# TEACHER'S GUIDE Overview 

continued
> i-Ready Classroom Mathematics lessons consist of three types of sessions: Explore, Develop, and Refine. The following is a walkthrough of the planning and support features within the Teacher's Guide for a Develop session. You will find many of the same features in the Explore and Refine sessions.

## Lesson Overview provides

 information for use in planning whole class instruction, small group differentiation, and independent learning opportunities.Content Objectives identify the mathematical learning goals for the lesson, while Language Objectives indicate the language students are expected to understand and produce as they work on those goals.

Prior Knowledge are opportunities to monitor understanding and identify students' learning needs.

Math Vocabulary is defined in the context of lessons, and academic words can be explored using the Academic Vocabulary Routine.

Learning Progression sets context for the mathematics of the lesson, providing information on how the content fits across and within grade levels-what students previously learned, what they are learning now, and what they will be learning next.

Lesson 10

## Overview | Model and Solve Multi-Step Problems

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)
SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect framework.*
This lesson provides additional support for:
4 Model with mathematics.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

* See page $1 q$ to learn how every lesson includes these SMP.


## Objectives

## Content Objectives

- Use equations with a letter standing for the unknown to represent multi-step word problems and solve these equations
- Interpret the remainder in a division word problem.
- Use estimation strategies to check that an answer is reasonable.


## Language Objectives

- Determine what is unknown in a word problem and represent it with a letter in an equation.
- Use the term remainder accurately when speaking and writing.
- Compare an estimate with a solution to check for reasonableness and record conclusions in writing.


## Prior Knowledge

- Add, subtract, multiply, and divide.
- Write equations to represent two-step word problems.
- Use a letter or symbol to represent an unknown quantity.
- Use parentheses to clarify order of operations.
- Understand estimation strategies, including rounding

Vocabulary

## Math Vocabulary

expression a group of one or more numbers, unknowns, and/or operation symbols that represents a quantity.
remainder the amount left over when one number does not divide another number a whole number of times.

Review the following key terms.
divide to separate into equal groups and find the number in each group or the number of groups
equation a mathematical statement that uses an equal sign (=) to show that two expressions have the same value.
estimate (verb) to give an approximate number or answer based on mathematical thinking.
multiply to repeatedly add the same number a certain number of times. Used to find the total number of items in equalsized groups.
reasonable something that makes sense when given facts are taken into account.
unknown the value you need to find to solve a problem.

## Academic Vocabulary

opinion what you think or believe about something.

Learning Progression

In Grade 3 students learned to write equations for two-step problems using whole numbers and the four operations.

In this lesson students write and solve equations for multi-step problems using letters to represent unknown quantities and check answers for reasonableness.

In Grade 5 students will add, subtract, multiply, and divide with fractions, mixed numbers, and decimals in word problems.


# TEACHER'S GUIDE Overview 

continued

Purpose provides a roadmap of what students will be learning and doing across the session.

Start establishes a clear and accessible entry point for each session, engaging students mathematically with prerequisite content. It frequently is an opportunity to have students engage in a math talk.

## Develop Academic Language

provides language support for all students and is especially useful in helping EL students use and produce academic language.

## Support Partner Discussion

provides teachers with prompts to help students engage in meaningful peer discourse.

## Make Sense of the Problem

uses a language routine to help students understand the problem. See the Language Routines section on the Teacher Toolbox (under the Program Implementation tab) for suggestions on how to integrate language routines, teacher moves, and conversation tips during instruction.

## LESSON 10|SESSION 3 ■■■ $\square$

Develop

## Purpose

- Develop strategies for writing and solving an equation to represent a multi-step problem.
- Recognize that the solution to a division problem with a remainder must be interpreted in the context of the question.

START CONNECT TO PRIOR KNOWLEDGE


Possible Solutions
All are division expressions.
$A$ and $D$ have no remainder.
$B$ and $C$ have a remainder.
$A$ and $C$ have the same number of equal groups.
WHY? Support students'facility with division to prepare them for interpreting remainders in word problem contexts.

## DEVELOP ACADEMIC LANGUAGE

WHY? Clarify the meaning of the multiple meaning word change.
HOW? Ask students to think about the everyday meaning of change and share examples such as changing classes. Have a volunteer read Apply It problem 9 and ask students to use the situation to identify the meaning of change. If needed, supply the definition: the amount of money returned to a customer when buying something.

## TRY IT

## SMP 1, 2, 4, 5, 6, 7, 8

Make Sense of the Problem
Before students work on Try It, use Say It Another Way to help them make sense of the problem. Ask students to listen as you read the problem. Then have a volunteer paraphrase what the problem is about. Ask students to turn and talk about the parts of the problem to decide if the paraphrase is complete and accurate.

Lesson 10

## Develop Solving Multi-Step Problems

Read and try to solve the problem below.
Students make origami animals with people at a senior center. They fold 24 butterflies and 27 whales. They put the origami in boxes, with 9 origami animals in each box. Write and solve an equation to find how many boxes the students need.

## TRY IT

Possible student work:
Sample A
$b=(24+27) \div 9$
$b=51 \div 9$
$51 \div 9=5 \mathrm{R} 6$
SESSION 3 ••• 。

Since there are 6 animals left over, the students
need $5+1=6$ boxes.
Sample B
$24+27=51$

| - ${ }^{\circ}$ | - ${ }^{\circ}$ | - ${ }^{\circ}$ | - ${ }^{\circ}$ | - ${ }^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
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| 9 | 18 | 27 | 36 | 45 | 51 |

## $1 \div 9=5 R 6$

The students need 6 boxes.


## Math Toolkit

- counter
- paper plate
- number lines
- index cards
- base-ten block
- base-ten blocks
- multiplication models

 because.


## DISCUSS IT

## Support Partner Discussion

Encourage students to share what did not work for them, as well as what did, as they discuss their solutions.
Support as needed with questions such as:

- How did you model the problem?
- How do you know whether your answer makes sense?

Common Misconception Look for students who may not recognize that the problem involves division and multiply by 9 instead of dividing by 9 . Have students restate the problem in their own words, drawing a diagram to support their explanation.

## Select and Sequence Student Strategies

One possible order for whole class discussion:

- counters, cups, or plates modeling a total of 51 in groups of 9 with 6 left over
- drawings or number lines modeling 51 in groups of 9 with 6 left over
- two separate equations using addition and division
- one equation using addition and division with a letter representing the unknown


## Common Misconception

identifies misconceptions that lead to errors in understanding, which can then be addressed in whole class discussion as students are prompted to explain their reasoning.

## Select and Sequence Student Solutions

gives a range of possible strategies-from concrete to representational to abstractfor use in monitoring student work and facilitating discourse. This information can be used to make decisions about which models and strategies to share and discuss as a class.

## Facilitate Whole Class Discussion

Guide students to Compare and Connect the representations. Prompt students to check that their explanations are clear by pausing and asking classmates if they have any questions.
Record students' drawings, models, or statements that show how to write an equation for the problem that students can refer to as they work.

ASK Where does your model show the number of origami animals? the number of animals each box holds? the number of boxes the students need? LISTEN FOR Students should recognize that accurate responses include representations showing adding 24 and 27 for a total number of animals, groups of 9 or dividing by 9 for the number of animals in each box, a letter standing for the unknown quotient, and a group of 6 or adding 1 to the quotient to show the number of boxes needed.

## Model It \& Solve It

If no student presented these models, have students analyze key features and then point out the ways each model represents:

- the 24 butterflies and 27 whales
- the 9 animals that each box holds
- the number of boxes needed to hold all the animals

ASK How are the numbers of butterflies and whales shown on the number line and in the equation? How is the number of animals each box holds shown? LISTEN FOR The lines labeled 24 and 27 above the number line and $24+27$ in the equation show the butterflies and whales. The jumps of 9 above the number line and $\div 9$ in the equation show the number of animals each box holds.

For a number line model, prompt students to identify how the labels relate to the information in the problem.

- Why does the number line start with 0 ?
-Why are jumps of 9 shown above the number line?
-Why does the number line end at 54 ?
For an equation, prompt students to identify how the equation represents the problem.
- What does the expression $(24+27)$ represent?
- Why is 51 divided by 9 ?
-What does the letter x represent?

Ask/Listen for are mathematical discourse questions followed by expected student responses that support and facilitate whole class discussion.

As students share their thinking, the discourse questions can be used to make connections between student approaches and different models and representations, prompt justifications and critiques of approaches and solutions, and check conceptual understanding.

The total number of animals is shown above the number line $(24+27)$.
The number line shows jumps of 9 because each box holds 9 animals.
Let $x$ equal the number of boxes needed. Remember to use
parentheses to show what to do first.

$$
x=(24+27) \div 9
$$

## SOLVE IT

You can solve the equation that represents the problem.
This is one way to represent the problem with equations.

## $x=(24+27) \div 9$

$x=51 \div 9$
When a number does not divide another number a whole number
of times, you have some left over. The amount left over is called a remainder, shown with an $R$.

$$
51 \div 9=5 \mathrm{R} 6
$$

## Deepen Understanding is a

 consistent opportunity to build conceptual understanding of a key lesson concept by extending mathematical discourse. The content connects a particular aspect of lesson learning to an SMP, showing how it use multiplication and division facts to help them divide.| ASK What do the jumps on the number line represent? How do they help you divide the total number of animals into boxes of 9 animals each?
LISTEN FOR The jumps show 9 times a number whose product is close to 51 .
ASK What does it mean that the last jump goes beyond the red line?
LISTEN FOR The last jump goes 3 past the total number of animals, 51. It represents another box of 9 animals. 5 jumps of 9 is 45 , and 6 more is 51 . So, 5 boxes hold 45 animals and another box is needed to hold 6 animals; $5+1=6$, so 6 boxes are needed to hold all the animals.

Standards for Mathematical
Practice (SMP) are infused throughout
the instructional model. looks in the classroom.

# TEACHER'S GUIDE Overview 

continued

## Monitor and Confirm

## Understanding is a way to ensure

 that students have made sense of mathematical learning goals.
## Facilitate Whole Class Discussion

provides a series of related discourse questions that illuminate the mathematical ideas of the lesson, prompting students to make connections and use that understanding to solve problems leading to abstract reasoning. These questions help students learn how to articulate a generalization of the mathematical concept.

## Hands-On Activities occur

consistently at strategic points in the lesson after teachers have acquired understanding of students' learning through observation and their work on questions in the Student Worktext. The activities support students who are unsure of the concept and are an opportunity for small group reteaching while other students work independently. Use of concrete objects lets students access understanding in a different way.

## LESSON 10|SESSION 3 ■■■ロ

## Develop

## CONNECT IT

## SMP 2, 4, 5, 6, 7

- Remind students that the one thing that is alike about all the representations is the numbers and a letter used to stand for the unknown.
- Explain that on this page students will use those numbers to interpret the meaning of the remainder in the solution to the division problem, use multiplication to check their answer, and use estimation to check the reasonableness of their answer.

Monitor and Confirm Understanding
(1)-4 Check for understanding that:

- the 5 in the solution represents 5 boxes of 9 animals each
- the remainder 6 is the number of animals left over
- 6 boxes are needed to hold all of the animals
- when using multiplication to check the solution, subtract 45 from 51 to get the remainder of 6 to add to the product of 9 and 5


## Facilitate Whole Class Discussion

(1) -5 Tell students that these problems will prepare them to provide the explanation required in problem 6
Be sure students understand that problem 5 is asking them to use estimation to check if the answer is reasonable.

ASK How do you make sure that the extra six animals are included in your answer when you check it for reasonableness?
LISTEN FOR You need to add 1 to the number of boxes to confirm 6 as the total number of boxes.
6. Look for the idea that the context of the problem determines the meaning of the remainder. In this problem, 6 boxes are needed to hold 51 animals. Five boxes holding 9 animals each hold 45 of the animals and the remaining 6 animals requires an additional box.

Reflect Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

## CONNECT IT

Now you will use the problem from the previous page to help you
understand how to solve multi-step problems.
(1) What does the 5 in the solution 5 R 6 mean? 5 full boxes with 9 animals each What does the R 6 mean? 6 animals left for another box
(2) How many animals are left over that do not make a full box? ........ 6 Is another box needed to hold the 6 leftover animals? ....... yes
3 How many boxes do the students need? ....... 6
4 Check the solution to the equation:
5 full boxes $\times \ldots$ animals per box $+\ldots 6$ animals $=51$ animals
(5) How could you estimate to make sure your answer is reasonable? Possible answer: $27 \div 9=3$; because 24 is a little less than $27,24 \div 9$ is a little less than $3.3+3$ would be 6 boxes, so the answer makes sense.
(6)

Explain why the solution to an equation is not always the answer to a problem when there is a remainder.
Possible answer: You have to think about what the remainder means. The remainder might make the answer go up by one, like it did in the origami problem.
(7) Reflect

Look back at your Try It, strategies by classmates, and Model It and Solve It. Which models or strategies do you like best for solving a
multi-step problem? Explain.
Students may respond that they like the strategy of using a number line
because they can show the numbers in the problem on the number line
and then look at the number line to help them write an equation to solve the problem.

## DIFFERENTIATION | EXTEND

Hands-On Activity
Use cups and counters to model division with remainders.
If students are struggling with modeling and solving a multi-step problem that involves division with a remainder, then use this activity to have them model the problem about origami animals.

## Materials For each pair: 6 cups and 51 counters

- Give each pair 24 counters for the 24 butterflies, 27 counters for the 27 whales, and 5 cups for 5 boxes that can hold 9 animals each.
- Ask students to model the animals and boxes in the problem. Facilitate discussion to conclude that they need to put 9 counters in each container to represent putting 9 animals in each box.
- Ask a volunteer to describe the result. Confirm with other groups that they got the same result. [5 cups with 9 counters each, with 6 counters left over]
- Ask students what they need in order to completely model the problem. [a cup for the leftover 6 counters to represent another box to hold the last 6 animals]
- Write $(24+27) \div 9=5 R 6$ on the board. Relate the numbers and operation symbols in the equation to the models of the counters and cups.

Apply It solutions at point of use give a correct response with explanations that include multiple approaches to solving the problem.

## Apply It

For all problems, encourage students to draw a number line or some other kind of model to support their thinking. Allow some leeway in precision as spacing the marks on a number line is difficult and precise intervals are not necessary.
(8) See Connect to Culture to support student engagement.
$f=200-(95+54+38), f=13$; Rounded estimates $100+50+40=190<200$. See possible work on the Student Worktext page.
(9) Yes. Possible explanation: Subtract the total cost of food from $\$ 200$ to get $\$ 13$, the amount Aun gets in change.

## CLOSE EXIT TICKET

(10) a. Selena can buy 6 puppets for each child; See possible work on the Student Worktext page. To solve the division problem $53 \div 8$, students may use the basic multiplication fact $8 \times 6=48$ and recognize that 5 is added to 48 to get 53 . b. Possible explanation: The remainder of 5 means that Selena will have $\$ 5$ left over after she buys 6 puppets for each of her 4 children.
Students' solutions should indicate understanding of:

- multiplying the number of children by the cost of each puppet to get the number to divide by
- representing 53 divided by 8 as a way to find out how many puppets she can buy for each child
- subtracting 48 from 53 to confirm that she has enough money and to determine how much money she has left
Error Alert If students confuse the meanings of the numbers in the problem, then have them label the numbers as $\$ 53,4$ children, and $\$ 2$ for each puppet.

Close: Exit Ticket is a quick
formative assessment of each day's learning and serves as an indicator of students' progress toward mastery or partial mastery of the learning goal of the session.

This is the last question on the Student Worktext page.

Error Alert gives insight into misconceptions that can lead to errors in calculation and provides on-the-spot remediation.

# TEACHER'S GUIDE Overview 

Additional Practice can be used as in-class small group work, after class work, or at-home learning.

Solutions are labeled as Basic, Medium, and Challenge to show the relative difficulty level in relation to the questions at hand or the standard in question. Use these to support independent practice or differentiation as needed.

Fluency \& Skills Practice provides ongoing opportunities for students to accurately, flexibly, and efficiently practice mathematical procedures and operations. This can be used as in-class small group work, after-class work, or at-home learning. Student pages are available in the optional Fluency and Skills Practice Book or on Teacher Toolbox. Download PDFs or editable versions, or assign to any LMS, including Google Classroom.

## LESSON 10 |SESSION 3 ■ ■ $\square$

## Addifional Practice

## Problem Notes

Assign Practice Solving Multi-Step Problems as extra practice in class or as homework.
(1) 6 tables $\times 4$ students per table +2 students $=$ 26 total students

- Basic
(2) $m=(45-24) \div 8 ; 21 \div 8=2$ R 5. Serafina needs to rake 3 more yards. To solve the division problem $21 \div 8$, students may use the basic division fact $16 \div 8=2$ and recognize that 5 needs to be added to 16 to get 21 , so there is a remainder of 5 .
Medium
$\qquad$ LESSON 10 SESSION 3


## Practice Solving Multi-Step Problems

Study the Example showing how to solve a multi-step problem with a remainder. Then solve problems 1-5.

## EXAMPLE

Mrs. Hunter has 12 students in one science class and 14 students in another She wants to combine both classes to do group work. Each table in the science room can seat 4 students. How many tables does Mrs. Hunter need?


Let $t$ equal the number of tables needed.

$$
\begin{aligned}
& t=(12+14) \div 4 \\
& t=26 \div 4 \\
& 26 \div 4=6 R 2
\end{aligned}
$$

$26 \div 4=6 R 2$
Mrs. Hunter needs 7 tables.

6 R 2 means:
.6 tables with 4 students each . 2 more students need another table
(1) Check the solution to the equation in the Example.

6 tables $\times{ }^{4}$... students per table $+\ldots 2$ students $=26$ total students
(2)
ans $\$ 8$ each time she rakes a yard. She
has earned $\$ 24$ so far. Write and solve an equation
to show how many more yards serafina needs to rake to earn enough to buy a music player that costs $\$ 45$. Show your work. Possible equation shown.
$m=(45-24) \div 8$
$m=21 \div 8$
$21 \div 8=2$ R 5

Solution Serafina needs to rake 3 more yards.


Fluency \& Skills Practice

## Assign Solving Multi-Step Problems ©

In this activity students practice writing equations to represent and solve multi-step problems. This is one approach that students can use to solve real-world problems involving more than one step. Although students are asked to use estimation to check only one of their answers, it may be helpful for students to use estimation to check all their answers.


## Interactive Practice

Assign your students additional digital practice, as needed.

## Cumulative Practice

Assign Cumulative Practice to review major content from previous units, as needed.
\&i-Ready Personalized Instruction
A personalized instruction path helps students reinforce prerequisites and build grade-level skills.

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## Additional Practice Opportunities

 include digital Learning Games, Interactive Practice, Cumulative Practice, and i-Ready Personalized Instruction.Demarco found 34 pieces of sea glass. To check, 19 rounds to 20 and $15+20=35$, so the answer 34 is reasonable.Medium
4. 8 adult tickets were sold. See possible equation and student work on the student page. Medium
(5) 4 children go to the play. See possible equation and student work on the student page. Challenge

## LESSON 10 SESSION 3

(3) Demarco found 15 pieces of sea glass on the beach one day. The next day he found 4 more pieces than he found the day before. Write and solve an equation to find how many pieces of sea glass he found altogether Estimate to check that your answer is reasonable. Show your work. Possible equation: $g=15+(15+4)$

## $=15+$ <br> $=34$

Solution Demarco found 34 pieces of sea glass. 19 rounds to 20 and $15+20=35$, so the answer 34 is reasonable since it is close to 35 .
(4) T

The table shows ticket prices at a movie
theater. Ticket sales to an afternoon show
were $\$ 106$. There were 10 child tickets sold.
Write and solve an equation to find how many
adult tickets were sold. Show your work.
Possible equation: $106=(10 \times 5)+(a \times 7)$
$106=50+(a \times 7)$
$106-50=a \times 7$
$56=a \times$
$8=a$
Solution 8 adult tickets were sold
Tickets to a play are $\$ 10$ for a child and $\$ 15$ for an adult
One adult spends $\$ 55$ to take a group of children to the play Write and solve an equation to find how many children
go to the play. Show your work.
Possible equation: $c=(55-15) \div 10$

$$
c=40 \div 10
$$

$$
c=4
$$

Solution 4 children go to the play. English learners can access and engage with grade-level mathematics.

