i-Ready Classroom Mathematics lessons consist of three types of sessions: Explore, Develop, and Refine. The following is a walkthrough of the planning and support features within the Teacher's Guide for a Develop session. You will find many of the same features in the Explore and Refine sessions.



Pacing Guide session-by-session pacing is used to plan daily instruction and practice.

Additional Practice is for use as in-class small group work, after class work, or at-home learning.

I

		LESSON 8	
		Overview	
Pacing Guide	MATERIALS	DIFFERENTIATION	
SESSION 1 Explore Proportional Re	elationships and Slope (35–50 min)		
 Start (5 min) Try It (5-10 min) Discuss It (10-15 min) 	Math Toolkit graph paper, straightedges	PREPARE Interactive Tutorial * RETEACH or REINFORCE Visual Model	Prepare students for the lesson content with <i>Interactive Tutorials</i> .
Connect It (10–15 min) Close: Exit Ticket (5 min)	Presentation Slides 🍾	Materials For display: 8 nickels, Activity Sheet Coordinate Plane: First Quadrant 🐂	
Additional Practice (pages 179–180)			Painforce understanding with Elyancy &
SESSION 2 Develop Showing That	the Slope of a Line Is Constant (4)	5–60 min)	Skills Practice, Apply It problems, and
 Start (5 min) Try It (10–15 min) Discuss It (10–15 min) Connect It (15–20 min) Close: Exit Ticket (5 min) 	Math Toolkit graph paper, straightedges Presentation Slides 🐂	RETEACH or REINFORCE Hands-On Activity Materials For each student: 1 tangram triangle, Activity Sheet Coordinate Plane: First Quadrant REINFORCE Fluency & Skills Practice	differentiated <i>Math Center Activities.</i> <i>Hands-On Activities</i> and <i>Visual Models</i> may also be useful in reinforcing mathematical concepts.
Additional Practice (pages 185–186)		EXTEND Deepen Understanding	
	<u>i</u>		
SESSION 3 Develop Finding the Slo	ope of a Line (45–60 min)		
 Start (5 min) Try It (10–15 min) Discuss It (10–15 min) Connect It (15–20 min) Close: Exit Ticket (5 min) 	Math Toolkit graph paper, straightedges Presentation Slides 🐂	RETEACH or REINFORCE Hands-On Activity Materials For each pair: 20 unit cubes, Activity Sheet 1-Centimeter Grid Paper	Reteach mathematical concepts using <i>Hands-On Activities</i> and <i>Visual Models</i> . Tools for Instruction also provide targeted skills instruction.
Additional Plactice (pages 191–192)			
SESSION 4 Refine Graphing Propo	rtional Relationships and Defining	Slope (45-60 min)	
 Start (5 min) Monitor & Guide (15–20 min) Group & Differentiate (20–30 min) Close: Exit Ticket (5 min) 	Math Toolkit Have items from previous sessions available for students.	RETEACH Hands-On Activity Materials For each pair: 1 geoboard, 5 rubber bands	Extend mathematical concepts with Deepen Understanding, Challenge Activities, and Enrichment Activities.
	Presentation Slides 🍾	REINFORCE Problems 4–7	
		EXTEND Challenge	
		i-Ready Personalized Instruction	Optional Add-On: Personalized
	:		Instruction resources provide students
Lesson 8 Quiz 🔭 or Digital Comprehension Check		RETEACH Tools for Instruction 🐂	with opportunities to strengthen grade-level skills by working on their personalized path.
		REINFORCE Math Center Activity	
		EXTEND Enrichment Activity 🔭	
			The Lesson Quiz or Digital
			Comprehension Check assesses
			students' progress toward mastery of
			where reteaching is needed
		1754	mere receacing is needed.
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reasoning.

LESSON 8 | SESSION 3 Develop

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- graph used to describe verbally how (x, y) values change, such as up 2, over 5
- (misconception) slope given as ⁵/₂ or 2.5, if students think slope cannot be less than 1
- reasoning used to identify unit rate
- graph used to find slope between the origin and another point on the line

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to refer to their models or diagrams as they justify their solutions.

Guide students to **Compare and Connect** the representations. After each strategy, allow student individual think time to process the ideas.

ASK How are the methods for finding the slope the same? How are they different?

LISTEN FOR Each method uses unit rate or the vertical change and horizontal change to find slope, by counting grid units, using subtraction, or knowing that slope is the unit rate.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK How are the strategies presented in the two Model Its alike?

LISTEN FOR Both focus on the relationship between vertical and horizontal change. Both use the same points to determine slope.

For the graph, prompt students to consider the rise over run quotient.

- Using any two points on the line, how does the rise divided by the run compare to the slope?
- How does this way of identifying slope connect the meaning visually and verbally?

For the formula, prompt students to consider the relationship between the points on the graph and the formula.

- How does the difference in the y-values in the formula relate to the rise shown in the graph?
- How does the difference in the x-values in the formula relate to the run shown in the graph?

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Explore different ways to find the slope of a line Ashwini's family is getting ready for the local Holi festival. Ashwini mixes yellow and red food coloring to make orange food coloring. She uses the graph to find how many ounces of yellow and red to mix. What is the slope of the line? What does the slope represent?



Model It

LESSON 8 SESSION 3

You can use the quotient of the vertical change and the horizontal change between any two points on the line to find the slope of a line.

This quotient representing slope, vertical change, is also called run



Model It

You can choose any two points on the line and use the slope formula to find the slope of a line.

For any two points on a line, (x_1, y_1) and (x_2, y_2) , the slope between these points can be found using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. In this formula, the letter *m* means slope. (10, 4) and (20, 8) are two points on the line.



DIFFERENTIATION | EXTEND

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Deepen Understanding

Prompt students to consider that a positive vertical and horizontal change leads to the same slope as a negative vertical and horizontal change. Have students use the graph from the first Model It. Ask them to consider moving from the right point to the left point. Have them draw the *run* arrow, which will point left, and the *rise* arrow, which will point down.

ASK How can you label the arrows to show the distance and the direction of the change? **LISTEN FOR** Label the run arrow -10. Label the rise arrow -4.

ASK What do you get if you calculate the slope using these rise and run values? How does it compare with the slope moving from the left point to the right point?

LISTEN FOR $\frac{-4}{-10}$ or $\frac{2}{5}$; The slope is the same.

ASK What conclusion can you make?

LISTEN FOR It does not matter which direction you move between two points to calculate the slope for this line. Moving from left to right, the rise and run are both positive. Moving from right to left, the rise and run are both negative. In either case, the slope is the same.

LESSON 8 Graph Proportional Relationships and Define Slope 188

Ask/Listen for are mathematical discourse questions followed by expected student responses that support and facilitate whole class discussion.

As students share their thinking, the discourse questions can be used to make connections between student approaches and different models and representations, prompt justifications and critiques of approaches and solutions, and check conceptual understanding.

Standards for Mathematical

Practice (SMP) are infused throughout the instructional model.

Deepen Understanding is a

consistent opportunity to build conceptual understanding of a key lesson concept by extending mathematical discourse. The content connects a particular aspect of lesson learning to an SMP, showing how it looks in the classroom.

Develop Finding the Slope of a Line

CONNECT IT

SMP 2, 4, 5,

Remind students that the relationship between the amounts of red and yellow food coloring is the same in both representations. Explain that they will now use the representations to reason about the formula for the slope of a line.

Before students begin to record and expand on their work in Model It, tell them that problems 1 and 2 will prepare them to provide the explanation asked for in problem 3.

Monitor and Confirm Understanding 1 – 2

- The slope, $\frac{2}{5}$, means that $\frac{2}{5}$ ounce of red is needed for each ounce of yellow.
- The vertical change, or rise, is the same as the difference of the *y*-values.
- The horizontal change, or run, is the same as the difference of the *x*-values.

Facilitate Whole Class Discussion

3 Look for the idea that any two points on a line determine the slope of the line.

ASK Why can you use the coordinates of any two points on the line in the slope formula?

LISTEN FOR The slope for a line is constant, so it is the same between any two points.

4 Look for understanding that thinking critically can help determine the points to use in the slope formula. The calculation may be easier if students use the origin or points with integer coordinates.

S Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

Use the problem from the previous page to help you understand how to find the slope of a line.

What is the slope of the line in the Try It problem? Why is it helpful to know the slope in this situation?
2: Possible answer: The slope tells you the number of ounces of red you

 $\frac{1}{5}$, rossible answer, the slope tens you the number of ounces of red y need to mix with each ounce of yellow.

2 Look at both **Model Its.** Use the quotient representing slope to explain why the slope formula makes sense. Possible answer: The vertical change between two points is the same as $y_2 - y_1$. The horizontal change between two points is the same as $x_2 - x_1$.

Find the slope of the line using two other pairs of points. Explain why you get the same value for the slope no matter which points you choose on the line.
 Possible answer: (0, 0) and (5, 2): 2-0/5-0 = 2/5;
 (30, 12) and (20, 8): 8-12/20-30 = (-4/-10) = 2/5;
 The slope between any two points on a line is the same.

- How do you decide which points to use to find the slope of a line? Possible answer: You might choose points that make the subtraction in the formula or counting on a graph easy, such as (0, 0) and points with integer coordinates.
- Seflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the slope of a line. Resoonses will vary. Check student responses.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity Use unit cubes to model slope.

If students are unsure about using the concept of slope, then use this activity to make connections between slope and the vertical and horizontal change of points on a line.

- Materials For each pair: 20 unit cubes, Activity Sheet 1-Centimeter Grid Paper in
 Have pairs make a first-quadrant coordinate plane on their grid paper, plot and label
- the points (0, 0) and (5, 4), and then draw a line through the points.
- Have students use the unit cubes to make the horizontal and vertical sides of a right triangle between the two points.
- Ask: How many cubes are along the vertical distance between the two points? [4]
- Ask: How many cubes are along the horizontal distance between the two points? [5]
- Ask: What is the slope of the line? $\left\lfloor \frac{4}{5} \right\rfloor$
- Have pairs plot and label (10, 8) and extend the line so it passes through this point; repeat bullets 2–5, then compare the two slopes.

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Monitor and Confirm

Understanding is a way to ensure that students have made sense of mathematical learning goals.

Facilitate Whole Class Discussion

provides a series of related discourse questions that illuminate the mathematical ideas of the lesson, prompting students to make connections and use that understanding to solve problems leading to abstract reasoning. These questions help students learn how to articulate a generalization of the mathematical concept.

Hands-On Activities occur

consistently at strategic points in the lesson after teachers have acquired understanding of students' learning through observation and their work on questions in the Student Worktext. The activities support students who are unsure of the concept and are an opportunity for small group reteaching while other students work independently. Use of concrete objects lets students access understanding in a different way.

Apply It solutions at point of use give a correct response with explanations that include multiple approaches to solving the problem.

LESSON 8 | SESSION 3 Develop

Apply It •

For all problems, encourage students to use mathematical reasoning to support their thinking and check their answers.

- 6 Students may solve this problem by using any two points in the table, although their calculation will be easier if they choose the origin as one point. Encourage students to check their answer by using a different pair of points.
- **7 a.** Students may also interpret slope to mean for every 1 cup of yogurt, use 2 cups of berries.
 - b. Students may also reason that the number of cups of berries is twice the number of cups of yogurt. So, for 9 cups of yogurt, Kaley needs 18 cups of berries.



Close: Exit Ticket is a quick formative assessment of each day's learning and serves as an indicator of students' progress toward mastery or partial mastery of the learning goal of the session.

This is the last question on the Student Worktext page.

Error Alert gives insight into misconceptions that can lead to errors in calculation and provides on-the-spot remediation.

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LESSON 8 Graph Proportional Relationships and Define Slope

Additional Practice can be used as in-class small group work, after class work, or at-home learning.

Solutions are labeled as Basic, Medium, and Challenge to show the

question. Use these to support independent practice or differentiation

as needed.

relative difficulty level in relation to the

questions at hand or the standard in

LESSON 8 | SESSION 3 Practice Finding the Slope of a Line

Problem Notes

Assign Practice Finding the Slope of a Line as extra practice in class or as homework.

- 1 Students should understand that any two points on the line may be used to calculate the slope. If the line passes through the origin, using (0, 0) as one point makes the calculations simpler. Medium
- 2 a. Students may also solve the problem using the slope formula. Medium
 - **b.** Students should recognize that the slope represents the change in time for each \$1 change in cost. So, you can rent the bike for 30 minutes for each dollar. Basic

LESSON 8 | SESSION 3 Name

- Practice Finding the Slope of a Line Study the Example showing how to find the slope of a line. Then solve problems 1–5. Example A scientist made this graph showing the numbe of times a hummingbird flaps its wings for different lengths of time. What is the slope of the line? What does the slope mean in this situation? Find two points on the line: (2, 140), (4, 280) Find the slope: $m = \frac{280 - 140}{4 - 2} = \frac{140}{2} = \frac{70}{1}$ The slope is $\frac{70}{1}$, or 70. The hummingbird flaps its wings 70 times per second.
- 1 Alejandro says that the slope of the line in the Example is $\frac{140}{2}$ because for points (2, 140) and (0, 0), the rise is 140 and the run is 2. Is Alejandro correct? Explain. Yes: Possible exc The vertical change is 140. T is 2. So the slope is $\frac{140}{2} = \frac{70}{1}$.
- 2 Chikelu wants to rent a bicycle. The graph shows the cost of renting a bicycle for different lengths of time a. What is the slope of the line? Show your work.
- work: See graph $\frac{\text{rise}}{\text{rup}} = \frac{120}{4} = \frac{30}{1}$

SOLUTION The slope is $\frac{30}{1}$, or 30.

b. What does the slope represent in this situation? The slope is the number of You get 30 minutes for \$1. s you can rent the bike for each doll

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Cost (\$)

Fluency & Skills Practice provides ongoing opportunities for students to accurately, flexibly, and efficiently practice mathematical procedures and operations. This can be used as in-class small group work, after-class work, or at-home learning. Student pages are available in the optional Fluency and Skills Practice Book or on Teacher Toolbox. Download PDFs or editable versions, or assign to any LMS, including Google Classroom.

Fluency & Skills Practice

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Finding the Slope of a Line 🍾 In this activity, students are given tables and graphs of lines, and they are asked to determine the slope and the meaning of the slope in the context of the problem.



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Additional Practice Opportunities

include digital Learning Games, Interactive Practice, Cumulative Practice, and i-Ready Personalized Instruction.

LESSON 8 | SESSION 3 **Additional Practice** a. Students may also solve the problem by LESSON 8 SESSION 3 using the slope formula. Medium 3 Colin wants to make purple paint. He finds **b.** Students may also interpret the slope as a graph online that shows how much red meaning that 3 pints of blue paint must be and blue paint he should use to make the mixed with 5 pints of red paint to make the shade of purple he wants a. What is the slope of the line? correct shade of purple. Basic Show your work. a. Students should recognize the numbers in Possible work the numerator as x-values and the numbers (0, 0) and (5, 3); rise = in the denominator as y-values. The SOLUTION _____ numerator and denominator should be reversed. Challenae b. What does the slope represent in this situation b. Students may solve the problem using the ossible answer: The slope means that $\frac{3}{5}$ pint of blue paint must be slope formula or by finding rise over run. ixed with 1 pint of red paint to make a certain shade of purple The most efficient method is to use the origin 4 Rafael wants to find the slope of the line. and a second point to calculate slope. However, a. This is Rafael's work. What mistake did he make? students may use any two points in the table $m = \frac{4-2}{100-50} = \frac{2}{50} = \frac{1}{25}$ for their calculation. Students should recognize that the unit rate represents the number of cars Possible answer: He used $\frac{x_2 - x_1}{x_2 - x_2}$ instead of $\frac{y_2 - y_1}{x_2 - x_2}$ made per hour. Medium b. What is the slope of the line? ²⁵/₁, or 25 Time (h) G A factory makes cars at a constant rate. The table shows the number of cars made for different lengths of time. A manager draws a graph of the number of cars made per hour. What is the slope of the line? What does the slope represent? Show your work. Time (h) 0 5 8 12 24 Number of Cars 0 130 208 312 624 Possible work: $\frac{208 - 130}{8 - 5} = \frac{78}{3} = \frac{26}{1}$ SOLUTION $\frac{26}{1}$, or 26; The slope represents the number of cars made per hour 192 Use with Session 4 Apply It

DIFFERENTIATION | ENGLISH LEARNERS

4

5

Medium



Levels 3-5: Listening/Speaking Have students read Apply It problem 2. Have students find the slope and tell what it means. Prepare students to respond to Pair/Share. Clarify the meaning of reverse as needed. Have students turn to a partner to restate the guestion in Pair/Share. Ask: What is the new situation you need to consider? What would you need to find for this situation? Provide

sentence starters to guide the conversation: • In the new situation, the coordinates

• For this new situation, I would need to _ Have partners take turns using reverse. reversing, and/or reversed to compare the original and new slope. Encourage students to use inverse if appropriate.

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DIFFERENTIATION | ENGLISH

LEARNERS helps teachers scaffold or amplify language in the next session so English learners can access and engage with grade-level mathematics.