# TEACHER'S GUIDE Overview 

continued
> i-Ready Classroom Mathematics lessons consist of three types of sessions: Explore, Develop, and Refine. The following is a walkthrough of the planning and support features within the Teacher's Guide for a Develop session. You will find many of the same features in the Explore and Refine sessions.

## Lesson Overview provides

information for use in planning whole class instruction, small group differentiation, and independent learning opportunities.

Content Objectives identify the mathematical learning goals for the lesson, while Language Objectives indicate the language students are expected to understand and produce as they work on those goals.

Prior Knowledge are opportunities to monitor understanding and identify students' learning needs.

Math Vocabulary is defined in the context of lessons, and academic words can be explored using the Academic Vocabulary Routine.

Learning Progression sets context for the mathematics of the lesson, providing information on how the content fits across and within grade levels-what students previously learned, what they are learning now, and what they will be learning next.

LESSON 8
Overview | Add with Negative Numbers

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)
SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect framework.* This lesson provides additional support for: 2 Reason abstractly and quantitatively.
7 Look for and make use of structure.

* See page is to learn how every lesson includes these SMP.


## Objectives

Content Objectives

- Add positive and negative integers, fractions, and decimals.
- Apply knowledge of zero pairs to add rational numbers.
- Represent addition of rational numbers on a number line.


## Language Objectives

- Read and interpret word problems involving addition of rational numbers to identify the question and important quantities.
- Describe in writing how models use zero pairs to show addition.
- Explain how a number line represents the addends and sum of a word problem using informal and mathematical language.
- Build on an idea or solution strategy during class discussion by giving examples that show that the idea makes sense.


## Prior Knowledge

- Understand that the sum of a number and its opposite is 0 .
- Use models to add positive and negative integers.
- Use models to add two negative integers.
- Add positive rational numbers.

Pacing Guide session－by－session pacing is used to plan daily instruction and practice．

Additional Practice is for use as in－class small group work，after class work， or at－home learning．


RETEACH or REINFORCE Hands－On Activity Materials For each pair： 20 integer chips

REINFORCE Fluency \＆Skills Practice＊ EXTEND Deepen Understanding
－Start（5 min）
－Try It（5－10 min）
－Discuss It（10－15 min）
－Connect It（10－15 min）
－Close：Exit Ticket（ 5 min ）
Additional Practice（pages 151－152）

SESSION 2 Develop Adding Positive and Negative Integers（45－60 min）
－Start（ 5 min ）
－Try lt（ $10-15 \mathrm{~min}$ ）
－Discuss It（10－15 min）
－Connect It（ $15-20 \mathrm{~min}$ ）
－Close：Exit Ticket（ 5 min ）
Additional Practice（pages 157－158）

N自 Math Toolkit grid paper，integer chips，number lines

Presentation Slides

SESSION 3 Develop Adding Positive and Negative Fractions and Decimals（45－60 min）
－Start（ 5 min ）
－Try It（ $10-15 \mathrm{~min}$ ）
－Discuss lt（ $10-15 \mathrm{~min}$ ）
－Connect It（ $15-20 \mathrm{~min}$ ）
－Close：Exit Ticket（5 min）
Additional Practice（pages 163－164）

N直 Math Toolkit grid paper，number lines

Presentation Slides \％

SESSION 4 Refine Adding with Negative Numbers（45－60 min）
－Start（ 5 min ）
－Monitor \＆Guide（15－20 min）
－Group \＆Differentiate（ $20-30 \mathrm{~min}$ ）
－Close：Exit Ticket（5 min）

N㑒 Math Toolkit grid paper，integer chips，number lines

Presentation Slides＂

Prepare students for the lesson content with Interactive Tutorials．

Reinforce understanding with Fluency \＆ Skills Practice，Apply It problems，and differentiated Math Center Activities． Hands－On Activities and Visual Models may also be useful in reinforcing mathematical concepts．

Reteach mathematical concepts using Hands－On Activities and Visual Models． Tools for Instruction also provide targeted skills instruction．

Extend mathematical concepts with Deepen Understanding，Challenge Activities，and Enrichment Activities．

Optional Add－On：Personalized Instruction resources provide students with opportunities to strengthen grade－level skills by working on their personalized path．

## The Lesson Quiz or Digital

 Comprehension Check assesses students＇progress toward mastery of lesson content and is a way to identify where reteaching is needed．
# TEACHER'S GUIDE Overview 

continued

Purpose provides a roadmap of what students will be learning and doing across the session.

Start establishes a clear and accessible entry point for each session, engaging students mathematically with prerequisite content. It frequently is an opportunity to have students engage in a math talk.

## Develop Academic Language

provides language support for all students and is especially useful in helping EL students use and produce academic language.

## Support Partner Discussion

provides teachers with prompts to help students engage in meaningful peer discourse.

## Make Sense of the Problem

uses a language routine to help students understand the problem. See the Language Routines section on the Teacher Toolbox (under the Program Implementation tab) for suggestions on how to integrate language routines, teacher moves, and conversation tips during instruction.

## LESSON 8 |SESSION 3 ■■■ <br> Develop Adding Positive and Negative Fractions and Decimals

## $\rightarrow$ Purpose

- Develop strategies for adding positive and negative fractions and decimals.
- Recognize that the strategies used for adding positive and negative integers can be extended to fractions and decimals.

START CONNECT TO PRIOR KNOWLEDGE


Possible Solutions
All are numbers between 2 and 3 ; $A$ has the greatest value; $B$ is the only mixed number; $C$ is the only fraction; $D$ is the only number that cannot be expressed as fourths.

WHY? Support students' ability to identify and compare fractions, mixed numbers, and decimals.

## - DEVELOP ACADEMIC LANGUAGE

WHY? Support students to build on a strategy or solution they agree with during discussion.
HOW? Encourage students to listen for strategies and solutions they agree with during partner or whole class discussion. Prompt them to give another example that shows that the strategy or solution makes sense. Students can use sentence starters such as:

- I agree that $\qquad$ and I also think that $\qquad$
- I can build on this idea by


## TRY IT

## Make Sense of the Problem

See Connect to Culture to support student engagement. Before students work on Try It, use Notice and Wonder to help them make sense of the problem. Some students may recognize that the problem is similar to those in the previous session but includes a mixed number.


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## DISCUSS IT

## Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- How is the problem similar to the problems you completed in the previous session?
-What simpler problem could you solve and then extend to find the solution?
- Common Misconception Listen for students who think that adding $-\frac{3}{4}$ will result in a sum that has a fractional $\frac{1}{4}$ part, such as $-13 \frac{1}{4}$. As students share their strategies, suggest that they solve the related problem without the fractional part: $-20+6$. Then ask them to explain how reintroducing the fraction $-\frac{3}{4}$ to the problem causes the sum to change.


## Common Misconception

identifies misconceptions that lead to errors in understanding, which can then be addressed in whole class discussion as students are prompted to explain their reasoning.

## Select and Sequence Student Strategies

 Select 2-3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion- using a number line to model $-20 \frac{3}{4}+6$
- (misconception) interpreting $-20 \frac{3}{4}$ as $(-20)+\frac{3}{4}$, resulting in a sum of $-14+\frac{3}{4}$, or $-13 \frac{1}{4}$
- adding the integers first and then adding the fraction
converting one or both addends to fractions, such as $-\frac{83}{4}$ and $\frac{24}{4}$, or to decimals, such as -20.75 and 6.0


## Facilitate Whole Class Discussion

Call on students to share selected strategies. After each presentation, allow individual think time to allow students to process the ideas.
Guide students to Compare and Connect the epresentations. Prompt students to build on ideas they agree with by giving other examples that show the comment makes sense
ASK How does each model show the scuba diver's initial position and the change in position?
LISTEN FOR They all show $-20 \frac{3}{4}$ as the initial position and an increase of 6 to the final position.

## Picture It \& Model It

If students presented these models, have students connect these models to those presented in class.
If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.
ASK How are the Picture It and Model It similar to each other? How are they different?

LISTEN FOR Both show the first addend, $-20 \frac{3}{4}$, and then increase it by 6 to find the sum

For the drawing, prompt students to relate the model to the problem situation

- How is it useful to use a vertical drawing for this problem instead of a horizontal drawing?
For the equations, prompt students to identify the purpose of the steps.
How is it useful to rewrite the mixed number as the sum of a whole number and a fraction?
When does the fraction get added?

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LESSON 8 | SESSION 3
Explore different ways to add positive and negative fractions and decimals.
On a cave dive in Oahu, a scuba diver swims at an elevation of $-20 \frac{3}{4} \mathrm{ft}$ relative to sea level. The scuba diver notices a sea turtle above them and rises 6 ft to take its picture. At what elevation does the scuba diver take the picture?

Picture It
You can make a drawing to make sense of the problem.


Model It


## DIFFERENTIATION | EXTEND

Deepen Understanding
Using the Structure of Numbers for Adding with Negative Addends
Prompt students to apply the properties of operations to explain strategies for adding with negative addends.
ASK How do you know that $-20 \frac{3}{4}$ is equal to $(-20)+\left(-\frac{3}{4}\right)$, and not $(-20)+\frac{3}{4}$ ? LISTEN FOR Since $20 \frac{3}{4}$ is between 20 and $21,-20 \frac{3}{4}$ must be between -20 and -21 . The sum ( -20 ) $+\frac{3}{4}$ would be between -20 and -19 .
ASK Instead of adding $-20 \frac{3}{4}+6$, could you get the same sum by adding $6+\left(-20 \frac{3}{4}\right)$ ? How do you know?
LISTEN FOR Yes. You can add numbers in any order
ASK If you rewrite numbers, can you still add them in any order? Explain using this problem LISTEN FOR If you add $6+\left(-20 \frac{3}{4}\right)$ by adding $6+(-20)+\left(-\frac{3}{4}\right)$ and $-20 \frac{3}{4}+6$ by adding $-20+6+\left(-\frac{3}{4}\right)$, you will still get the same answer of $-14 \frac{3}{4}$.

Ask/Listen for are mathematical discourse questions followed by expected student responses that support and facilitate whole class discussion.

As students share their thinking, the discourse questions can be used to make connections between student approaches and different models and representations, prompt justifications and critiques of approaches and solutions, and check conceptual understanding

## Standards for Mathematical

Practice (SMP) are infused throughout the instructional model.

## Deepen Understanding is a

consistent opportunity to build conceptual understanding of a key lesson concept by extending mathematical discourse. The content connects a particular aspect of lesson learning to an SMP, showing how it looks in the classroom.

# TEACHER'S GUIDE Overview 

continued

## Monitor and Confirm

Understanding is a way to ensure that students have made sense of mathematical learning goals.

## Facilitate Whole Class Discussion

provides a series of related discourse questions that illuminate the mathematical ideas of the lesson, prompting students to make connections and use that understanding to solve problems leading to abstract reasoning. These questions help students learn how to articulate a generalization of the mathematical concept.

## Hands-On Activities occur

 consistently at strategic points in the lesson after teachers have acquired understanding of students' learning through observation and their work on questions in the Student Worktext. The activities support students who are unsure of the concept and are an opportunity for small group reteaching while other students work independently. Use of concrete objects lets students access understanding in a different way.
## LESSON $8 \mid S E S S I O N 3 \quad \square \square \square \square$ Develop Adding Positive and Negative Fractions and Decimals

## CONNECTIT

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those similarities to reason about strategies for adding positive and negative fractions and decimals.
Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the comparison asked for in problem 4.

## Monitor and Confirm Understanding (1)-

- The picture shows that the diver goes towards sea level, but does not reach it.
- The strategy for representing a mixed number as the sum of a whole number and a fraction can be extended to decimals.


## Facilitate Whole Class Discussion

(3) Students may also rewrite the sum as $(-21)+\frac{1}{4}+(-12)+\left(-\frac{1}{4}\right)$. This strategy allows for a zero pair of $\frac{1}{4}$ and $-\frac{1}{4}$, and shows the sum is equal to $-21+(-12)=-33$.
4. Look for the idea that the same strategies used for adding integers, such as modeling on a number line and finding zero pairs, can be extended to adding fractions and mixed numbers.
ASK What extra steps do you need to follow to add $-20 \frac{3}{4}+6$ compared to adding $-20+6$ ? LISTEN FOR You need to account for the fractional part and remember that it is negative. For example, $-20+6=-14$, and then you add $-\frac{3}{4}$.
(5) Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT
> Use the problem from the previous page to help you understand how to add positive and negative fractions and decimals.
(1) Look at Picture It. At what elevation does the diver take the picture? How does the picture help you understand that the elevation is negative?
$-14 \frac{3}{4} \mathrm{ft}$; Possible explanation: The arrow starts at $-20 \frac{3}{4} \mathrm{ft}$. Then, the arrow shows the diver moving up 6 ft . But the diver does not pass 0 , or sea level. That means the elevation is negative.
(2) Look at Model It. Suppose the problem used -20.75 instead of $-20 \frac{3}{4}$. How could you use this strategy?
You could write -20.75 as $(-20)+(-0.75)$ to make it the sum of an integer and a decimal.
(3) The sum of $-20 \frac{3}{4}$ and $-12 \frac{1}{4}$ is -33 . How can you break apart the mixed numbers to find the sum?
Possible answer: You can rewrite this as $-20+\left(-\frac{3}{4}\right)+(-12)+\left(-\frac{1}{4}\right)$. This is the same as $-32+(-1)$, which is -33 .
4. How is adding positive and negative fractions or decimals similar to adding positive and negative integers? How is it different?
Posssible answer: They are similar because you can use a number line to find the sum of both. They are different because you have to think about fractional units as well as whole units.
(5) Reflect Think about all the models and strategies you have discussed today Describe how one of them helped you better understand how to solve the Try It problem.
Responses will vary. Check student responses.

## DIFFERENTIATION | RETEACH or REINFORCE

## Hands-On Activity

Model the addition of negative fractions.
If students are unsure about adding negative fractions or mixed numbers, then use this activity to model this type of addition.
Materials For each pair: 2 copies of Activity Sheet Fraction Bars 畨

- Have students label both sets of fraction bars, one with positive fractions and one with negative fractions.
- Have students use the fraction bars to model the sum of a positive fraction and a negative fraction with the same denominator, such as $\frac{3}{4}$ and $-\frac{1}{4}$.
- Ask: How can you use zero pairs in your model? [A positive $\frac{1}{4}$ can form a zero pair with the $-\frac{1}{4}$. The remaining fraction $\frac{2}{4}$, or $\frac{1}{2}$, is the sum.]
- Have pairs suggest and model other examples of addition using zero pairs.
- Ask: How can you use your model to add fractions with unlike denominators, such as $\frac{3}{4}$ and $-\frac{1}{2}$ ? [Use the fraction bars to replace one or both addends so they have the same denominator. Then the zero pair is easier to find.]

Apply It solutions at point of use give a correct response with explanations that include multiple approaches to solving the problem.

## Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; students should recognize that a number line can reasonably represent fractional distances with approximate divisions of units into halves, fourths, or tenthsStudents may also break apart both decimals into an integer and a decimal and then add the integers and decimals separately. Students may also represent all or some of the calculations on a number line.
(7) Students may choose to break apart the arrow for adding $-5 \frac{1}{2}$ into a series of jumps, such as $-\frac{1}{4},-5$, and $-\frac{1}{4}$. The first jump ends at -13 , an integer. The second jump adds the integer part of $-5 \frac{1}{2}$. The third jump adds the remaining $-\frac{1}{4}$ and ends at the sum.

Close: Exit Ticket is a quick
formative assessment of each day's learning and serves as an indicator of students' progress toward mastery or partial mastery of the learning goal of the session.

This is the last question on the Student Worktext page.

Error Alert gives insight into misconceptions that can lead to errors in calculation and provides on-the-spot remediation.

# TEACHER'S GUIDE Overview 

continued

Additional Practice can be used as in-class small group work, after class work, or at-home learning.

Solutions are labeled as Basic, Medium, and Challenge to show the relative difficulty level in relation to the questions at hand or the standard in question. Use these to support independent practice or differentiation as needed.

## LESSON $8 \mid S E S S I O N 3 \quad \square \square \square \square$ <br> Practice Adding Positive and Negative Fractions and Decimals

## Problem Notes

Assign Practice Adding Positive and Negative Fractions and Decimals as extra practice in class or as homework.
(1) Students may also support part or all of their calculations with a number line model. They may also solve the problem by breaking apart each decimal into integer and decimal components. Medium

Students may identify pairs of addends in which one addend is positive and one addend is negative. If the first addend is less than -21.5 , then the second addend must be a positive number. Challenge

LESSON 8 | SESSION 3
Practice Adding Positive and Negative
Fractions and Decimals
> Study the Example showing addition of positive and negative decimals. Then solve problems 1-5.

## Example

Oren conducts a chemistry experiment. The starting temperature of a mixture is $-14.8^{\circ} \mathrm{C}$. The temperature of the mixture rises by $8.5^{\circ} \mathrm{C}$. What is the new temperature of the mixture?
To find the new temperature, add the change to the starting temperature.
$-14.8+8.5=-14+(-0.8)+8+0.5$
$=-14+8+(-0.8)+0.5$
$=-6+(-0.8)+0.5$
$=-6+(-0.3)$
$=-6.3$
The new temperature of the mixture is $-6.3^{\circ} \mathrm{C}$.
(1) The next day, the temperature of the mixture in the Example rises by another $7.2^{\circ} \mathrm{C}$. What is the new temperature? Show your work.
Possible work:
$-6.3+7.2=-6.3+6.3+0.9$
$=0.9$

SOLUTION The new temperature is $0.9^{\circ} \mathrm{C}$.
(2) Write two addition equations that each have a sum of -21.5 Possible answers:
$-10+(-11.5)=-21.5 \quad 20+(-41.5)=-21.5$

Fluency \& Skills Practice provides ongoing opportunities for students to accurately, flexibly, and efficiently practice mathematical procedures and operations. This can be used as in-class small group work, after-class work, or at-home learning. Student pages are available in the optional Fluency and Skills Practice Book or on Teacher Toolbox. Download PDFs or editable versions, or assign to any LMS, including Google Classroom.

Fluency \& Skills Practice
Adding Positive and Negative Fractions and Decimals *
In this activity, students will first determine whether the sum of two rational numbers is positive or negative. They will then find the sum for the exercises with negative answers.


Assign Cumulative Practice to review major content from previous units, as needed.
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A personalized instruction path helps students reinforce prerequisites and build grade-level skills.

Additional Practice Opportunities include digital Learning Games, Interactive Practice, Cumulative Practice, and i-Ready Personalized Instruction.
3. Students may use a number line to represent the addition or may break apart each mixed number into the sum of an integer and a fraction. Basic


Students may also apply the strategy of rewriting each mixed number as a sum of an integer and a fraction and then adding the integers and fractions separately. Students will need to express the fractional parts with a common denominator, 6 , but may do so at different stages in the solution process. Medium
Students may reason that the sum of $-75 \frac{3}{4}$ and $-16 \frac{3}{8}$ is the opposite of the sum of $75 \frac{3}{4}$ and $16 \frac{3}{8}$ and solve that way: $-\left(75 \frac{3}{4}+16 \frac{3}{8}\right)=-92 \frac{1}{8}$. Medium

LESSON 8 | SESSION 3
(3) What is the sum of 13.75 and -11.25 ? Show your work.
3 What is the sum
Possible work:
$13.75+(-11.25)=13+0.75+(-11)+(-0.25)$

$$
\begin{aligned}
& =13+(-11)+0.75+(-0.25) \\
& =2+0.5 \\
& =2.5
\end{aligned}
$$

SOLUTION 2.5

$$
\begin{aligned}
& \text { (4) Find } 1 \frac{2}{3}+\left(-2 \frac{1}{2}\right) \text {. Show your work. } \\
& \text { Possible work: } \\
& \begin{aligned}
1 \frac{2}{3}+\left(-2 \frac{1}{2}\right) & =1 \frac{4}{6}+\left(-2 \frac{3}{6}\right) \\
& =1 \frac{4}{6}+\left(-1 \frac{4}{6}\right)+\left(-\frac{2}{6}\right)+\left(-\frac{3}{6}\right) \\
& =\left(-\frac{2}{6}\right)+\left(-\frac{3}{6}\right) \\
& =-\frac{5}{6}
\end{aligned}
\end{aligned}
$$

SOLUTION
(5) A naval submarine travels at an elevation of $-75 \frac{3}{4} \mathrm{ft}$ relative to sea level. Then its elevation changes by $-16 \frac{3}{8} \mathrm{ft}$. What is the new elevation of the naval submarine? Show your work.
Possible work:

$$
-75 \frac{3}{4}+\left(-16 \frac{3}{8}\right)=-75 \frac{6}{8}+\left(-16 \frac{3}{8}\right)
$$

$$
=-91+\left(-\frac{9}{8}\right)
$$

$$
=-91+-1+-\frac{1}{8}
$$

$$
=-92 \frac{1}{8}
$$

SOLUTION The new elevation is $-92 \frac{1}{8} \mathrm{ft}$.
DIFFERENTIATION | ENGLISH
LEARNERS helps teachers scaffold or amplify language in the next session so English learners can access and engage with grade-level mathematics.

