Lesson Overview

LESSON 12

Multiply by Two-Digit Numbers

CCSS Focus

Domain
Number and Operations in Base Ten

Cluster
B. Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Objectives

Content Objectives
- Multiply a two-digit number by a two-digit number.
- Use area models and partial products to multiply.
- Use estimation to determine whether an answer is reasonable.

Language Objectives
- Read aloud multiplication problems.
- Draw an area model to multiply.
- Write a solution to a multiplication problem using partial products.
- Tell how each part of an area model relates to the factors, partial products, and product of a multiplication problem.

Prerequisite Skills
- Recall basic multiplication facts.
- Know properties of operations.
- Understand place value.
- Understand and use area models.
- Multiply whole numbers of up to four digits by a one-digit whole number.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **estimate (verb)** to give an approximate number or answer based on mathematical thinking.
- **factor** a number that is multiplied.
- **factors of a number** whole numbers that multiply together to get the given number.
- **multiple** the product of a given number and any other whole number.
- **multiplication** an operation used to find the total number of items in a given number of equal-sized groups.
- **multiply** to repeatedly add the same number a certain number of times. Used to find the total number of items in equal-sized groups.
- **partial products** the products you get in each step of the partial-products strategy. You use place value to find partial products. For example, the partial products for 124×3 are 3×100 or 300, 3×20 or 60, and 3×4 or 12.
- **product** the result of multiplication.
- **reasonable** something that makes sense when given facts are taken into account.

Learning Progression

In Grade 3 students developed an understanding of multiplication.

In the previous Grade 4 lesson students used arrays of base-ten blocks, area models, and partial products, as well as their understanding of place value, to multiply three- and four-digit numbers by one-digit numbers.

In this lesson students use properties of operations, area models, and partial products to multiply two-digit numbers by two-digit numbers. This helps build a foundation for Grade 5 when students will use the standard multiplication algorithm.

In Grade 5 students will multiply three-digit numbers by two-digit numbers as well as multiply decimals. Students will continue to use area models, properties of operations, and place-value understanding to multiply as they become familiar with the standard multiplication algorithm.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 **Explore** *45–60 min*	**Multiplying by Two-Digit Numbers** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close & Exit Ticket *5 min*	**Additional Practice** Lesson pages 255–256
SESSION 2 **Develop** *45–60 min*	**Multiplying by Two-Digit Numbers** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close & Exit Ticket *5 min*	**Additional Practice** Lesson pages 261–262 **Fluency** Multiplying by Two-Digit Numbers
SESSION 3 **Refine** *45–60 min*	**Multiplying by Two-Digit Numbers** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* copy of Start slide (Session 2)
Activities	*Per pair:* base-ten blocks (2 hundreds flats, 14 tens rods, 55 ones units), play money (93 $1 bills, 62 $10 bills, and 6 $100 bills)
Math Toolkit	base-ten blocks, counters, cups, paper plates, grid paper
Digital Math Tool	Multiplication Models

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 3
- Lesson 8 Use Order and Grouping to Multiply
- Lesson 9 Use Place Value to Multiply
- Lesson 12 Multiplication and Division Facts

RETEACH

Tools for Instruction

Grade 3
- Lesson 8 Use Order and Grouping to Multiply
- Lesson 9 Use Place Value to Multiply
- Lesson 12 Write Multiplication and Division Facts

Grade 4
- Lesson 12 Multiply by Two-Digit Numbers

REINFORCE

Math Center Activity

Grade 4
- Lesson 12 Multiplying by Two-Digit Numbers

EXTEND

Enrichment Activity

Grade 4
- Lesson 12 Display of Cans

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*

Grade 4
- Multiply Two-Digit Numbers by Two-Digit Numbers
- Practice: Multiply Two-Digit Numbers

Learning Games
- Prerequisite: *Match*
- Prerequisite: *Cupcake*
- Prerequisite: *Pizza*

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox

Multiply by Two-Digit Numbers

LESSON 12

Dear Family,

This week your child is learning to multiply two-digit numbers by two-digit numbers.

Your child is learning to multiply a two-digit number by another two-digit number, such as 17 × 38.

One way to multiply two-digit numbers is to use an area model. With this strategy, you multiply using the place value of each digit.

The area model below shows the number **17** as **10** + **7** at the left of the rectangle as its width and the number **38** as **30** + **8** along the top of the rectangle as its length. First, find the individual products that represent each individual area. Then add the products together to find the total area. The total area is the product of 17 and 38.

	30	+ 8
10	10 × 30 1 ten × 3 tens = 3 hundreds 300	10 × 8 1 ten × 8 = 8 tens 80
+ 7	7 × 30 7 × 3 tens = 21 tens 210	7 × 8 = 56

300 + 210 + 80 + 56 = 646

17 × 38 = 646

Invite your child to share what he or she knows about multiplying by two-digit numbers by doing the following activity together.

251

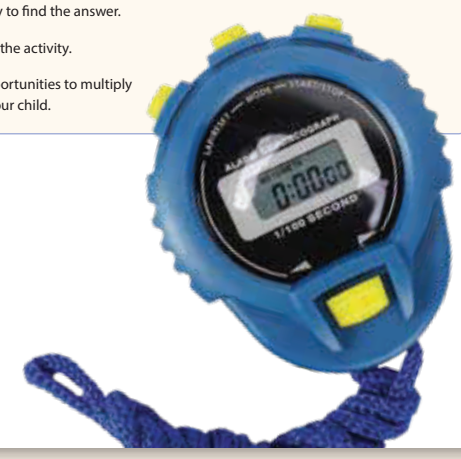
ACTIVITY MULTIPLYING BY TWO-DIGIT NUMBERS

Do this activity with your child to multiply two-digit numbers.

Materials timer or watch with a second hand

- Together with your child, think of things that can be counted in one minute, such as the number of times you clap your hands or the number of steps you walk.
- Choose one idea. Have one person do the activity while the other person uses a timer or watch to time the activity for one minute.
- The person doing the activity counts how many. Count carefully. Stop counting when the person with the timer says "Stop!" For example, you might clap your hands for one minute and count 92 claps.
- Have your child use that number to figure out how many could be counted in 15 minutes. For example, to find out the number of times you might clap your hands in 15 minutes, your child would find: 15 × 92.
- Have your child multiply to find the answer.
- Switch roles and repeat the activity.

Look for other real-life opportunities to multiply two-digit numbers with your child.



252

Goal

The goal of the Family Letter is to reinforce the multiplication of two-digit numbers by two-digit numbers and to encourage students to use their background knowledge of multiplying a two-digit number by a one-digit number to find a product.

- When multiplying by two-digit numbers, an area model can be used to represent the value of each digit in the two-digit numbers to find the product.

Activity

Multiplying two-digit numbers by two-digit numbers is a skill used in everyday life. Look at the *Multiplying by Two-Digit Numbers* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to use examples of items they find at home to begin conversations about multiplying two-digit numbers by either one-digit or two-digit numbers. Encourage students to begin the conversations with the "I see" problems below, and then take turns with family members to make new multiplication problems.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *I see 10 branches. I see 14 leaves on each branch. How many leaves do I see?*
- *I see 12 windows in 6 different apartments. How many windows do I see?*
- *I see 11 stacks of pennies. There are 22 pennies in each stack. How many pennies do I see?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- To make the ***Try It*** problem relevant to students, encourage them to write a word problem that involves finding the product of 14 and 13. For example: *At the car dealership, there are 14 rows of new cars. In each row, there are 13 cars. How many cars are at the car dealership?* Encourage students to write their own word problems to help them build mental pictures for solving the problem. Ask them to share their word problems with partners.

Session 2 **Use with *Additional Practice Example*.**

- Explain to students that the student in the word problem takes guitar lessons to become a better guitar player. Ask students if they take lessons or practice skills to become better at doing something they enjoy. Remind students that they may have activities that they practice at home or at school. Explain to students that they can practice skills without taking private lessons. Provide examples or share a skill you have practiced.
- For example, students who like to cook may spend time with family members preparing meals to become better cooks. Students who like soccer may spend time kicking soccer balls to become better players. Students who like math may spend time solving word problems to become better mathematicians. Rewrite the word problem so that it is relevant to students by using a skill or interest they are familiar with.

Session 3 **Use with *Apply It* problem 3.**

- Display a picture of a deli and a tray of sandwiches. Ask students what they would call the deli and tray. In different parts of the United States, delis may be referred to as sandwich shops, sub shops, hoagie shops, or bodegas. Also, students may use the terms *platter*, *plate*, or *sheet* instead of *tray*. To improve student understanding of the word problem, use the terms students are familiar with, or use the familiar terms as definitions in the word problem. For example, say: *A deli, or, as we would say, a sandwich shop, is preparing trays, or platters, of sandwiches.*

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Reading Use with ***Try It***. Write the following information on sentence strips. • *Break apart the number 13 into 10 and 3.* • *Multiply 14 by 10.* • *Multiply 14 by 3.* • *Add the partial products.* Display the sentence strips. Think aloud about the process for solving the equation $14 \times 13 =$ _____. Read each strip and then complete the task. Shuffle the sentence strips. Have students read them and put them in order. Encourage students to use the sentence strips as they work through the process for solving the problem.	**Listening/Reading** Use with ***Try It***. Work with students to develop a process to find the product. Ask questions to help students organize their thoughts: • *How could you break apart a number?* • *What do you do after you break apart the number?* • *After you multiply the numbers, what do you do?* Write students' information on sentence strips and then shuffle them. Have students read the strips and put them in order. Encourage students to refer to the sentence strips as they solve the problem.	**Reading/Writing** Use with ***Try It***. Provide the following questions for partners to use as they develop a process to solve the problem: • *How would you break apart a number? How does it help you to multiply?* • *What do you do after you break apart the number?* • *After you multiply the numbers, what do you do?* • *Why do you add the partial products?* Ask partners to write their process on sentence strips, shuffle them, and exchange them with another group. Encourage partners to read the strips, put them in order, and follow the process to solve the problem.

LESSON 12

SESSION 1 Explore

Purpose In this session, students draw on their knowledge of multiplying multi-digit numbers by one-digit numbers and of using place value and breaking apart numbers to multiply. Students share models to explore how to multiply a two-digit number by another two-digit number. They will look ahead to use basic facts and patterns of zeros in factors and products to multiply by multiples of 10 and use an area model to multiply 2 two-digit numbers.

Start

Connect to Prior Knowledge

Why Support students' facility with multiplying two-digit numbers by one-digit numbers to prepare students to multiply 2 two-digit numbers.

How Have students multiply two-digit numbers by one-digit numbers.

Multiply.

1. 23 × 3 = ...
2. 47 × 2 = ...
3. 61 × 4 = ...

Solutions

1. 69
2. 94
3. 244

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand the meaning of the word *product*.

DISCUSS IT

Support Partner Discussion

To reinforce students' prior work with partial products, encourage students to use the term *partial product* as appropriate as they talk to each other.

Look for, and prompt as necessary for, understanding that:

- each two-digit number has a ones place and a tens place
- you can break apart each number and multiply to find partial products
- you can add partial products to find the product

LESSON 12 SESSION 1 ● ○ ○

Explore Multiplying by Two-Digit Numbers

You have learned how to multiply two-digit numbers by one-digit numbers, how to multiply one-digit numbers by multiples of 10, and how to break apart numbers by place value to multiply. Use what you know to try to solve the problem below.

What is the product of 14 and 13?

Learning Target

- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A

14 × 13 = 14 × (10 + 3)
= (14 × 10) + (14 × 3)
= 140 + 42
= 182

$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline 12 \\ +\ 30 \\ \hline 42 \end{array}$$

Math Toolkit

- base-ten blocks
- counters
- cups
- paper plates
- grid paper
- multiplication models

Sample B

	10	+ 4
10	10 × 10 = 100	10 × 4 = 40

10 × 14 = 100 + 40 = 140

	10	+ 4
3	3 × 10 = 30	3 × 4 = 12

3 × 14 = 30 + 12 = 42

140 + 42 = 182
So, 14 × 13 = 182.

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I am not sure how to find the answer because . . .

253

Common Misconception Look for students who multiply one number by the other without taking into account the place values of the digits, such as multiplying 14 by 3 and by 1 rather than by 3 and by 10. As students present solutions, have them specify the value of each digit in the factors 14 and 13.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- base-ten blocks, counters, or drawings modeling 14 groups of 13
- breaking apart one factor into tens and ones and multiplying to find partial products
- one or two area models showing partial products
- vertical multiplication of two-digit numbers showing partial products

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models represent 13 fourteen times?*

Listen for There are 14 groups of 13; 13 is added fourteen times; 13 is broken apart into 10 and 3, and each is multiplied by 14; 13 and 14 are each broken apart into tens and ones and multiplied by the tens and ones in the other factor.

CONNECT IT

1 LOOK BACK

Look for understanding of breaking apart one or both factors, multiplying to find partial products, and adding partial products to find the product.

Hands-On Activity

Use base-ten blocks to understand multiplying 2 two-digit numbers.

If . . . *students are unsure about multiplying a two-digit number by another two-digit number,*

Then . . . *use this activity to provide a concrete model of two-digit by two-digit multiplication.*

Materials For each pair: base-ten blocks (1 hundreds flat, 12 tens rods, 55 ones units)

- Present the problem 11 × 15. One partner uses base-ten blocks, and the other records the multiplication.
- Have one partner represent 15 (1 tens rod and 5 ones units) and then add more base-ten blocks to represent 15 eleven times.
- Have the partner count the number of ones units in the 11 groups as the other partner records 11 × 5 ones = 11 × 5. The first partner states the total [55 ones units = 55], and the other partner confirms it by recording the multiplication 11 × 5 = 55.
- Then have the first partner count the number of tens rods in the 11 groups as the other partner records 11 × 1 ten = 11 × 10. The first partner counts the total [11 tens rods = 110], and the other partner confirms it by recording the multiplication 11 × 10 = 110.
- Have the first partner regroup the blocks to use the fewest blocks possible. [1 hundreds flat, 6 tens rods, 5 ones units] The other partner confirms the total by recording 55 + 110 =165.
- Repeat for 11 × 13 and 12 × 14.

2 LOOK AHEAD

Point out that multiplying two-digit numbers involves breaking apart numbers by place value, using basic facts, and multiplying by multiples of 10.

Students should be able to multiply 2 two-digit numbers by completing an area model and adding partial products to find the product.

CONNECT IT

1 LOOK BACK

Explain how you found the product of 14 and 13.

Possible answer: I broke apart the 13 and multiplied 14 by 10 and 14 by 3. Then I added the products to get 182.

2 LOOK AHEAD

To multiply a two-digit number by another two-digit number, you need to understand how to multiply by multiples of 10.

a. Fill in the blanks to show how to multiply by multiples of 10.

Expression	Think of it as . . .	Think of it as . . .	Product
3 × 2	3 × 2 ones	6 ones	6
3 × 20	3 × 2 tens	6 tens	60
30 × 20	3 tens × 2 tens	6 hundreds	600
	3 × 10 × 2 × 10		
	3 × 2 × 10 × 10		
	6 × 100		

b. Complete the area model. Then add the four partial products to find 25 × 32.

	30	+ 2
20	20 × 30 = 600	20 × 2 = 40
+ 5	5 × 30 = 150	5 × 2 = 10

600 + 150 + 40 + 10 = 800

3 REFLECT

Suppose you want to find 30 × 30. How can you use a basic fact and breaking apart numbers to find the product of these multiples of 10?

Possible answer: Break apart 30 × 30 into 3 × 10 × 3 × 10. Change the order of factors to 3 × 3 × 10 × 10. 3 × 3 = 9; 10 × 10 = 100; 9 × 100 = 900

Close: Exit Ticket

3 REFLECT

Look for understanding of representing each multiple of 10 as a one-digit number multiplied by 10 and changing the order of the factors to use the basic multiplication facts 3 × 3 = 9 and 10 × 10 = 100 to find the product.

Common Misconception **If** students do not recognize that they can break apart each factor in 30 × 30 as the product of 3 × 10, **then** review how the factors 30 and 20 in the expression 30 × 20 in problem 2a are broken apart to show using a basic multiplication fact and multiplying by multiples of 10 to find the product.

Real-World Connection

Encourage students to think of everyday situations in which people may need to multiply a two-digit number by another two-digit number. Examples include calculating the number of minutes in ten or more hours and calculating the number of pennies, nickels, or dimes in ten or more dollars.

Solutions

Support Vocabulary Development

1 Ask students what *partial products* are, and when they've used them in the past to solve multiplication problems.

To guide student responses, ask questions and provide examples, as needed. Possible questions include:

- *What do you know about the words* part *and* product*?*
- Display 24 × 4. Say: *We used partial products to multiply two-digit numbers by one digit-numbers.* Have students identify the numbers and digits and guide them through the steps. Ask: *What is the first partial product? What do you do next?* Ask students to add this and other examples to the concept map.

2 Have students describe to a partner their process for completing the area model. Ask questions to guide student thinking:

- *Why are the numbers 18 and 24 broken apart by place value?*
- *When the numbers 18 and 24 are broken apart, what will you do next?*
- *When the partial products are found, what is the next step?*

Supplemental Math Vocabulary

- *place value*
- *product*
- *multiply*

Name: _____ LESSON 12 SESSION 1

Prepare for Multiplying by Two-Digit Numbers

1 Think about what you know about multiplication. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?

A strategy used to multiply multi-digit numbers

What I Know About It

I can use an area model to help find partial products for a multiplication problem.

partial products

Examples

	10	+ 5
10	10 × 10 = 100	10 × 5 = 50

10 × 15 = 100 + 50
= 150

Examples

12 × 15 = 12 × (10 + 5)
= (12 × 10) + (12 × 5)
= 120 + 60
= 180

Examples

```
  37
 × 5
  35   5 × 7 ones
+ 150  5 × 3 tens
 185
```

2 Complete the area model. Then add the four partial products to find 18 × 24.

	20	+ 4
10	10 × 20 = 200	10 × 4 = 40
+ 8	8 × 20 = 160	8 × 4 = 32

200 + 160 + 40 + 32 = 432

255

3 Assign problem 3 to provide another look at solving a problem by multiplying two-digit numbers.

This problem is very similar to the problem about finding the product of 14 and 13. In both problems, students are asked to multiply a two-digit number by another two-digit number. The question asks students to find the product of 16 and 12.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:
Students may use any method to solve the multiplication problem. 16 × 12 = 192
Basic

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

What is the product of 16 and 12?

Possible student work:

$$\begin{aligned} 16 \times 12 &= 16 \times (10 + 2) \\ &= (16 \times 10) + (16 \times 2) \\ &= 160 + 32 \\ &= 192 \end{aligned}$$

$$\begin{array}{r} 16 \\ \underline{\times\ 2} \\ 12 \\ \underline{+\ 20} \\ 32 \end{array}$$

Solution 16 × 12 = 192

4 Check your answer. Show your work.

Possible student work:

	10	+ 6
10	10 × 10 = 100	10 × 6 = 60
+ 2	2 × 10 = 20	2 × 6 = 12

$$\begin{aligned} 100 + 60 + 20 + 12 &= 100 + 80 + 12 \\ &= 100 + 92 \\ &= 192 \end{aligned}$$

16 × 12 = 192

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Reading/Writing Read ***Connect It*** problem 6 to students. Ask: *What strategies did you use for multiplying a two-digit number by a two-digit number?* [area model, partial products] Encourage students to confirm their responses by finding examples in ***Picture It*** and ***Model It***. Write student responses and then read them to students. Ask: *Which strategy do you like best for multiplying a two-digit number by a two-digit number?* Provide a sentence frame for students to use for their written responses: *I like using _____ to solve the multiplication equations*. Have students form pairs and practice reading their sentences to partners.

Levels 2–4

Speaking/Writing Read ***Connect It*** problem 6 with students. Have students brainstorm strategies used to multiply a two-digit number by a two-digit number. [area model, partial products, other strategies recommended by students] Ask: *How is an area model used to multiply a two-digit number by a two-digit number?* Have students explain their process for finding the product. Ask: *Which strategy do you like best for multiplying a two-digit number by a two-digit number? Why?* Provide a sentence frame: *I like using _____ to solve the multiplication equations because _____*. Have students use the sentence frame to provide oral responses before writing their responses.

Levels 3–5

Speaking/Writing Have students read ***Connect It*** problem 6 with partners. Ask partners to brainstorm strategies used to multiply a two-digit number by a two-digit number and explain their process for using each strategy. Have partners make T-charts for each strategy with the headings *Advantages* and *Disadvantages*. When partners have completed their T-charts, have them read the information to other partner groups. Encourage them to add information to their T-charts, as needed. Ask: *Which strategy do you like best for multiplying a two-digit number by a two-digit number? Why?* Call on pairs to share their T-charts with the class and to explain their thought process.

LESSON 12

SESSION 2 Develop

Purpose In this session, students solve a problem that requires finding the product of 2 two-digit numbers. Students model the multiplication either on paper or with manipulatives to find the product. The purpose of this problem is to have students develop a strategy for multiplying two-digit numbers by two-digit numbers.

Start

Connect to Prior Knowledge

Materials For each student: copy of Start slide

Why Support students' facility with breaking apart numbers by place value to multiply as well as multiplying with multiples of 10.

How Have students find the product of 2 two-digit numbers by completing expressions to break apart one factor and use partial products to find the product.

Complete to find 12 × 32.

12 × (..... + ...)

(12 × .....) + (12 × .....)

..... + ...

.....

Possible Solution

12 × (30 + 2)

(12 × 30) + (12 × 2)

360 + 24

384

Develop Language

Why Clarify the meaning of the term *row*.

How Explain to students that the word *row* can be a straight line of people or things that are next to one another. Remind students that they line up in a row, or straight line, one after another, when they go to the cafeteria or library. Ask students to give real-world examples of rows they may see at home or in school. Have students close their eyes and visualize rows of chairs in a school auditorium or cafeteria and then describe to partners what they see in their mental images.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that there are 16 rows of chairs and that each row has 28 chairs in it.

Ask *What are you trying to find out? What do you know?*

LESSON 12 SESSION 2 ● ● ○

Develop Multiplying by Two-Digit Numbers

Read and try to solve the problem below.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs. Each row has 28 chairs. How many folding chairs are set up for the play?

TRY IT

Math Toolkit
- base-ten blocks
- grid paper
- multiplication models

Possible student work:

Sample A

```
  16      16      320
 × 20    × 8    + 128
 120      48      448
+ 200    + 80
 320     128
```

There are 448 chairs set up for the play.

Sample B

	20	+ 8
10	10 × 20 = 200	10 × 8 = 80
+ 6	6 × 20 = 120	6 × 8 = 48

16 × 28 = 200 + 80 + 120 + 48
= 448

There are 448 folding chairs set up for the play.

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

257

DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *partial product* and *hundreds, tens*, and *ones* as they discuss their solutions.

Support as needed with questions such as:

- *How did you break apart the numbers 16 and 28?*
- *How did you find the total that represents the product?*

Common Misconception Look for students who find only some, not all, of the partial products. Remind students that they must multiply the value of each digit in 16 by the value of each digit in 28.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- base-ten blocks or drawings modeling 16 groups of 28
- breaking apart one factor into tens and ones and multiplying to find partial products
- area model showing partial products
- vertical multiplication of two-digit numbers showing partial products

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the tens and ones in 16? the tens and ones in 28? multiplication of the tens and ones in each number by the tens and ones in the other? the partial products? the product?*

Listen for Students should recognize that accurate responses include representations showing 16 and 28 broken apart into tens and ones, multiplying the value of each place in each number by the value of each place in the other to find partial products, and adding partial products to find the product.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of tens and ones in 16 and 28
- multiplying the value of each place in 28 by the value of each place in 16
- partial products of 10 × 20, 10 × 8, 6 × 20, and 6 × 8

Ask *How do the models show breaking apart 16 and 28 into tens and ones? using place value to multiply? partial products?*

Listen for The area model shows 16 as 10 + 6 on the side of the model and 28 as 20 + 8 on the top; the other model shows the ones and tens in 16 and 28 next to the partial products. Both show the 2 tens and the 8 ones in 28 multiplied by the 1 ten and the 6 ones in 16 to find four partial products that are added to find the product.

For an area model, prompt students to identify how multiplication by place value is shown.

- *Where is the number of rows of chairs in the model?*
- *Why is the number of rows of chairs shown this way?*
- *Where is the number of chairs in each row?*
- *Why is the number of chairs shown this way?*

For partial products, prompt students to identify how multiplication by place value is shown.

- *How is multiplication of 28 by 16 represented?*
- *How are the partial products represented?*
- *Why do you think there are four partial products?*

Explore different ways to understand multiplying a two-digit number by a two-digit number.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs. Each row has 28 chairs. How many folding chairs are set up for the play?

PICTURE IT

You can use an area model to multiply two-digit numbers.

To solve this problem, multiply 28 by 16.

	20	+ 8
10	10 × 20 = 200	10 × 8 = 80
+ 6	6 × 20 = 120	6 × 8 = 48

200 + **80** + **120** + **48** = ?

MODEL IT

You can also multiply two-digit numbers using partial products.

```
   28
 × 16
   48 ⟶ 6 ones × 8 ones
  120 ⟶ 6 ones × 2 tens
   80 ⟶ 1 ten × 8 ones
 + 200 ⟶ 1 ten × 2 tens
    ?
```

Deepen Understanding

Partial Products

SMP 7 Use structure.

When discussing the partial products model, prompt students to consider the effect of changing the order of the factors. Display the vertical multiplication problem 16 × 28 with the order of the factors reversed. Work together with students to solve the problem.

Ask *Compare this multiplication with the multiplication shown in the* Model It. *What is the same about the partial products? What is different?*

Listen for The partial products are the same but in a different order.

Ask *Why does it make sense that the partial products are the same?*

Listen for You multiply the same ones and tens digits in 16 by the same ones and tens digit in 28, so the four partial products are the same.

Generalize *Do you think this is true for multiplying any two-digit number by any other two-digit number?* Have students explain their reasoning. Listen for understanding that you can multiply 2 two-digit numbers in any order and the partial products will be the same.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use those numbers to show how both models represent multiplying 2 two-digit numbers by breaking apart the numbers and using place value to multiply in order to find the partial products and the product.

Monitor and Confirm

1 – 3 Check for understanding that:

- 16 and 28 are broken apart by place value in both the area model and the partial products model
- each of the four sections in the area model represents one partial product
- each step in the partial products model shows the product in one section of the area model
- the sum of the four partial products, 448, is the product of 28 × 16

Support Whole Class Discussion

4 – 5 Be sure students understand that problem 4 is asking them about breaking apart the factor 28 in a different way and that problem 5 is asking how to check that the answer to a two-digit multiplication problem is reasonable.

Ask *Why might you be likely to break apart 28 into 10 + 10 + 8 instead of into 20 + 8? How does this affect the partial products you get and the product?*

Listen for It is easy to multiply by 10. The partial products are different, but their sum, the product, is the same. You can break apart a factor in different ways to multiply, and the product stays the same.

Ask *How is it helpful to use easier numbers to check the reasonableness of your answer to a two-digit multiplication problem?*

Listen for Multiplying with easier numbers is a quick way to check whether your answer makes sense.

5 Look for the idea that you can use nearby multiples of 10 as factors because it is easy to multiply by multiples of 10.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply a two-digit number by a two-digit number.

1 Why is the area model divided into four sections?

Each number in the expanded form of one factor is multiplied by each number in the expanded form of the other factor. Each section shows a product.

2 How do the four steps in the multiplication using partial products in **Model It** relate to the four sections in the area model in **Picture It**?

Each step shows the product in one section of the area model.

3 What is the sum of the partial products and also the product of 28 and 16?

448

4 Would the product change if 20 + 8 on the top of the area model were changed to 10 + 10 + 8? Explain.

No, the product would be the same. Possible explanation: Instead of a partial product of 200, you would have two partial products of 100. Instead of a partial product of 120, you would have two partial products of 60. The sum of all the partial products would still be the same.

5 How could you estimate to check the reasonableness of your answer to 28 × 16 by multiplying with easier numbers?

Possible answer: 30 × 10 = 300 and 30 × 20 = 600. My answer is between 300 and 600, so it is reasonable.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for multiplying a two-digit number by a two-digit number? Explain.

Some students may like drawing an area model because they can add the area of each section to find the product. Other students may like using partial products because they can add them to find the product.

Hands-On Activity

Use base-ten blocks to multiply 2 two-digit numbers.

If . . . *students are unsure about multiplying two-digit numbers using partial products,*

Then . . . *use this activity to have them multiply using base-ten blocks.*

Materials For each pair: base-ten blocks (2 hundreds flats, 14 tens rods, 19 ones units)

- Distribute base-ten blocks to each pair. Have partners use the steps below to model 23 × 14 using base-ten blocks in a way similar to that of an area model.
- Model the factor 23 on a flat surface by displaying 2 tens rods and 3 ones units side by side in a single horizontal row.
- Model the factor 14 by displaying 1 tens rod and 4 ones units in a single vertical column to the left of and just beneath the horizontal row showing 23.
- Find each partial product by filling the area inside with the corresponding values: 2 hundred flats, 3 tens rods, 8 tens rods, and 12 ones units.
- The product is the value of the inside blocks. [200 + 110 + 12 = 322]
- Repeat the activity for another two-digit multiplication, such as 24 × 12.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision of student-drawn models.

7 567; See completed area model on the Student Worktext page; Students may add the partial products in any order to find the product.

8 851; See possible work on the Student Worktext page; Students may also make an area model to find the product.

Close: Exit Ticket

9 **A**; $10 \times 64 = 640$

D; $(30 + 2) \times 20 = (30 \times 20) + (2 \times 20) = 600 + 40 = 640$

E; $(40 \times 10) + (40 \times 6) = 400 + 240 = 640$

Error Alert **If** students choose B, **then** have them break apart each factor in 60×40 and rewrite the product as $(6 \times 10) \times (4 \times 10)$ and then change the order of the factors to $6 \times 4 \times 10 \times 10$, resulting in 24×100, to recognize that the product is 2,400, not 640.

APPLY IT

Use what you just learned to solve these problems.

7 Complete the area model below. Then add the partial products to find the product of 27 and 21. Show your work.

Possible student work:

	20	+ 7
20	$20 \times 20 =$ **400**	$20 \times 7 =$ **140**
+ 1	$1 \times 20 =$ **20**	$1 \times 7 =$ **7**

400 + 140 + 20 + 7 = 567

Solution **567**

8 Find 37×23. Show your work.

Possible student work:

```
  37
×  23
  21
  90
 140
+ 600
 851
```

Solution **851**

9 Select all the expressions that have a product of 640.

Ⓐ 10×64 Ⓑ 60×40

Ⓒ 80×80 Ⓓ $(30 + 2) \times 20$

Ⓔ $(40 \times 10) + (40 \times 6)$

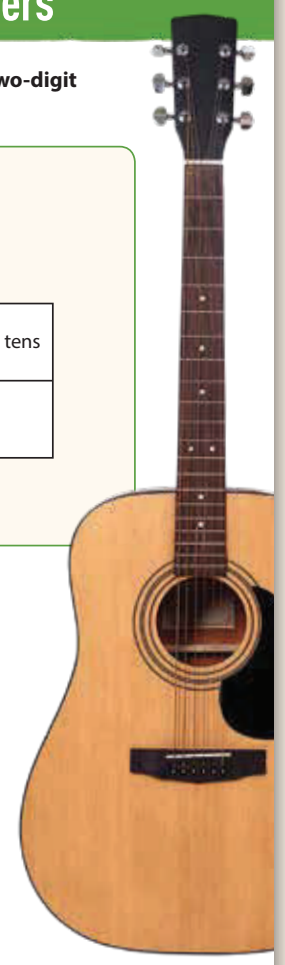
Solutions

1. See completed multiplication using partial products on the student page.
Basic

2. $71 \times 48 = 2{,}800 + 560 + 40 + 8 = 3{,}408$;
Students may add partial products in any order.
See possible student work showing an area model on the student page.
Medium

Name: _____ **LESSON 12** **SESSION 2**

Practice Multiplying by Two-Digit Numbers



Study the Example showing how to multiply a two-digit number by a two-digit number to solve a word problem. Then solve problems 1–6.

EXAMPLE

Aaron spends 35 minutes at each guitar lesson. He has 12 guitar lessons. How many minutes does Aaron spend at his guitar lessons?

Use an area model to multiply 35 by 12.

	30	+ 5
10	10 × 30 1 ten × 3 tens = 3 hundreds **300**	10 × 5 1 ten × 5 = 5 tens **50**
+ 2	2 × 30 2 × 3 tens = 6 tens **60**	2 × 5 = **10**

300 + **50** + **60** + **10** = **420**

Aaron spends 420 minutes at his guitar lessons.

1. Look at the Example above. Use partial products to multiply 35 by 12. Fill in the blanks.

```
      3 5
    × 1 2
    -----
      1 0  →  2 ones × 5 ones
    [6] [0] →  2 ones × 3 tens
      5 0  →  1 ten × 5 ones
  + [3][0][0] →  1 ten × 3 tens
    -------
    4 2 0
```

2. Show how to use an area model to multiply 71 by 48.

	70	+ 1
40	40 × 70 = 2,800	40 × 1 = 40
+ 8	8 × 70 = 560	8 × 1 = 8

Possible model shown. Students may add partial products in any order.

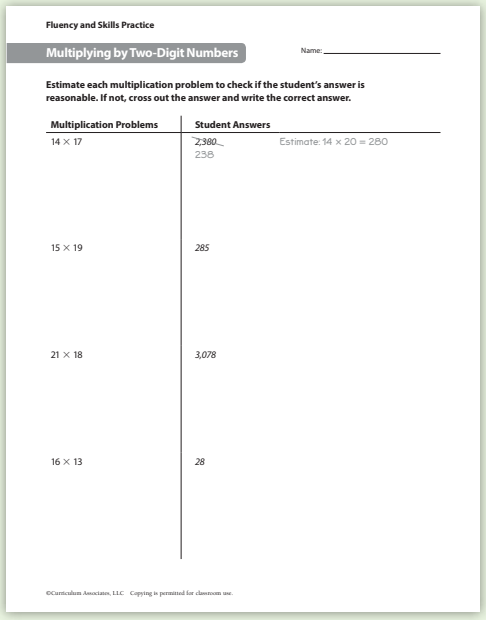
71 × 48 = **2,800** + **560** + **40** + **8** = **3,408**

Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying by Two-Digit Numbers*

In this activity students use estimation to check that two two-digit numbers have been multiplied correctly. Students may use rounding or another estimation strategy to check the reasonableness of the given answer. If students do not think the answer is reasonable, then they multiply to find the correct product. Students may apply the same process in real-world situations. For example, suppose a bowling team buys 12 T-shirts at $14 each. The team may want to check that the total price they are charged is reasonable.



Fluency and Skills Practice

Multiplying by Two-Digit Numbers

Name: _____

Estimate each multiplication problem to check if the student's answer is reasonable. If not, cross out the answer and write the correct answer.

Multiplication Problems	Student Answers
14 × 17	~~2,380~~ 238 Estimate: 14 × 20 = 280
15 × 19	285
21 × 18	3,078
16 × 13	28

©Curriculum Associates, LLC Copying is permitted for classroom use.

3 3,408; See multiplication with partial products on the student page.
Medium

4 **A** (**True**);
D (**False**);
F (**False**);
G (**True**)
Medium

5 1,120 glass tiles; 28 rounds to 30; 30 × 40 = 1,200. 1,200 is close to 1,120. My answer is reasonable. Students may use an area model or partial products to show that 28 × 40 = 1,120.
Medium

6 1,560 minutes; Students may show two different models such as area models and partial products or show various groupings to multiply. See possible student work on the student page.
Challenge

3 Show how to use partial products to multiply 48 by 71.

```
   48
 × 71
    8
   40
  560
+ 2,800
  3,408
```

48 × 71 = 3,408

4 Tell whether each equation is *True* or *False*.

	True	False
18 × 42 = (10 × 40) + (10 × 2) + (8 × 40) + (8 × 2)	Ⓐ	Ⓑ
60 × 15 = (6 × 10) + (6 × 5)	Ⓒ	Ⓓ
37 × 22 = (30 × 20) + (30 × 20) + (7 × 20) + (7 × 20)	Ⓔ	Ⓕ
99 × 11 = (1 × 9) + (1 × 90) + (10 × 9) + (10 × 90)	Ⓖ	Ⓗ

5 Mr. Greene prepares 28 bags of glass tiles for his art class. He puts 40 glass tiles in each bag. How many glass tiles does Mr. Greene use? Estimate to check that your answer is reasonable. Show your work.

Students may use an area model or partial products to show that 28 × 40 = 1,120.

Solution **1,120 glass tiles; 28 rounds to 30; 30 × 40 = 1,200. 1,200 is close to 1,120. My answer is reasonable.**

6 Stephanie has 6 classes a day at school. Each class is 52 minutes long. She goes to school 5 days a week. How much time does she spend in class each week? Show two different ways to solve this problem. Show your work.

Answers will vary. Possible answer:
Way 1: first multiply 6 × 5, then multiply 30 × 52.
Way 2: multiply 6 × 52, then 312 × 5.

Students may show 2 different models for multiplying (area model and partial products) or show various groupings to multiply.

Solution **1,560 minutes**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Writing Use with ***Apply It*** problem 9. Display the following questions:

1. *What will the problem be about?*
2. *What 2 two-digit numbers will be used?*
3. *What will you find out?*

Read the first question. Ask students to make suggestions. If needed, show pictures, such as cookies on platters or books on shelves. Read the second question. Ask students to identify 2 two-digit numbers. Read the third question. Model using the following sentence frames to write a word problem: *I have 42 cookies on 13 plates. How many cookies do I have in all?* Ask students to make original problems using the sentence frames.

Levels 2–4

Writing/Speaking Have students work in pairs. Read ***Apply It*** problem 9 with students. Provide the following questions to guide pairs in writing their own word problem:

1. *Who is your problem about?*
2. *What is your problem about?*
3. *What 2 two-digit numbers will you multiply in the problem?*
4. *What do you want to find out?*

When partners have completed writing their word problems, have them explain to each other how they will find the answer. Suggest students use sequencing words (*first, next, then*) to help them organize their thoughts. Call on pairs to read their word problems to the class.

Levels 3–5

Writing/Reading Use with ***Apply It*** problem 9. Have students write word problems, referring to the following guiding questions as needed:

1. *Who is your problem about?*
2. *What is your problem about?*
3. *What 2 two-digit numbers will you multiply in the problem?*
4. *What do you want to find out?*

Have students exchange their word problems. Say: *Read your partner's word problem. Then explain to your partner how you will find the answer.* When students have completed the activity, have them record their word problems and explanations in their math journal.

LESSON 12

SESSION 3 Refine

Purpose In this session, students solve problems involving multiplying a two-digit number by another two-digit number, then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: base-ten blocks (4 tens rods, 11 ones units)

Why Confirm understanding of multiplying a two-digit number by a two-digit number.

How Have students find the product of 36 and 15 using any strategy they want.

Find the product of 36 and 15.

Solution
540

LESSON 12 SESSION 3 • • •

Refine Multiplying by Two-Digit Numbers

Complete the Example below. Then solve problems 1–9.

EXAMPLE

What is the product of 73 and 58?

Look at how you could show your work using partial products.

```
    73
  × 58
    24 ⟶ 8 ones × 3 ones
   560 ⟶ 8 ones × 7 tens
   150 ⟶ 5 tens × 3 ones
+ 3,500 ⟶ 5 tens × 7 tens
```

Solution 4,234

The student added the partial products to find 73 × 58.

PAIR/SHARE
How else could you solve this problem?

APPLY IT

1 Find the product of 15 and 24. Show your work.

Possible student work:

```
    15
  × 24
    20 ⟶ 4 ones × 5 ones
    40 ⟶ 4 ones × 1 ten
   100 ⟶ 2 tens × 5 ones
+  200 ⟶ 2 tens × 1 ten
   360
```

Solution 360

Should you multiply 15 × 24 or 24 × 15?

PAIR/SHARE
How did you decide which method to use to help you solve the problem?

263

Error Alert

If the error is . . .	Students may . . .	To support understanding . . .
51	have added.	Remind students that *product* means multiplication.
54	have found all partial products as ones digits multiplied by ones digits.	Have students use base-ten blocks to show 36 as 30 + 6 and 15 as 10 + 5. Guide students to draw an area model to show each partial product.
270	have incorrectly found the tens by tens partial product as 3 × 10.	Remind students that when multiplying tens by tens, the result is 30 × 10 = 300, not 3 × 10 = 30.
440	have incorrectly added partial products.	Remind students that they need to regroup 14 tens as 1 hundred and 4 tens when adding partial products.

EXAMPLE

4,234; Using partial products to multiply is shown as one way to solve the problem. Students could also solve the problem by using an area model.

Look for Add the four partial products of 24, 560, 150, and 3,500 to find the product.

APPLY IT

1. 360; Students could solve the problem by multiplying the value of each digit in each factor by the value of each digit in the other factor to get four partial products and then add the partial products to find the product.
   DOK 1

   Look for You can multiply the factors 15 and 24 in any order.

2. 384; Students could solve the problem by using an area model to find partial products of 300, 60, 20, and 4 and adding the partial products to get a product of 384.
   DOK 1

   Look for An area model may represent the factors 12 and 32 as 10 + 2 multiplied by 30 + 2 with four sections showing partial products of 300, 60, 20, and 4.

3. **C**; Students could solve the problem by using partial products to multiply 48 by 23.

   Explain why the other two answer choices are not correct:

   B is not correct because the product is the sum of the four partial products, 24 + 120 + 160 + 800, not the sum of only two partial products, 24 + 800.

   D is not correct because the product is the sum of 3 × (40 + 8) and 20 × (40 + 8), not the sum of 3 × (4 + 80) and 20 × (4 + 80).
   DOK 3

2. What is the product of 12 and 32? Show your work.

   Possible student work:

	30	+ 2
10	10 × 30 = 300	10 × 2 = 20
+ 2	2 × 30 = 60	2 × 2 = 4

   300 + 60 + 20 + 4 = 384

   Solution 384

Could you use an area model to help solve the problem?

PAIR/SHARE
How could you check your answer for reasonableness?

3. A deli is preparing trays of sandwiches. There are 48 trays. Each tray has 23 sandwiches. How many sandwiches are there?

   Ⓐ 240
   Ⓑ 824
   Ⓒ 1,104
   Ⓓ 1,932

How could partial products help you solve this problem?

Nathan chose Ⓐ as the correct answer. How did he get that answer?

Nathan multiplied 2 by 40 and 2 by 8 instead of multiplying 20 by 40 and 20 by 8.

PAIR/SHARE
Does Nathan's answer make sense?

4 **D**; Find the number of minutes in 3 hours, 60 × 3 = 180. Multiply the number of blinks each minute by 180, 16 × 180 = 2,880.
DOK 2

Error Alert Students who choose A multiplied the number of blinks each minute by the number of hours, without taking into account the number of minutes in an hour.

5 4,277; Multiply the value of each digit in 47 by the value of each digit in 91 to get the four partial products of 7, 630, 40, and 3,600 and then add the partial products to get a product of 4,277.
DOK 2

6 **A**; The rectangular array represents the factor 45 as rows of 40 + 5 and the factor 15 as columns of 10 + 5. The total number of squares in the array represents the solution to 45 × 15.

D; The expression breaks apart the factors 15 and 45 by place value and represents 45 × 15 as (10 × 40) + (10 × 5) + (5 × 40) + (5 × 5).

E; The number line represents 45 × 15 as 15 jumps of 45.
DOK 2

4 A person blinks about 16 times per minute. About how many times does a person blink in 3 hours? [*Hint:* 1 hour = 60 minutes]

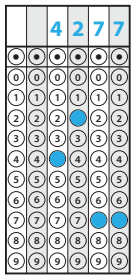
Ⓐ 48

Ⓑ 96

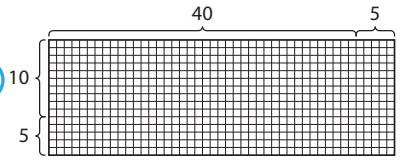
Ⓒ 960

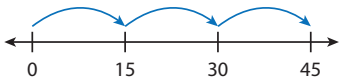
Ⓓ 2,880

5 What is the product of 47 and 91?



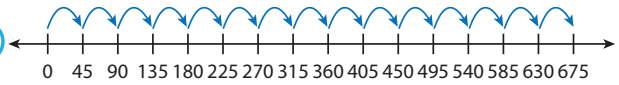
6 Which models below could represent the solution to the problem 45 × 15?

Ⓐ


Ⓑ


Ⓒ (4 × 1) + (4 × 5) + (5 × 1) + (5 × 5)

Ⓓ (10 × 40) + (10 × 5) + (5 × 40) + (5 × 5)

Ⓔ


Differentiated Instruction

RETEACH

Hands-On Activity
Use play money to understand multiplying two-digit numbers.

Students struggling with multiplying two-digit numbers

Will benefit from additional work with concrete representations of place value

Materials For each pair: play money (93 $1 bills, 62 $10 bills, and 6 $100 bills)

- Distribute $1, $10, and $100 bills to partners. Present multiplication problems in which partners multiply two-digit whole dollar amounts by two-digit numbers, such as $23 × 31 and $26 × 14.
- Have students count out the dollar amount using the least number of bills. Then have them count out the number of additional sets represented by the other factor. Students find and record the total amount of $1s and $10s. Then have students exchange ten $1s for one $10 and ten $10s for $100 as needed. Connect this to finding partial products using place value. [$23 × 31 = $713; $26 × 14 = $364]
- Have students do the same multiplication problem on paper to see the connection.

EXTEND

Challenge Activity
Solve two-step word problems involving multiplication.

Students who have achieved proficiency

Will benefit from deepening understanding by solving two-step word problems that involve multiplication

Have students solve the following problem.

- Amelia earns $12 for each hour she works as a math tutor. She works as a tutor for 16 hours one week. She also earns $25 that week for watering her neighbor's garden. How much does Amelia earn altogether that week? [$217]

7 Ian's equation: 43 × 23 = 989, Tia's equation: 43 × 24 = 1,032; Students may solve the problem by substituting numbers beginning with 20 in Tia's equation to identify 24 as the least number to yield a four-digit product. The number 1 less, 23, is then the greatest number to yield a three-digit product.
DOK 2

8 490 minutes; Multiply the value of each digit in 14 by the value of each digit in 35 to get the four partial products of 20, 50, 120, and 300 and then add the partial products to get a product of 490. An area model may also be used to find the partial products and product.
DOK 2

7 Complete each equation below using a factor between 20 and 30 so that:

- The missing factor in Ian's equation will give the greatest possible three-digit product.
- The missing factor in Tia's equation will give the least possible four-digit product.

Ian's equation: 43 × 23 = 989

Tia's equation: 43 × 24 = 1,032

8 Mo has 14 tutoring sessions. Each session is 35 minutes long. How many minutes does Mo spend in the 14 sessions? Show your work.

Possible student work:

```
   14
 × 35
   20
   50
  120
+ 300
  490
```

Solution 490 minutes

9 MATH JOURNAL

Write a word problem you can solve by multiplying 2 two-digit numbers. Solve the problem and show how to find the answer.

Possible answer: Tara buys 18 packages with 12 muffins each. How many muffins does she buy in all? Multiply 18 and 12 using partial products: (2 × 8) + (2 × 10) + (10 × 8) + (10 × 10) = 16 + 20 + 80 + 100 = 216. Tara buys 216 muffins.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

REINFORCE

Problems 4–9

Multiply by two-digit numbers.

All students will benefit from additional work with multiplying by two-digit numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding of how to represent in words a situation involving multiplication of 2 two-digit numbers as well as how to break apart two-digit numbers by place value to multiply and how to multiply by multiples of ten.

Error Alert **If** students write an accurate word problem that involves multiplying 2 two-digit numbers but make an error in calculating the product, **then** have students draw an area model to represent the problem, break apart each factor by place value and find the four partial products and then add the partial products to find the product.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener.

Lesson Quiz

Teacher Toolbox

Tested Skills

Assesses 4.NBT.B.5

Problems on this assessment form require students to be able to use a variety of models and strategies based on place value to multiply two-digit numbers by two-digit numbers. Students will also need to be familiar with writing two-digit numbers in expanded form, and using the commutative and associative properties of multiplication and addition.

Alternately, teachers may assign the **Digital Comprehension Check** online to assess student understanding of this material.

Error Alert Students may:

- decompose two-digit numbers incorrectly.
- multiply by the digits of a number without considering the place value of each digit.
- confuse the ones and tens places.
- multiply partial products instead of adding them.

Solutions

1 **B (No);**
C (Yes);
E (Yes);
H (No);
I (Yes)
2 points
4.NBT.B.5, *DOK 2*

Lesson 12 Quiz

Solve the problems.

1 Decide if each expression can be used to find 15 × 24. Choose *Yes* or *No* for each expression. *(2 points)*

	Yes	No
(5 × 4) + (10 × 4) + (5 × 2) + (10 × 2)	Ⓐ	Ⓑ
(24 × 10) + (24 × 5)	Ⓒ	Ⓓ
(5 × 4) + (10 × 4) + (5 × 20) + (10 × 20)	Ⓔ	Ⓕ
(10 × 2) + (10 × 4) + (5 × 2) + (5 × 4)	Ⓖ	Ⓗ
(20 × 10) + (20 × 5) + (4 × 10) + (4 × 5)	Ⓘ	Ⓙ

2 What is the product of 48 and 23? Show your work. *(2 points)*

Possible student work:

```
   48
 × 23
   24
  120
  160
+ 800
1,104
```

Solution 1,104

3 What multiplication equation is shown by the model below? Write your answer in the blanks. *(2 points)*

12 × 23 = 276

Choice Matrix Scoring Rubric

2 points	1 point	0 points
All answers are correct	1 incorrect answer	2 or more incorrect answers

Fill-in-the-Blank Scoring Rubric

2 points	1 point	0 points
All answers are correct	1 incorrect answer	2 incorrect answers

Short Response Scoring Rubric

Points	Expectations
2	Response contains the following: • Correct computations, solutions, and/or calculations. (1 point) • Well-organized, clear, and concise work demonstrating thorough understanding of math concept and/or procedures. (1 point)
1	Response contains the following: • Mostly correct solution(s). • Shows partial or good understanding of math concepts and/or procedures.
0	Response contains the following: • Incorrect solution(s). • No attempt at finding a solution. • No effort to demonstrate an understanding of math concepts and/or procedures.

2 1,104; See possible work on the student page.
2 points
4.NBT.B.5, *DOK 1*

3 $12 \times 23 = 276$ or $23 \times 12 = 276$
2 points
4.NBT.B.5, *DOK 2*

4 **C**; Students could solve the problem by decomposing 27 and 13 into tens and ones, and identifying the partial products.
A is not correct because place value was not taken into account when decomposing 27 and 13 into tens and ones.
B is not correct because the numbers forming the partial products are added instead of multiplied.
D is not correct because two of the partial products are 70 and 200, not 7 and 2.
1 point
4.NBT.B.5, *DOK 2*

5 720 corn plants; See possible work on student page.
2 points
4.NBT.B.5, *DOK 2*

Lesson 12 Quiz continued

4 Which model could represent the solution of 27×13? ***(1 point)***

Ⓐ

	2 +	7
1 +	1×2	1×7
3	3×2	3×7

Ⓑ $(20 + 10) + (20 + 3) + (7 + 10) + (7 + 3)$

Ⓒ

	20 +	7
10 +	10×20	10×7
3	3×20	3×7

Ⓓ

```
  27
× 13
  21
  60
   7
 + 2
```

5 A farmer's field has 16 rows of corn plants. There are 45 plants in each row. How many corn plants are in the field? Show your work. ***(2 points)***

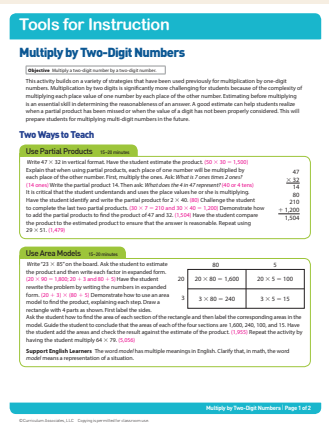
Possible student work:

```
  16
× 45
  30
  50
 240
+ 400
 720
```

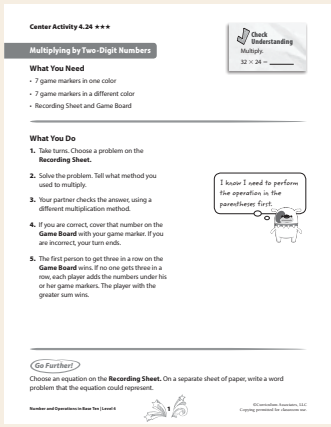
Solution **720 corn plants**

Differentiated Instruction **Teacher Toolbox**

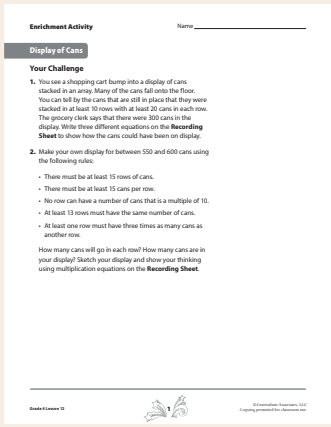
RETEACH: Tools for Instruction


Tools for Instruction
Multiply by Two-Digit Numbers
Two Ways to Teach
Use Partial Products
Use Area Models

REINFORCE: Math Center Activities


Center Activity 4.24
Multiplying by Two-Digit Numbers
What You Need
What You Do
Go Further!

EXTEND: Enrichment Activities


Enrichment Activity
Display of Cans
Your Challenge