

Finding Equivalent Fractions

Name: _____

The answers to problems 1–6 are mixed up at the bottom of the page.
Cross out the answers as you complete the problems.

1. $\frac{1}{3} = \frac{\square}{6}$
2. $\frac{2}{3} = \frac{\square}{6}$
3. $\frac{1}{2} = \frac{\square}{6}$
4. $\frac{4}{8} = \frac{\square}{2}$
5. $\frac{3}{4} = \frac{\square}{8}$
6. $\frac{1}{4} = \frac{2}{\square}$

7. Draw a model to show why your answer to problem 2 is true.

8. Draw a model to show why your answer to problem 4 is true.

Answers

1 2 3 4 6 8

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

4 See Student Worktext page for shading; $\frac{1}{2} = \frac{4}{8}$
Medium

5 See Student Worktext page for shading; $\frac{2}{3} = \frac{4}{6}$
Medium

6 See Student Worktext page for models; $\frac{1}{2} = \frac{2}{4}$
Medium

7 See Student Worktext page for models; $\frac{4}{6} = \frac{2}{3}$
Medium

8 Answers will vary. Possible answers include $\frac{2}{2}$; $\frac{3}{3}$; $\frac{6}{6}$; and $\frac{8}{8}$; Explanations show recognition that any fraction where the numerator and denominator are the same number equals 1 whole.
Challenge

Shade the models to show equivalent fractions in problems 4 and 5. Then fill in the blanks to write equivalent fractions.

4 

$\frac{1}{2}$ = $\frac{4}{8}$

5 

$\frac{2}{3}$ = $\frac{4}{6}$

Draw lines and shade to show equivalent fractions in problems 6 and 7. Then fill in the blanks to write equivalent fractions.

6 

$\frac{1}{2}$ = $\frac{2}{4}$

7 

$\frac{4}{6}$ = $\frac{2}{3}$

8 What is a fraction equivalent to $\frac{4}{4}$? Explain how you know.

Answers will vary. Possible answer: $\frac{2}{2}$; If I shaded $\frac{4}{4}$ of a shape, I would shade the whole shape. If I shaded $\frac{2}{2}$ of the same shape, I would also shade the whole shape. $\frac{4}{4}$ and $\frac{2}{2}$ are both equal to 1 whole, so $\frac{4}{4} = \frac{2}{2}$.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 6. Display the graphic. Ask: *How many parts are in each triangle?* [2] *What unit fraction names each part?* [one half] Label $\frac{1}{2}$ and $\frac{2}{2}$ on the first triangle. Have students replicate. Point to, say, and have students repeat: *one half, two halves*. Point to the next triangle. Label $\frac{3}{2}$, $\frac{4}{2}$ and say: *three halves, four halves*. Have students replicate and repeat. Say: *Label the last triangle*. Have students chorally count the halves. Display and chorally complete:

- $\frac{2}{2}$ = *1 whole*
- $\frac{4}{2}$ = *2 wholes*
- $\frac{6}{2}$ = *3 wholes*

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 6. Display the graphic. Ask: *How many parts are in each triangle?* [2] *What unit fraction names each part?* [one half] Label $\frac{1}{2}$ and $\frac{2}{2}$ on the first triangle. Have students replicate. Point to, say, and have students repeat: *one half, two halves*. Point to the next triangle. Label $\frac{3}{2}$ and say: *three halves*. Have students replicate and repeat with the fourth half. Say: *Finish labeling the halves*. Have students chorally count the halves. Display:

- $\frac{2}{2}$ = *1 whole*
- $\frac{4}{2}$ = *2 wholes*
- $\frac{6}{2}$ = *3 wholes*

Have pairs complete the fractions.

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 6. Display the graphic. Ask: *How many parts are in each triangle?* [2] *What unit fraction names each part?* [one half] Label $\frac{1}{2}$ and $\frac{2}{2}$ on the first triangle. Have students replicate. Point to, say, and have students repeat: *one half, two halves*. Say: *Finish labeling the halves*. Have students chorally count the halves. Display:

- $\frac{2}{2}$ = *1 whole*
- $\frac{4}{2}$ = *2 wholes*
- $\frac{6}{2}$ = *3 wholes*

Have students form pairs and complete the fractions. Call on students to say each equation.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

SESSION 3 Develop

Purpose In this session students solve a problem that requires writing a fraction to represent a whole number. Students model the quantity with area models, fraction bars, or number lines to find the numerator and denominator of the fraction. The purpose of this session is to develop strategies for writing whole numbers as fractions.

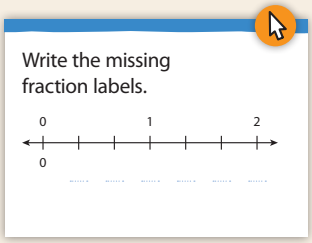
Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *Number Lines*

Why Support students' facility with labeling a number line with fractions involving whole numbers.

How Have students label a number line with thirds from 0 to 2.



Solution

$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}$

Develop Language

Why Clarify that *cut* and *divide* can be synonyms.

How Display the word *cut*. Say: *In the problem, Kacey cuts boards into fourths.* Have students find a synonym for *cut* in ***Picture It***. [divided] Clarify that the present tense of *divided* is *divide*. Reread the problem and substitute *divides* for *cuts*. Say: *Both* cut *and* divide *in this context mean* to separate. Provide the sentence frame: *Kacey _____ the boards into fourths.*

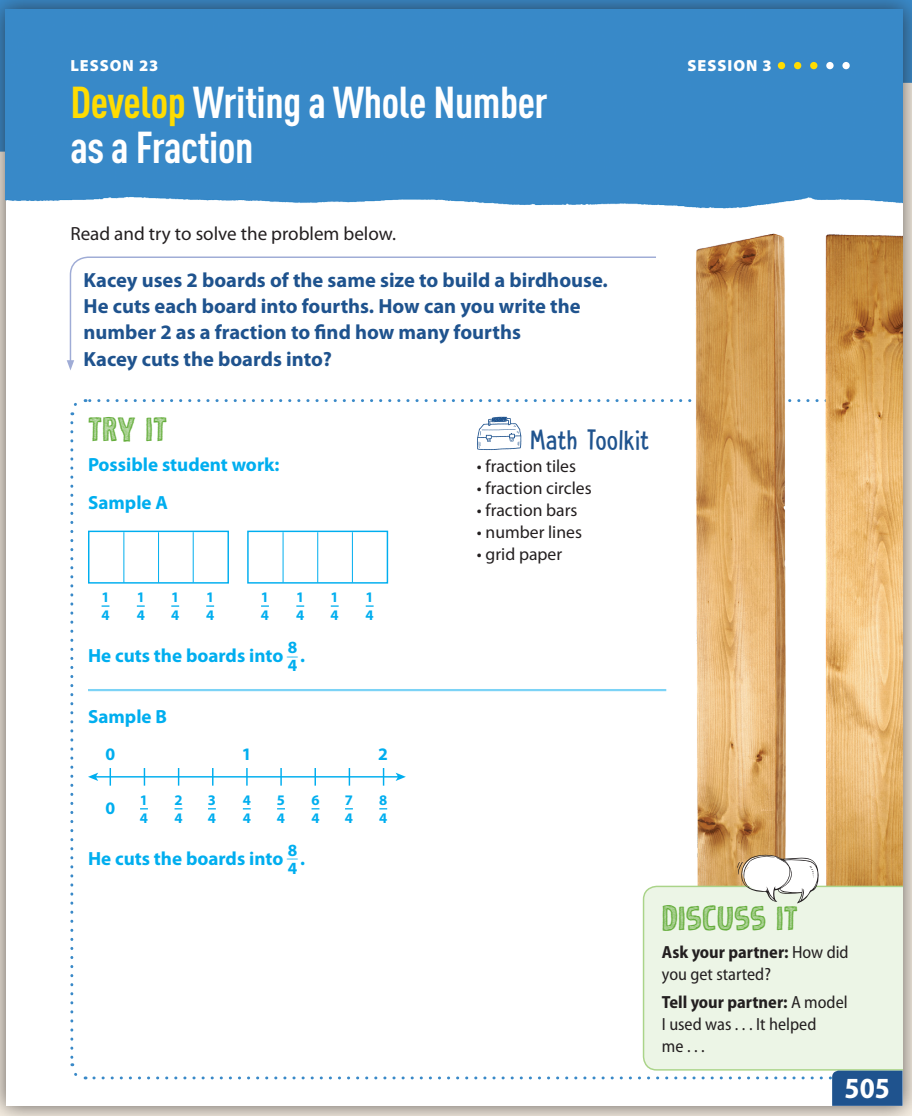
Have students practice the sentence using *cuts, divides* and *separates*.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that there are 2 boards and each one is cut into fourths. They need to write 2 as a fraction in fourths.

Ask *How many whole boards did Kacey start with? How many parts did he cut each board into?*



DISCUSS IT

Support Partner Discussion

Encourage students to use the term *fourths* as they discuss their solutions.

Support as needed with questions such as:

- *Why did you choose the model you did?*
- *How did you use your model to get the final answer?*

Common Misconception Look for students who do not understand that there is more than one whole being divided into fourths.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction tiles or fraction circles
- drawings or area models showing fourths
- number lines showing fourths
- solutions without visual models

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does each model show that Kacey had 2 boards? How does each model show that each board was cut into fourths?*

Listen for Area models should show two wholes, each divided into four equal parts. Number lines should go up to 2 and have tick marks at every fourth.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- two wholes
- each whole divided into fourths

Ask *How many wholes are there? How are they divided? Why are $\frac{4}{4}$ and $\frac{8}{4}$ shown in red on the number line?*

Listen for Each fraction bar represents one whole and each whole is divided into 4 equal parts, or fourths. The red labels on the number line are the whole numbers.

For the fraction bars, prompt students to identify how many wholes there are and how many parts in each.

- *How many boards did Kacey start with?*
- *How many parts did he cut each board into?*

For the number line, prompt students to describe how wholes and parts are represented.

- *How are the tick marks labeled?*
- *How could you tell which tick marks represented whole numbers if "1" and "2" were not labeled?*

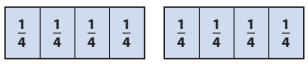
Explore different ways to understand writing a whole number as a fraction.

Kacey uses 2 boards of the same size to build a birdhouse. He cuts each board into fourths. How can you write the number 2 as a fraction to find how many fourths Kacey cuts the boards into?

PICTURE IT

You can use models to help you write a whole number as a fraction.

The fraction bars below show 2 wholes, each divided into fourths.



Each part is $\frac{1}{4}$ of a whole. There are eight $\frac{1}{4}$s in all.

MODEL IT

You can use a number line to help you write a whole number as a fraction.

This number line shows whole numbers on the top and fourths on the bottom.

Notice that each whole number has an equivalent fraction with a denominator of 4.

Deepen Understanding

Modeling Fractions Greater Than 1

SMP 6 Attend to precision.

When discussing the area model, prompt students to take note of how many fraction bars are used to model this one quantity.

Ask *Why is there more than 1 fraction bar to model just one number?*

Listen for The number modeled is 2, or 2 wholes, so there needs to be a fraction bar for each whole.

Ask *The problem states that Kacey cut the boards into fourths; however, there are more than 4 parts. Why?*

Listen for It is not the total amount that is divided into 4 parts, but each individual whole. There is more than one whole, so there are more than 4 parts.

Ask *Why is the denominator 4 instead of 8?*

Listen for The denominator is the number of parts that make up one whole, not all the wholes combined. Each whole is divided into 4 parts, so the denominator is 4.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to write a fraction equivalent to 2.

Monitor and Confirm

1 – 2 Check for understanding that:

- there are 4 parts in 1 whole
- there are 2 wholes
- there are 8 parts in all

Support Whole Class Discussion

3 Be sure students understand that the fraction represents the total number of parts (fourths) in both wholes.

Ask *Explain why each part of the two boards is labeled $\frac{1}{4}$. How did you find the numerator and denominator of the fraction you wrote in problem 3?*

Listen for Each part is $\frac{1}{4}$ because it takes 4 of them to make a whole. The numerator is the total number of parts you have (8) and the denominator is the number of parts it takes to make 1 whole (4).

4 Look for understanding that the steps for writing a whole number as a fraction are the same as the steps for writing other fractions: identify the number of parts in a whole (the denominator) and the number of parts you have (the numerator).

5 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to write a whole number as a fraction.

1 Look at the models in **Picture It**. How many equal parts are shown in 1 whole? Explain how you know. **Possible answer: There are 4 parts in 1 whole. Each fraction bar has 4 parts.**

2 How many equal parts are shown in 2 wholes? Explain how you know.

Possible answer: There are 8 parts in 2 wholes. I counted the number of parts in both fraction bars.

3 Complete the sentences to show the fraction that is equivalent to 2.

Use words: Two wholes equals **eight fourths**.

Use a fraction: 2 = $\frac{8}{4}$.

How many fourths does Kacey cut the boards into? **eight fourths**

4 Explain how to find a fraction equivalent to a whole number.

Possible answer: You look at how many parts are in one whole, and that is the denominator. Then you count how many parts there are in all of the wholes together, and that is the numerator.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for writing a whole number as a fraction? Explain.

Some students may prefer using a number line or an area model so that they can count the parts and visually check that they combine to make the given number of wholes. Other students may be fluent enough with their math facts to calculate the numerator by multiplying.

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Hands-On Activity

Use fraction circles to write whole numbers as fractions.

If . . . *students have trouble writing whole numbers as fractions,*

Then . . . *use this activity to let them build fractions with concrete materials.*

Materials For each pair: 4 sets of fraction circles, Activity Sheet *Multiplication Table*

- Have one student from each pair use one-fourth pieces to model 1 whole and have the other student write the fraction modeled. [$\frac{4}{4}$] Tell them to switch roles and repeat the process for 2 wholes, 3 wholes, and 4 wholes. They should write the fractions in order. [$\frac{4}{4}, \frac{8}{4}, \frac{12}{4}, \frac{16}{4}$]
- Ask one partner to locate the column of the multiplication table under the number 4 and read the first 4 numbers in the column. [4, 8, 12, 16]
- Have the other partner read the numerators of the fractions they wrote, in order. [4, 8, 12, 16]
- Repeat the activity using another denominator, such as 2, 3, 6, or 8.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

6 $\frac{6}{2}$; 6 halves = 3 wholes

7 $\frac{16}{8}$; 16 eighths = 2 wholes

Close: Exit Ticket

8 See Student Worktext page; Area models should show 3 wholes each divided into 6 equal parts. Number lines should go from 0 to 3 and have a tick mark at every sixth.

Students' solutions should indicate understanding that:

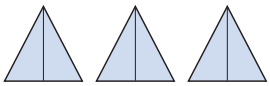
- there are 3 wholes
- each whole is divided into 6 equal parts
- there are 18 equal parts in all

Error Alert **If** students draw a model that shows 6 wholes divided into thirds, **then** ask them to point out the denominator and the whole number mentioned in the problem and discuss how each of those numbers should be used to create the model.

APPLY IT

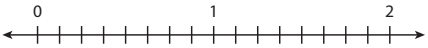
Use what you just learned to solve these problems.

6 Use the model below to write a fraction equivalent to 3.



Solution $\frac{6}{2}$

7 Louisa has 2 ribbons that are the same length. She cuts each one into eighths. Use the number line below to help you write the number 2 as a fraction to show how many eighths she cuts the ribbons into.



Solution $\frac{16}{8}$

8 Draw a model to show $3 = \frac{18}{6}$. Show your work.

Possible student model:

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Solutions

1. $1 = \frac{3}{3}$
   Basic

2. $2 = \frac{6}{3}$
   Basic

3. $3 = \frac{9}{3}$
   Basic

4. $4 = \frac{12}{3}$
   Medium

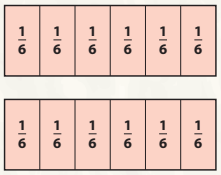
Name: _____ **LESSON 23 SESSION 3**

Practice Writing a Whole Number as a Fraction

Study the Example showing different ways to write whole numbers as fractions. Then solve problems 1–13.

EXAMPLE

Mrs. Clark cuts 2 same-sized pieces of colored paper into sixths to make strips for paper chains. How many strips does she make?



1 whole = six $\frac{1}{6}$s

$1 = \frac{6}{6}$

2 wholes = twelve $\frac{1}{6}$s

$2 = \frac{12}{6}$

Each strip is $\frac{1}{6}$ of a whole piece of paper.

Mrs. Clark makes 12 strips.

Write the whole numbers as fractions in problems 1–4.

1. $1 = \frac{3}{3}$
2. $2 = \frac{6}{3}$
3. $3 = \frac{9}{3}$
4. $4 = \frac{12}{3}$

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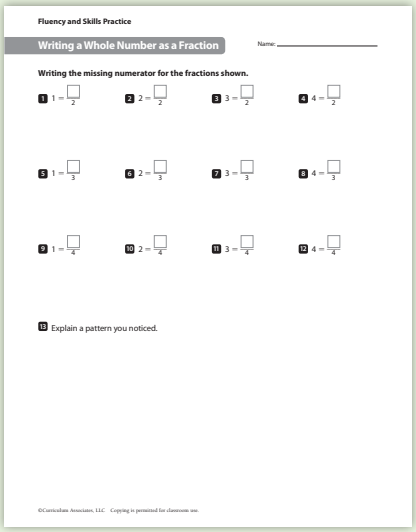
Fluency & Skills Practice

Teacher Toolbox

Assign Writing a Whole Number as a Fraction


Fluency and Skills Practice

Writing a Whole Number as a Fraction

Name: _____

Writing the missing numerator for the fractions shown.

1. $1 = \frac{\square}{2}$
2. $2 = \frac{\square}{2}$
3. $3 = \frac{\square}{2}$
4. $4 = \frac{\square}{2}$
5. $1 = \frac{\square}{3}$
6. $2 = \frac{\square}{3}$
7. $3 = \frac{\square}{3}$
8. $4 = \frac{\square}{3}$
9. $1 = \frac{\square}{4}$
10. $2 = \frac{\square}{4}$
11. $3 = \frac{\square}{4}$
12. $4 = \frac{\square}{4}$
13. Explain a pattern you noticed.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

5 $1 = \frac{4}{4}$
Basic

6 $2 = \frac{8}{4}$
Basic

7 $3 = \frac{12}{4}$
Basic

8 $0 = \frac{0}{4}$
Medium

9 8
Medium

10 2
Medium

11 $3 = \frac{24}{8}$
Medium

12 $3 = \frac{12}{4}$
Medium

13 See Student Worktext page. Area models and fraction bars should each show 2 wholes divided into 4 equal parts. Number lines should go from 0 to 2 with a tick mark at every fourth.
Challenge

Use this number line to solve problems 5–8.

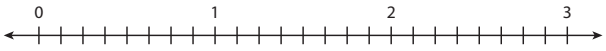
5 $1 = \frac{4}{4}$

6 $2 = \frac{8}{4}$

7 $3 = \frac{12}{4}$

8 $0 = \frac{0}{4}$

Use this number line to solve problems 9–11.



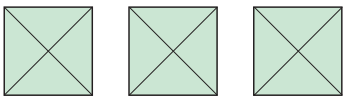
9 One whole is equal to 8 eighths.

10 16 eighths is equal to 2 wholes.

11 $3 = \frac{24}{8}$

12 Use the model below to write a fraction equivalent to 3.

$3 = \frac{12}{4}$



13 Draw a model to show $2 = \frac{8}{4}$.

Students' models will vary. They may draw number lines labeled with whole numbers and fourths. They may also draw 2 same-sized shapes divided into 4 equal pieces, with all 4 pieces in each shape shaded.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 10 aloud. Arrange students in groups of four. Give each group a piece of yarn and four strips of paper folded in half and stapled one inch from the fold. Say: *You are going to make a number line. In your group, decide what whole numbers represent Oscar's loaves of bread.* When students share, display 0 through 3. Say: *Zero is the first whole number. Label one strip 0. The numerator for the first loaf is 1. The denominator is 1 because the loaf is whole.* Label a strip $\frac{1}{1}$. Slide the strips onto the yarn. Have students replicate. Say: *Use your strips. Discuss and say fractions to represent the other two whole loaves of bread. Make a number line with the yarn and paper strips.*

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 10 aloud. Arrange students in groups of four. Say: *You are going to make a number line.* Give each group a piece of yarn and four strips of paper folded in half and stapled one inch from the fold. Ask: *What number will start your number line?* [0] *Label one strip 0.* Model how to slide the strips onto the yarn. Say: *In your group, decide what fraction represents the first loaf.* When students respond, write $\frac{1}{1}$ below the staple. Say: *Write a fraction on each strip of paper to represent each loaf. Make a number line with the yarn and paper strips.* When the number lines are complete, guide a discussion about equal placement of the strips along the piece of yarn.

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 10. Arrange students in groups of four. Ask: *What model will you use to solve the problem?* [number line] Give each group a piece of yarn and four strips of paper folded in half and stapled one inch from the fold. Say: *What number will start your number line?* [0] *Label one strip 0.* Model how to slide the strips onto the yarn. Say: *Write a fraction on three strips of paper to represent each loaf of bread. Then make a number line with the yarn and paper strips.* When the number lines are complete, facilitate a discussion about equal placement of the strips along the piece of yarn.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Purpose In this session students solve a problem that requires writing a whole number as a fraction with a denominator of 1. Students model the quantity either on paper or with manipulatives. The purpose of this session is to have students develop strategies for writing whole numbers as fractions with a denominator of 1.

Start

Connect to Prior Knowledge

Materials For each student: 2 sets of fraction tiles

Why Reinforce the idea that there are different ways to write a whole number as a fraction.

How Have students model and write three different fractions that are equivalent to 2, given the denominators.

Use fraction tiles to model the whole number 2 with halves, thirds, and fourths. Then write the missing numerators.

$2 = \frac{\square}{2}$ $2 = \frac{\square}{3}$ $2 = \frac{\square}{4}$

Solutions
$\frac{4}{2}, \frac{6}{3}, \frac{8}{4}$
Look for tile models
4 one-halves,
6 one-thirds, and
8 one-fourths.

Develop Language

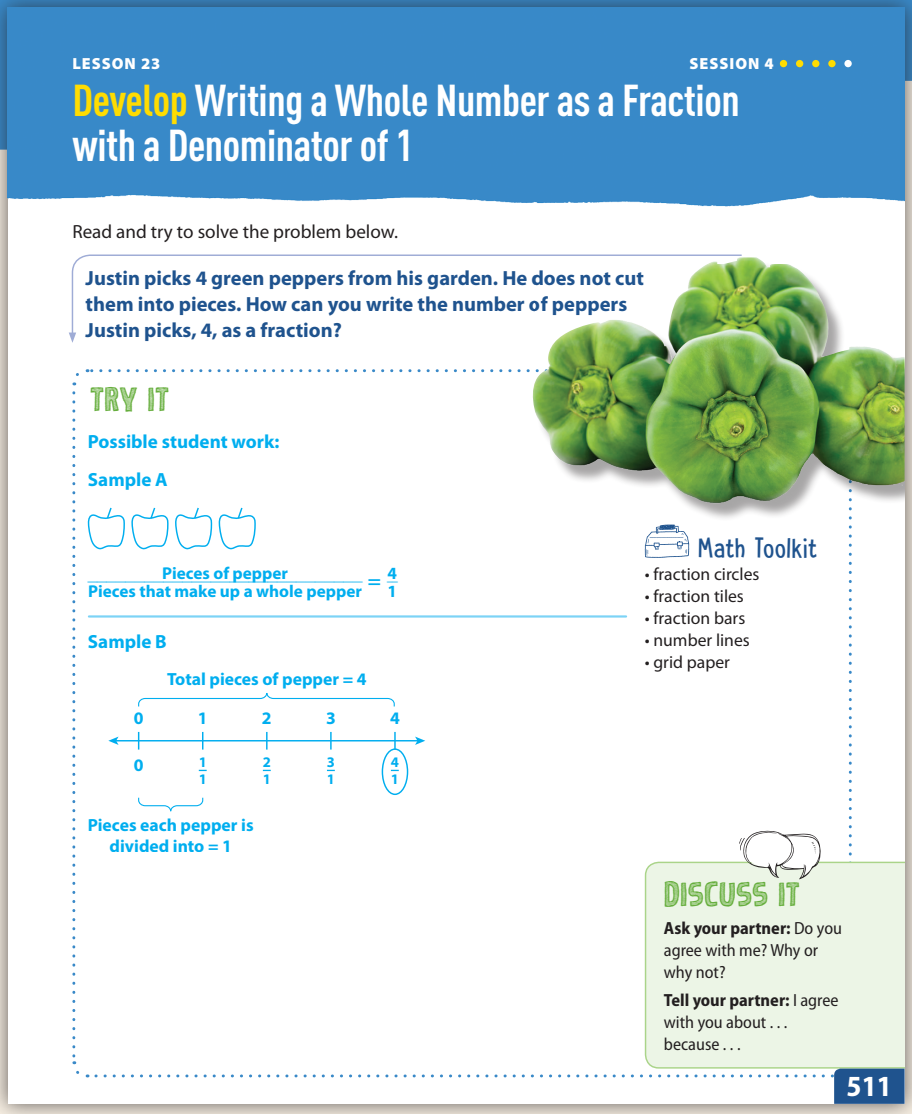
Why Clarify how to read a whole number represented as a fraction.

How Display $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. Say: *You have learned how to read fractions.* Have students read the fractions chorally. Display $\frac{2}{1}$, $\frac{3}{1}$, and $\frac{4}{1}$. Say: *Each of these fractions represents a whole number. These fractions are read as two over one, three over one, and four over one. The numerator is over the denominator when you write a fraction.* Point to and read each fraction. Have students repeat.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify a complete pepper as both the part and the whole.



LESSON 23 SESSION 4 • • • • •

Develop Writing a Whole Number as a Fraction with a Denominator of 1

Read and try to solve the problem below.

Justin picks 4 green peppers from his garden. He does not cut them into pieces. How can you write the number of peppers Justin picks, 4, as a fraction?

TRY IT

Possible student work:

Sample A

$\frac{\text{Pieces of pepper}}{\text{Pieces that make up a whole pepper}} = \frac{4}{1}$

Sample B

Total pieces of pepper = 4

0 1 2 3 4

0 $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$

Pieces each pepper is divided into = 1

Math Toolkit
- fraction circles
- fraction tiles
- fraction bars
- number lines
- grid paper

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *whole* as they discuss their solutions.

Support as needed with questions such as:

- *How did you decide what the denominator should be?*
- *How did you decide what the numerator should be?*

Common Misconception Look for students who are confused by the idea that "1 part = 1 whole" or that think the pepper is divided into 0 parts since it is not divided at all. When referring to the denominator, use the phrase "number of parts that make up a whole" rather than "number of parts a whole is divided into."

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction tiles or fraction circles
- drawings or area models showing wholes
- number lines showing wholes

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How can you tell from each model that there are 4 peppers? How does each model show that the peppers are not cut up, but left whole?*

Listen for Drawings and fraction tiles or circles will show 4 wholes that are not divided into parts. A number line will show 0 through 4, with no tick marks between whole numbers.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- there are 4 wholes
- the wholes are not divided into parts

Ask *How many parts are there? How can you tell that one part makes a whole?*

Listen for The picture shows 4 circles and the number line goes up to 4. The circles are not divided into parts, so each circle = 1 part = 1 whole. There are no tick marks dividing the wholes on the number line, so each whole has only 1 part.

For an area model, prompt students to identify the number of wholes in the problem and that the wholes are not divided into parts.

- *How can you tell how many whole peppers Justin has?*
- *How can you tell that the peppers were left whole instead of cut into pieces?*

For a number line, prompt students to think about how far the number line goes up to and where the tick marks are.

- *How many wholes does the number line show?*
- *What quantity does the space between tick marks represent?*

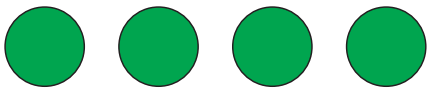
Explore different ways to understand writing a whole number as a fraction with a denominator of 1.

Justin picks 4 green peppers from his garden. He does not cut them into pieces. How can you write the number of peppers Justin picks, 4, as a fraction?

PICTURE IT

You can use models to help you write a whole number as a fraction with a denominator of 1.

Each circle stands for 1 green pepper.

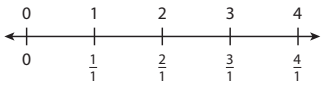


They are not divided into pieces, so each whole has one part.

MODEL IT

You can use a number line to help you write a whole number as a fraction with a denominator of 1.

This number line shows whole numbers on the top and fractions on the bottom.



Notice that each whole number has an equivalent fraction. The spaces between whole numbers are not divided into parts. Each whole number has one part, so the denominator of each equivalent fraction is 1.

Deepen Understanding

Wholes on a Number Line

SMP 8 Use repeated reasoning.

As you discuss the number-line model, prompt students to think about the space between each whole number as "a whole" just as in an area model.

Ask *What patterns do you see in the fraction labels for the whole numbers?*

Listen for The numerator is the same as the whole number.

Ask *What are the fraction labels at each whole number if the whole is divided into 2 parts?* $\left[\frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}\right]$ *What patterns do you see in these labels?*

Listen for The numerator is two times the whole number.

Ask *What are the labels and patterns if the whole is divided into 3 parts?*

Listen for $\frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{12}{3}$; The numerator is three times the whole number.

Generalize Elicit a general pattern: When you divide the whole into a number of equal parts, the numerators of the whole numbers will be multiples of the number of equal parts the whole is divided into.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to develop a strategy for writing whole numbers as fractions with a denominator of 1.

Monitor and Confirm

1–4 Check for understanding that:

- there are 4 parts
- 1 part = 1 whole
- the numerator shows how many parts you have
- the denominator shows how many parts make a whole

Support Whole Class Discussion

5 Tell students that this problem will prepare them to provide the explanation required in problem 6.

Be sure students understand why there is only 1 part in the whole.

Ask *How many separate parts are there? How many parts make 1 whole?*

Listen for There are 4 parts. Each part is a whole pepper, so it only takes 1 part to make a whole.

6 Look for the understanding that the numerator is the whole number and the denominator is 1, because it only takes 1 part to make a whole.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to write a whole number as a fraction with a denominator of 1.

1 Look at the models in **Picture It**. Explain how you know each whole has only 1 part.

The wholes are not divided into parts, so there is only 1 part.

2 How many parts do the 4 green peppers make? **4**

3 What does the numerator of a fraction show?

how many parts are being described

4 What does the denominator of a fraction show?

how many parts are in the whole

5 Write a fraction equivalent to 4. Use the fraction below to help you.

$\frac{\text{number of parts described}}{\text{number of equal parts in the whole}}$ **$\frac{4}{1}$**

6 Explain how to write a whole number as a fraction with a denominator of 1.

Possible answer: Write the whole number as the numerator and use 1 for the denominator. The whole number is the number of parts described. The denominator is the number of parts in the whole, which is 1.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for writing a whole number as a fraction with a denominator of 1? Explain.

Some students may prefer using diagrams or number lines if they are still working on the concept of having only one part in the whole. Others may be comfortable enough to simply write a fraction with the whole number as the numerator and 1 as the denominator without using a visual model.

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Visual Model

Generalize the writing of equivalent fractions for whole numbers.

If . . . *students struggle with writing fractions for undivided wholes,*

Then . . . *use this illustration to help them apply the same steps for all fractions.*

- Draw 3 same-sized circles on the board and divide each into 2 equal parts. Elicit that each part is called a *half*. Draw 3 more same-sized circles, but do not divide these. Explain that because they are not divided, each part is called a *whole*. Elicit that both models show the whole number 3.
- Together, write the fraction shown by the first model. [$\frac{6}{2}$] Ask a volunteer to explain the process and record the steps on the board. [Count the number of equal parts in each whole to find the denominator of the fraction. Count the total number of equal parts to find the numerator.]
- Follow the same steps to write the fraction shown by the second model. [$\frac{3}{1}$] Point to each numerator and denominator as you tell students, *6 halves equals 3 wholes.*
- Repeat the activity for $\frac{8}{4}$ and $\frac{2}{1}$.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

8 $\frac{6}{1}$; The wholes are not divided into more than one part, so the denominator is 1. There are 6 parts, or 6 wholes, so the numerator is 6.

9 See Student Worktext page; Area models or fraction bars should show 5 wholes that are not divided. Number lines should go from 0 to 5 and have tick marks only at whole numbers.

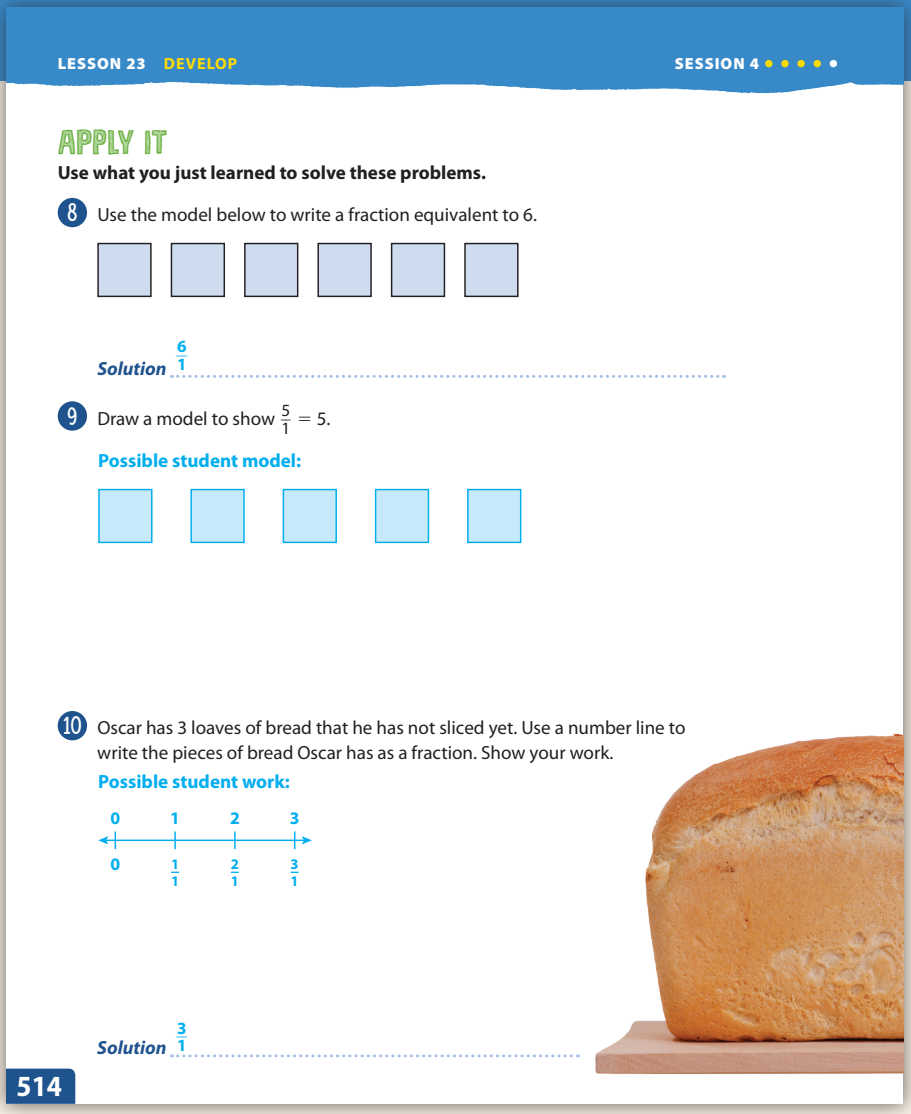
Close: Exit Ticket

10 $\frac{3}{1}$; The number line should go up to 3 and have tick marks only at whole numbers.

Students' solutions should indicate understanding that:

- there are 3 wholes
- the wholes are not divided into smaller parts
- 1 part = 1 whole

Error Alert **If** students write $\frac{3}{3}$, **then** review the definition of denominator. Use fraction tiles or area models to show $\frac{3}{1}$ and $\frac{3}{3}$ and discuss the difference between the two quantities.



LESSON 23 DEVELOP SESSION 4

APPLY IT

Use what you just learned to solve these problems.

8 Use the model below to write a fraction equivalent to 6.

Solution $\frac{6}{1}$

9 Draw a model to show $\frac{5}{1} = 5$.

Possible student model:

10 Oscar has 3 loaves of bread that he has not sliced yet. Use a number line to write the pieces of bread Oscar has as a fraction. Show your work.

Possible student work:

0 1 2 3

0 $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$

Solution $\frac{3}{1}$

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Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

Solutions

1. 4
   Basic
2. 2
   Basic
3. 5
   Basic
4. 8
   Medium
5. $\frac{2}{1}$
   Basic
6. $\frac{5}{1}$
   Basic
7. $\frac{1}{1}$
   Basic
8. $\frac{7}{1}$
   Medium

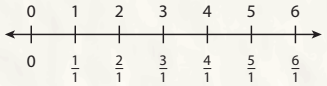
Name: _____ LESSON 23 SESSION 4

Practice Writing a Whole Number as a Fraction with a Denominator of 1

Study the Example showing how to write a whole number as a fraction with a denominator of 1. Then solve problems 1–14.

EXAMPLE

The spaces between whole numbers on this number line are not divided into smaller parts. So, each whole has only 1 part.



The number line shows that $\frac{3}{1}$ is equal to 3.

$\frac{3}{1}$ is a fraction name for 3.

Write the whole number for each fraction in problems 1–4.

1. $\frac{4}{1} =$ 4
2. $\frac{2}{1} =$ 2
3. $\frac{5}{1} =$ 5
4. $\frac{8}{1} =$ 8

Write a fraction with a denominator of 1 for each whole number in problems 5–8.

5. $2 =$ $\frac{2}{1}$
6. $5 =$ $\frac{5}{1}$
7. $1 =$ $\frac{1}{1}$
8. $7 =$ $\frac{7}{1}$

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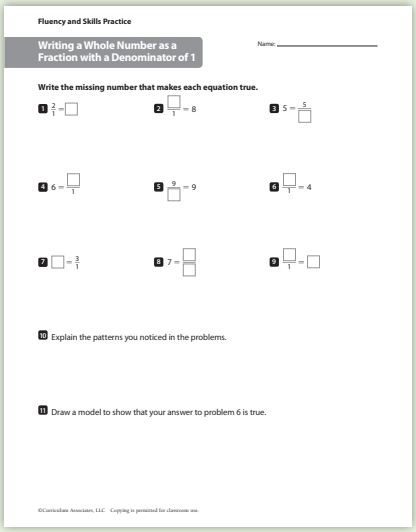
Fluency & Skills Practice

Teacher Toolbox

Assign Writing a Whole Number as a Fraction with a Denominator of 1


Fluency and Skills Practice

Writing a Whole Number as a Fraction with a Denominator of 1

Name: _____

Write the missing number that makes each equation true.

1. $\frac{2}{1} = \frac{\square}{\square}$
2. $\frac{\square}{1} = 8$
3. $5 = \frac{5}{\square}$
4. $6 = \frac{\square}{1}$
5. $\frac{9}{\square} = 9$
6. $\frac{\square}{1} = 4$
7. $\frac{\square}{\square} = \frac{3}{1}$
8. $7 = \frac{\square}{\square}$
9. $\frac{\square}{1} = \frac{\square}{\square}$
10. Explain the patterns you noticed in the problems.
11. Draw a model to show that your answer to problem 6 is true.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

9 9
Medium

10 10
Medium

11 $\frac{12}{1}$
Medium

12 $\frac{18}{1}$
Medium

13 See Student Worktext page; Use the whole number as the numerator and 1 as the denominator.
Challenge

14 See Student Worktext page; Explanations should show understanding that equivalent fractions with different denominators would indicate that the wholes are divided differently but the amount would be the same.
Challenge

Write the whole number for each fraction in problems 9 and 10.

9 $\frac{9}{1} =$ 9

10 $\frac{10}{1} =$ 10

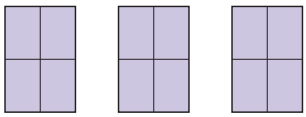
Write a fraction with a denominator of 1 for each whole number in problems 11 and 12.

11 $12 =$ $\frac{12}{1}$

12 $18 =$ $\frac{18}{1}$

13 Explain how to write a whole number as a fraction with a denominator of 1.
Possible answer: Write the whole number as the numerator of the fraction and use 1 as the denominator.

14 Bella says this model shows 3 wholes. She says it shows that if you write the whole number 3 as a fraction, you have to write $3 = \frac{12}{4}$.
How can you explain to Bella that there are other ways to write 3 as a fraction?



Possible answer: I would tell her there are different ways to write a whole number as a fraction. The denominator could be 1 instead of 4 if you don't divide the whole into pieces. So, you could write 3 as $\frac{3}{1}$. You could also divide each whole into different numbers of parts. If you divided each whole into 2 parts, you would get $3 = \frac{6}{2}$.

Vocabulary

numerator the number above the line in a fraction; it tells how many equal parts are being described.

denominator the number below the line in a fraction; it tells how many equal parts are in the whole.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 5
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Apply It*** problem 2 and have students form pairs. Ask: *What whole number do you need to show as a fraction?* [5] Give each student a tile. Model how to trace the tile. Say: *Trace the tile five times in a row. Divide each square in half. Point to the first square. There are two halves in this square.* Display $\frac{2}{2}$. Say: *There are two halves in each square.* Label the halves. Say: *Let's count the halves.* Have students chorally count by ones or twos. Display and say: $\frac{10}{2}$. Have students trace five more tiles. Ask: *How many equal pieces do you want to make in each square?* Field students' suggestions and provide support as needed. Repeat the process.	**Listening/Speaking** Read ***Apply It*** problem 2. Pair students and give each pair a tile. Ask: *What whole number do you need to show as a fraction?* [5] Model and say: *Trace the tile five times in a row. Divide each square in half.* Point to each and ask: *How many halves are in this square?* [2] Label each square $\frac{2}{2}$. Ask: *How many halves are in all the squares?* [10] Display $\frac{10}{2}$. Say: *Trace five more squares. Divide the first square in fourths. What fraction represents the whole square?* $\left[\frac{4}{4}\right]$ Say: *Work with your partner. Divide the squares into fourths. Label each square. Write a fraction that shows how many fourths are in all five squares.* Call on pairs to say the fraction.	**Listening/Speaking** Have pairs read ***Apply It*** problem 2. Give each student a tile. Ask: *What whole number do you need to show as a fraction?* [5] Model and say: *Trace the tile five times in a row. Divide each square in half.* Point to each and ask: *How many halves are in this square?* [2] Label each square $\frac{2}{2}$. Ask: *How many total halves are in all the squares?* [10] Display: $\frac{10}{2}$. Say: *Trace five more squares.* Say: *Work with your partner. Decide how you will divide the five squares equally. Label each square. Write a fraction to represent all of the squares.* Call on pairs to share their fractions and discuss their strategy.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

LESSON 23

SESSION 5 Refine

Purpose In this session students solve word problems involving equivalent fractions, then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

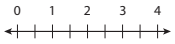
Check for Understanding

Materials For each student: Activity Sheet *Number Lines*

Why Confirm understanding of finding equivalent fractions.

How Have students find a fraction equivalent to $\frac{4}{1}$ given a number line that shows halves.

Use the number line to write a fraction that is equivalent to $\frac{4}{1}$.



Possible Solution

$\frac{8}{2}$

LESSON 23 SESSION 5

Refine Finding Equivalent Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Caleb and Hannah buy two melons that are the same size. Caleb cuts his melon into fourths. Hannah cuts her melon into eighths. Hannah eats $\frac{4}{8}$ of her melon. Caleb eats an equal amount of his melon. What fraction of his melon does Caleb eat?

Look at how you could show your work using a model.

Solution Caleb eats $\frac{2}{4}$ of his melon.

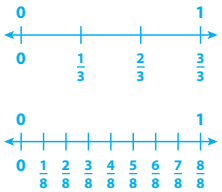
The student used solid lines to show fourths. She used dashed lines to show how to divide fourths to make eighths.

PAIR/SHARE
How could you solve this problem using a number line?

APPLY IT

1 Matt says $\frac{3}{3}$ is equivalent to 1. Elisa says $\frac{8}{8}$ is equivalent to 1. Who is correct? Show your work.

Possible student work using number lines:



Solution They are both correct.

How many thirds are in 1 whole? How many eighths are in 1 whole?

PAIR/SHARE
What is another fraction that is equivalent to 1?

517

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
students label the halves but do not clearly show that $\frac{4}{1}$ and $\frac{8}{2}$ are the same point on the number line and, therefore, are equivalent	not understand how to show that two fractions on a number line are equivalent or they may be confused as to what the fraction $\frac{4}{1}$ means.	Review with students the meaning of $\frac{4}{1}$. Be sure students understand that the denominator names the number of parts in one whole. Since the denominator is 1, there is 1 part. So, each part is one whole. The numerator tells how many parts, or wholes, there are. Ask students to write the fraction for the whole numbers on the top part of the number line $\left[\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}\right]$. Direct students to draw a point at $\frac{4}{1}$ and explain in words that $\frac{4}{1}$ and $\frac{8}{2}$ name that same point.
$\frac{8}{1}$, $\frac{8}{4}$, or $\frac{8}{8}$	not understand what the denominator of the fraction represents.	Review the meaning of *denominator*. Be sure students understand that the denominator names the number of parts in one whole. Have them identify one whole on the number line and count the number of parts it is divided into.
$\frac{2}{2}$, $\frac{4}{2}$, or $\frac{2}{8}$	not understand what the numerator of the fraction represents.	Review the meaning of *numerator*. Be sure students understand that the numerator tells how many parts there are.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

EXAMPLE

Caleb eats $\frac{2}{4}$ of his melon; The model shown is one way to solve the problem. Students could also solve the problem by drawing a number line labeled with both eighths and fourths and finding that $\frac{2}{4}$ is located at the same point as $\frac{4}{8}$.

Look for If fourths and eighths are shown in the same model, then there should be some way to distinguish one from the other.

APPLY IT

1. They are both correct; Students could also solve the problem by drawing a whole divided into thirds and a whole divided into eighths and finding that both $\frac{3}{3}$ and $\frac{8}{8}$ are equal to 1 whole.
DOK 3

   Look for There are 3 thirds in 1 whole and 8 eighths in 1 whole.

2. Possible answers include $\frac{5}{1}$, $\frac{10}{2}$, $\frac{15}{3}$, $\frac{20}{4}$, $\frac{30}{6}$, and $\frac{40}{8}$; Students could also solve the problem by drawing number lines from 0 to 5 labeled with two different unit fractions.
DOK 3

   Look for Each model should show 5 wholes all divided into the same number of equal parts.

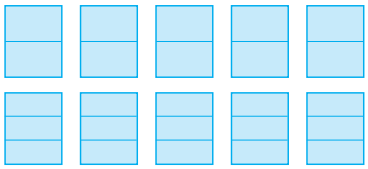
3. **D**; Students could solve the problem by identifying the fraction that shows the same amount on a fraction model, or is at the same location on a number line as $\frac{3}{6}$.

   Explain why the other two answer choices are not correct:

   B and **C** are not correct because neither fraction shows the same amount on a fraction model, nor is at the same location on a number line, as $\frac{3}{6}$.
DOK 3

2. Write two fractions that are equivalent to 5. Show your work.

Possible student work using models:



There will be 5 wholes in all. Think about how many parts will be in each whole.

PAIR/SHARE
How did you decide what denominators to use in your fractions?

Solution **Possible answers: $\frac{10}{2}$, $\frac{15}{3}$**

3. Kaia ate $\frac{3}{6}$ of a banana. Zoie ate an equivalent amount. Which fraction shows how much of a banana Zoie ate?

Ⓐ $\frac{1}{3}$

Ⓑ $\frac{2}{3}$

Ⓒ $\frac{5}{8}$

Ⓓ $\frac{1}{2}$

Find $\frac{3}{6}$ on a number line. What is another fraction that names the same location?

Landon chose Ⓐ as the correct answer. How did he get that answer?

Possible answer: He found a fraction that is equivalent to $\frac{2}{6}$, not $\frac{3}{6}$.

PAIR/SHARE
Does Landon's answer make sense?

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

4 **D**; $\frac{1}{3} = \frac{2}{6}$
DOK 1

Error Alert Students may choose the model in choice C because there are 2 shaded parts and 6 unshaded parts, or they may choose the model in choice B because it shows a fraction equivalent to the fraction shown in choice C.

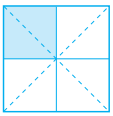
5 Possible answer: $\frac{2}{8}$; Model should show 1 whole divided into 4 equal parts with 1 part shaded, and then each of the 4 parts further divided into smaller equal parts.
DOK 3

6 **A** (**Yes**);
D (**No**);
F (**No**)
DOK 3

4 Which model shows a fraction equivalent to $\frac{2}{6}$?

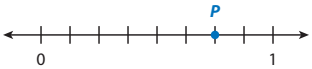
5 Draw a model to find a fraction equivalent to $\frac{1}{4}$. Show your work.

Possible student model:

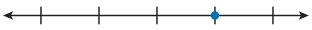
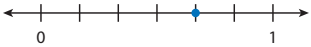


$\frac{1}{4}$ is equivalent to $\frac{2}{8}$.

6 Look at point *P* on the number line.



Does the point on each number line below represent a fraction equivalent to the fraction shown by point *P*?

	Yes	No
	Ⓐ	Ⓑ
	Ⓒ	Ⓓ
	Ⓔ	Ⓕ

Differentiated Instruction

RETEACH

Hands-On Activity
Use paper models to find equivalent fractions.

Students struggling with the concept of naming fractions greater than 1

Will benefit from additional work creating equivalent fractions.

Materials For each student: Activity Sheet *1-Inch Grid Paper*

- Have students draw three 2 inch-by-4 inch rectangles side by side so that the shorter sides lie along the same two grid lines. Identify each rectangle as a whole and have them write a fraction for the model. $\left[\frac{3}{1}\right]$
- Have students draw a line down the center of each rectangle and write a new fraction for the model. $\left[\frac{6}{2}\right]$ Then have students draw a line across the center of each rectangle and write a new fraction for the model. $\left[\frac{12}{4}\right]$
- Now have students draw a line along the remaining untraced gridlines in all 3 rectangles. Ask them to write another fraction for the model. $\left[\frac{24}{8}\right]$

EXTEND

Challenge Activity
Find equivalent unit fractions.

Students who have achieved proficiency

Will benefit from deepening understanding of finding equivalent fractions.

Materials For each student: Activity Sheet *1-Inch Grid Paper*

- Have students draw a rectangle that is 2 units long and 1, 2, 3, or 4 units wide, and shade one whole row or column of the rectangle. Have them write a fraction for the model, using the grid squares as equal parts.
- Have students divide the rectangle into equal parts so that the shaded region is exactly one part of the rectangle. Have them write a unit fraction for the shaded region.
- Repeat the activity with a different rectangle.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

7 **A** (**Yes**);
D (**No**);
E (**Yes**);
G (**Yes**)
DOK 3

8 $\frac{9}{3}$; The number line should be labeled from $\frac{1}{3}$ to $\frac{9}{3}$ with $\frac{9}{3}$ at the same point as 3.
DOK 3

7 Does the point on each number line represent one whole?

	Yes	No
0, $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$	Ⓐ	Ⓑ
0, $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$	Ⓒ	Ⓓ
0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, 1	Ⓔ	Ⓕ
0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$	Ⓖ	Ⓗ

8 Use the number line to find a fraction equivalent to 3. Show your work.

Possible student work:

3 is equivalent to $\frac{9}{3}$.

9 MATH JOURNAL

Write two fractions equivalent to 4 using the denominators 1 and 3. Use a number line to show how you found your answers.

Check that student number lines show fractions equivalent to 4 with denominators of 1 and 3.

$\frac{4}{1}$, $\frac{12}{3}$

SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–9

Find equivalent fractions.

All students will benefit from additional work with equivalent fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding of writing a whole number as a fraction with a denominator of 1 by using the whole number as the numerator, and an understanding that equivalent fractions are located at the same point on a number line.

Error Alert **If** students write $\frac{4}{3}$, **then** review the definition of *numerator* with them. Check that their number lines show thirds and if so, have them count the number of thirds that correspond to the whole number 4.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.

Teacher pages have been reduced. Actual book size is 10 1/4" x 12".

LESSON 23

Lesson Quiz

Teacher Toolbox

Tested Skills

Assesses 3.NF.A.3b, 3.NF.A.3c

Problems on this assessment form require students to be able to use fraction models and number lines to identify and write equivalent fractions, including those that are greater than or equal to one whole. Students will also need to be familiar with different ways to model fractions, placing fractions on number lines, and the difference between a whole and a part in various types of fraction models.

Error Alert Students may:

- think that any two fractions with the same numerators are equivalent.
- count the number of tick marks on a number line rather than the distance from zero.
- think that $\frac{n}{n}$, $\frac{n}{1}$, and/or $\frac{1}{n}$ represent the same amount.

Solutions

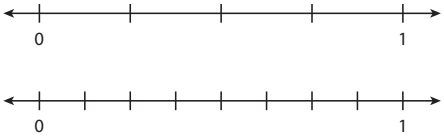
1 C, D
2 points
DOK 3

2 $\frac{7}{1}$
1 point
DOK 3

Lesson 23 Quiz

Solve the problems.

1 Which pairs of equivalent fractions can be shown on the number lines below? Select all the correct answers.



Ⓐ $\frac{1}{4} = \frac{1}{8}$

Ⓑ $\frac{3}{4} = \frac{7}{8}$

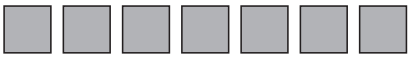
Ⓒ $\frac{2}{4} = \frac{4}{8}$

Ⓓ $\frac{1}{4} = \frac{2}{8}$

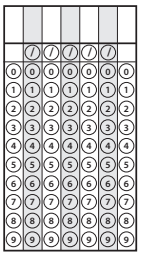
Ⓔ $\frac{2}{4} = \frac{2}{8}$

Ⓔ $\frac{2}{4} = \frac{1}{8}$

2 Jane draws this model to show a fraction equivalent to 7.



Write the equivalent fraction.



Extended Response Scoring Rubric (4 points)	
Points	**Expectations**
4	Response has the correct solution(s) and includes well-organized, clear, and concise work demonstrating thorough understanding of mathematical concepts and/or procedures.
3	Response contains mostly correct solution(s) and demonstrates a strong understanding of mathematical concepts and/or procedures.
2	Response shows partial to limited understanding of mathematical concepts and/or procedures.
1	Response contains incorrect solutions(s), shows poorly organized and incomplete work and explanations, and demonstrates limited understanding of mathematical concepts and/or procedures.
0	Response shows no attempt at finding a solution and no effort to demonstrate an understanding of mathematical concepts and/or procedures.

Multiple Select Scoring		
2 points	**1 point**	**0 points**
All answers are correct	1 incorrect answer	2 or more incorrect answers

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3 C
1 point
DOK 3

4 **Part A**
Possible answer: I would draw a second number line with 0 and 1 directly below the 0 and 1 of this number line. Then I would divide my number line into thirds and look to see what fraction on the given number line is directly above $\frac{1}{3}$ on my number line.
4 points

Part B
$\frac{2}{6}$
1 point
DOK 3

3 Steve says that the model below shows $\frac{3}{1}$.

Oscar says that the model shows $\frac{3}{3}$.

Which sentence explains who is correct?

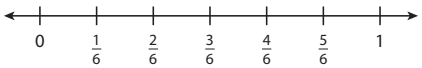
Ⓐ Steve is correct because the model shows 3 wholes, and $\frac{3}{1} = 3$.

Ⓑ Neither is correct because the model shows 1 whole, and $\frac{3}{1}$ and $\frac{3}{3}$ are both equal to 3.

Ⓒ Oscar is correct because the model shows 1 whole, and $\frac{3}{3} = 1$.

Ⓓ Both are correct because the model show 3 wholes, and $\frac{3}{1}$ and $\frac{3}{3}$ are both equal to 3.

4 ***Part A*** Explain how to draw a point on the number line at the fraction equivalent to $\frac{1}{3}$.



Part B Which fraction on the number line in Part A is equivalent to $\frac{1}{3}$?

Differentiated Instruction

Teacher Toolbox

RETEACH

Tools for Instruction

Students who require additional support for prerequisite or on-level skills

Will benefit from activities that provide targeted skills instruction

- Grade 3, Lesson 23

REINFORCE

Math Center Activities

Students who require additional practice to reinforce concepts and skills and deepen understanding

Will benefit from small group collaborative games and activities (available in three versions—on-level, below-level, and above-level)

- Grade 3, Lesson 23

EXTEND

Enrichment Activities

Students who have achieved proficiency with concepts and skills and are ready for additional challenges

Will benefit from group collaborative games and activities that extend understanding

- Grade 3, Lesson 23

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