



i-Ready[®] Research Base for Instruction

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Overview

i-Ready combines a valid and reliable adaptive K–12 diagnostic, individualized K–8 student online instruction and teacher-led instruction in a single product. *i-Ready*'s adaptive diagnostic pinpoints student needs and has research that shows that *i-Ready* can predict performance on state assessments. *i-Ready Diagnostic* results point to a combination of online instruction, downloadable teacher-led lessons unique to each student's assessment result, and to on-grade level instruction within *Ready*®. Results from the diagnostic create easy-to-understand, individualized instructional plans that differentiate instruction and support blended learning. Easy-to-read reports and ongoing progress monitoring ensure that students are on track to meet end-of-year growth.

The design basis for assessing and building students' reading and mathematics foundation is supported by research as described on the following pages.

Integrating Assessment with Instruction

Adaptive Diagnostic Assessment: *i-Ready Diagnostic*

i-Ready Diagnostic allows for regular assessments of students' reading progress. Assessments can be administered up to three times throughout the year to measure gains and to monitor all students.

i-Ready Diagnostic is an adaptive assessment. Adaptivity provides a much more comprehensive analysis of progress in critical reading skills than an on-level benchmark assessment. *i-Ready Diagnostic* measures strengths and weaknesses in five domains of reading. It also drills down to the level of individual skills to help teachers understand the reason behind students' difficulties.

Finally, *i-Ready Diagnostic* supports educators in collecting and analyzing student data. It lays the foundation for sound instructional decision-making by:

- providing data to monitor growth
- delivering an individualized online instruction plan for every student
- recommending next steps for classroom instruction as well as priorities for instructional grouping

The Research

Assessment is an integral part of instruction, providing educators with the information they need to understand students' strengths and weaknesses and to adjust instruction accordingly.

- Regular assessments of students' reading progress have been proven effective in numerous scientifically based research studies (e.g., Fuchs & Fuchs, 1999; Shinn, 1998).
- "Timely, reliable assessments indicate which children are falling behind in critical reading skills so teachers can help them make better progress in learning to read" (Torgesen, 2006).
- Student achievement increases when teachers track their progress, identify those in need of additional instruction, and design stronger instructional programs (Conte & Hintze, 2000; Fuchs, Fuchs, Hamlett, & Ferguson, 1992; Mathes, Fuchs, & Roberts, 1998).
- A comprehensive assessment system integrates assessment and instruction, so that educators can continually use data to ensure they are meeting the needs of all students (National Center on Response to Intervention, 2010; Smith, 2010).
- Regular progress monitoring is vital to track student growth and determine which students need additional help or intervention (National Joint Committee on Learning Disabilities, 2008; Fisher & Ivey, 2006; Stecker, Fuchs, & Fuchs, 2005).
- Technology is an important tool for assessment. For teachers, technology can minimize loss of instructional time by providing an efficient method of collecting and analyzing student data (Bransford, Brown, & Cocking, 2003).

Online Instructional Modules: *i-Ready Instruction*

As Carol Anne Tomlinson (1995) describes, teaching begins where students are, not at the front of a curriculum guide. Today's students need individual support to meet grade-level requirements.

The online instructional modules in *i-Ready Instruction* provide explicit instruction in skills, based on the results of students' assessments. The instructional modules appeal to different learning styles as well as to different learning abilities. Instruction comes to life and is presented in a fun, exciting environment.

The Research

- Learning is best achieved by adjusting the curriculum and presentation of information to learners rather than expecting learners to adjust themselves to the curriculum (Hall, 2002; Tomlinson, 1995; Tomlinson, 1999).
- “By allowing options that accommodate different thinking patterns, teachers help all students not only achieve planned learning goals but also own these goals in a way that’s all theirs” (Carolan & Guinn, 2007).
- Progress monitoring should produce a clear profile of students’ strengths, weaknesses, and needs, and be linked to targeted follow-up instruction and intervention (Carnegie Council on Advancing Adolescent Literacy, 2010; National Joint Committee on Learning Disabilities, 2008).
- Assessment data should track student growth, identify students who are not demonstrating adequate progress and need more intensive intervention, and determine the efficacy of instructional programs (National Center on Response to Intervention, 2010).
- Effective differentiated instruction engages in continuous progress monitoring and translates the results generated from these assessments into effective reading instruction (Foorman & Moats, 2004).

Real-World Scenarios

The instructional modules in *i-Ready Instruction* are centered on “out of school” interdisciplinary topics that immediately draw students in and keep them engaged. Relevant, real-world scenarios, examples, and themes help students build connections between the skills they are learning and their personal experiences both inside and outside the classroom.

Learning activities stimulate background knowledge that comes from previous lessons or from earlier experiences. Before reading a selection, students have opportunities to link new information in the selection with what they already know.

The Research

- Integrating students’ interests and related skills when teaching helps them to see how what they are learning prepares them for their future (Bellon & Oates, 2002; McCombs & Vakilia, 2005; Palloff & Pratt, 1999; Shin, 2006; Vandergrift, 2002).
- Students learn better when new knowledge is connected to things they already know and understand (Hiebert & Carpenter, 1992; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997).
- Theme-based, integrated learning experiences engage young children in meaningful and functional literacy events, focus on real-life experiences by providing socially interactive settings, and provide an organizational framework for language acquisition (Bergeron, Wermuth, Rhodes, & Rudenga, 1996).

Explicit Instruction

Explicit instruction throughout *i-Ready Instruction* is systematic, clear, and precise. Topics are carefully sequenced and broken down into small, constituent parts and taught individually. This involves explanation, demonstration, and practice in a structured environment.

i-Ready Instruction provides tutorials and practice activities that model instruction and guide practice to develop conceptual understanding of skills. As noted in the research below, explicit instruction has been found effective for all the areas of reading that *i-Ready Diagnostic & Instruction* assesses and instructs.

The Research

- “Direct instruction is appropriate instruction for all learners, all five components of reading, and in all settings (whole group, small group, and one-on-one)” (Florida Center for Reading Research, 2006).
- Explicit instruction is crucial for students with disabilities and those at risk to retain new skills (Swanson, 2001).
- Explicit instruction proves to be an effective, research-based instructional strategy that improves students’ understanding of reading strategies and vocabulary acquisition (Kamil, Borman, Dole, Kral, Salinger, & Torgesen, 2008).
- Research finds that, like other students, those who are considered disadvantaged and have diverse needs benefit most from early and explicit teaching of word recognition skills, including phonics (Adams & Engelmann, 1996).
- Teachers should provide explicit phonemic awareness instruction including clear explanations, modeling of tasks, and opportunities for student practice (Foorman & Torgesen, 2001; Cunningham, 1990).
- Systematic and explicit phonics instruction contributes more to reading growth than instruction with a non-systematic program or no phonics instruction at all (Adams & Engelmann, 1996; Hall, 2002).
- Students need explicit instruction and systematic practice to learn irregular words (Vellutino & Scanlon, 2002).
- “By giving students explicit instruction in vocabulary, teachers help them learn the meaning of new words and strengthen their independent skills of constructing the meaning of text” (Kamil et al., 2008).
- Students, including those with learning disabilities and English language learners, benefit from explicit comprehension strategy instruction (Francis, Rivera, Lesaux, Kieffer, & Rivera, 2006; Nokes & Dole, 2004; Duke & Pearson, 2002).
- Strategies to understand and interpret narrative and expository text structures need to be explicitly taught (Duke, 2010; Readence, Bean, & Baldwin, 2004; Duke & Pearson, 2002).

Immediate Feedback

As a part of the scaffolded instruction in *i-Ready Instruction*, students receive immediate corrective feedback that is specific and purposeful. They are told why their answers are correct or incorrect without fear of judgment or penalty. In addition, a quiz at the end of each lesson quickly assesses student learning and provides immediate, informative, and encouraging feedback.

The Research

- When students receive direct instruction about the reasons why an answer is correct or incorrect, they demonstrate long-term retention and understanding of newly learned content (Rohrer & Pashler, 2007).
- “As an alternative or adjunct to traditional reading instruction, computer-assisted instruction can offer students the opportunity to receive customized support, learn at a comfortable pace, and encourage the active processing of text” (Kamil, 2003).
- Modeling and corrective feedback are essential for struggling readers, particularly those with learning disabilities, to learn to read and respond to text (Swanson, Wexler, & Vaughn, 2009; Vaughn & Roberts, 2007).

Frequent Interactivity

Students today have a lower threshold for boredom. They are often multi-tasking in their lives away from school. In the classroom, instruction must keep them actively involved in their learning experience, which will also lead to deeper understanding. The instructional modules in *i-Ready Instruction* promote active student participation in learning. Students interact with the program every 30 seconds or less by clicking on answers, using online tools, typing in text or numbers, and manipulating models.

The Research

- “The more ways in which the learner is involved in task related activity, the stronger the learning” (Biggs, 1991).
- Independent work; individualized, one-on-one instruction; and computer-assisted instruction have been found to be the most effective strategies that lead to increased levels of academic engagement (Greenwood et al., 2002; Marston et al., 1995).
- “Kids want a multi-sensory experience. Not only do they find it more entertaining, but they also find it a more engaging environment” (Druin et al., 1999).
- Students are engaged by computer-assisted instruction and the ability to control activities on screen. Studies have found that students frequently ask to use computer-assisted programs and remain on task for longer periods of time (Distel, 2001; Hitchcock & Noonan, 2000).

Gradual Release of Responsibility

Lev Vygotsky’s zone of proximal development—the range of reading skills that children cannot do alone but can with some assistance—can be addressed with the gradual release of responsibility model. First, teachers model the skill. Then they slowly release responsibility with guided practice that starts with a great amount of teacher support, which is gradually reduced. Finally, when it is believed students have mastered the skill, they move to independent practice.

Each instructional module in *i-Ready Instruction* is structured with a tutorial that provides modeled and guided instruction, a practice activity that supports and reinforces student learning, and a quiz for independent practice and assessment.

The Research

- There is widespread agreement that scaffolding plays an essential and vital role in fostering comprehension (Clark & Graves, 2005).
- All struggling readers, including English language learners and students with special needs, benefit from highly scaffolded instruction and gradual release of responsibility in comprehending challenging texts (Fisher & Frey, 2008; Francis, Rivera, Lesaux, Kieffer, & Rivera, 2006; Duke & Pearson, 2002).
- “Scaffolded instruction optimizes student learning by providing a supportive environment while facilitating student independence” (Larkin, 2002).

Reading Skills Assessed and Taught

Foundational Skills

The Common Core State Standards grouped the very basic and essential building blocks of literacy together as foundational skills—print concepts, phonological awareness, phonics and word recognition, and fluency. In addition, state standards typically emphasize phonemic awareness, phonics, and fluency, which were also identified as three of the five essential components of reading by the National Reading Panel (2000).

i-Ready Diagnostic & Instruction assesses and teaches the foundational skills of phonological awareness, phonics, and high-frequency words.

Phonological Awareness

Phonological awareness is the understanding that oral language can be divided into smaller parts. This can include segmenting, or breaking up, sentences into words, along with segmenting words into syllables (*sadness* into /sad//ness/), into onset and rime (bat into /b/ /at/), or into individual phonemes (*skip* into /s/ /k/ /i/ /p/). Phonological awareness also includes the understanding that these sounds can be manipulated (substitute the /m/ sound in *mat* with the /c/ sound to make *cat*).

Phonological awareness is a critical prerequisite for learning phonics. The ability to distinguish discrete sounds is a critical first step in understanding how each sound maps onto a particular spelling pattern.

The Research

- Students who develop phonological awareness are better prepared to develop other reading skills, including phonics and spelling (Adams, Foorman, Lundberg, & Beeler, 1998; Chard, Simmons, & Kame'enui, 1998; Adams, 2001; Goswami, 2000, 2001).
- Students with strong phonological awareness are likely to become good readers, whereas students with weak phonological skills are likely to become poor readers (Blachman, 2000; Liberman, Shankweiler, & Liberman, 1989).
- Phonemic awareness is a reliable predictor of later reading achievement (Bishop, 2003; Ehri, Nunes, Willows, Schuster, Yaghoub-Zadeh, & Shanahan, 2001).
- “The most common cause of children’s early difficulties in acquiring accurate and fluent word recognition skills involves individual differences in their phonological knowledge and skills” (Torgesen, 2002).
- Training in phonological awareness supports early reading development for all students—both those with disabilities and those without (Fletcher, Shaywitz, Shankweiler, Katz, Liberman, Stuebing, Francis, Fowler, & Shaywitz, 1994).
- Segmenting and blending are the phonological awareness skills that are most effective in supporting phonics instruction (Chard & Dickson, 1999).

How is Phonological Awareness assessed in *i-Ready Diagnostic*?

In *i-Ready Diagnostic*, test items use both audio and visual support to assess children’s ability to distinguish and manipulate the sounds in spoken language. The stems, which comprise questions or directions, are read aloud to children, as are the individual answer choices. Children can use an audio icon to hear items and answer choices repeated. Many items are supported by art.

Most items focus on segmenting and blending because these skills are the most important building blocks for phonics instruction. Children are asked to segment and blend syllables, onset and rime, and individual phonemes. Other items assess children’s ability to manipulate phonemes by deleting, adding, or substituting sounds in spoken words.

How is Phonological Awareness taught in *i-Ready Instruction*?

i-Ready Instruction provides explicit instruction in phonological awareness. Students learn to distinguish and manipulate the individual sounds in spoken words. The instructional modules focus on skills that include:

- rhyming
- segmentation
- blending
- deletion
- addition
- substitution

The instructional modules combine audio, art, animation, and interactivity to support the development of the critical skills that students need in order to benefit from phonological awareness instruction. In addition to practice opportunities, interactive quizzes assess progress on each skill.

Phonics

Phonics is the understanding of how sounds relate to letters, or groups of letters. Phonics skills include both decoding (reading written words) and encoding (spelling spoken words). As children develop as readers, they become more accurate in recognizing the relationship between sounds and spelling patterns. They also become more fluent, which means that they can read and spell words more automatically.

The Research

- For early readers at risk of developing reading problems, phonics instruction has been shown to help prevent reading difficulties (National Reading Panel, 2000).
- “Systematic phonics instruction helps students learn to read more effectively than nonsystematic phonics or no phonics instruction” (National Reading Panel, 2000).
- According to research, if students do not master phonics by the end of first grade, they will continue to struggle in other areas of reading (Torgesen, Rashotte, & Alexander, 2001).
- When students can read words accurately and automatically they are able to focus on text comprehension because less mental energy is required to decode words and more mental energy can be devoted to making meaning from text (Adams, 1990; Snow, Burns, & Griffin, 1998; Freedman & Calfee, 1984; LaBerge & Samuels, 1974).

How is Phonics assessed in *i-Ready Diagnostic*?

i-Ready Diagnostic assesses children’s ability to recognize sound-spelling correspondences. Test items use both audio and visual support. Some items, which comprise questions or directions, are read aloud, and children are asked to choose among written answer choices. Other items are written, and children are asked to choose among answer choices that are read aloud. As with phonological awareness, children can use an audio icon to hear items and answer choices repeated. Many items are supported by art.

Items focus on a range of high-utility skills, including:

- letter recognition
- one-to-one letter-sound correspondences
- CVC and CCVC words, as well as other one-syllable words
- consonant digraphs
- final e conventions

- *r*-controlled vowels
- inflectional endings
- vowel teams (digraphs and diphthongs)
- two-syllable words
- three, four, and five syllable words
- words with prefixes and suffixes

How is Phonics taught in *i-Ready Instruction*?

The instructional modules in *i-Ready Instruction* provide an interactive, rich-media environment for the systematic and explicit phonics instruction that reading research recommends.

The sequence of instruction begins with consonant and short vowel sounds. As children learn to build words, instruction follows in skills such as initial consonant blends, final e conventions, consonant digraphs, vowel digraphs, *r*-controlled vowels, and diphthongs. The instructional modules also teach word-study and word-analysis skills, such as using syllabification, recognizing prefixes and suffixes, and identifying compound words. In addition to practice opportunities, interactive quizzes assess progress on each skill.

What sets *i-Ready Instruction* apart from many other instructional programs is that the emphasis is on teaching students to use various phonics skills strategically in the context of sentences, stories, letters, and expository paragraphs. Students are taught how to use both graphophonic (letter and sound) cues as well as semantic and syntactic (meaning and language-based) cues to figure out unknown words. They are taught to figure out difficult words by rereading sentences and using what they know about letters and sounds as well as by considering the context of the sentence. Students are also taught how to self-monitor their reading on a metacognitive level.

The goal of these instructional modules is to help students find strategies that are useful to them with the goal of making them independent readers.

High-Frequency Words

While there are hundreds of thousands of words in the English language, approximately 100 of them account for perhaps 50 percent of the words read and used. Called high-frequency words, many of these words have irregular spellings—that is, they don’t follow regular sound-spelling conventions. These words must be explicitly taught and memorized. Because they appear so often in text, readers need to learn to recognize high-frequency words automatically. Being able to read these words quickly and easily is critical to fluency and comprehension.

Words assessed and taught in *i-Ready Diagnostic & Instruction* are drawn from the Dolch Basic Word List (Dolch, 1941) and the Fry Instant Word List (Fry, 1999).

The Research

- When students learn high-frequency words thoroughly and reliably, they demonstrate smoother, less effortful reading and perhaps a greater inclination to read independently (which may also increase the number of words they instantly recognize) (O’Connor, 2007).
- Developing readers who cannot instantly identify high-frequency words are unlikely to become fluent due to the common occurrence of these words (Pikulski, 2006).
- “When children at an early age learn to recognize and automatically spell the most frequently occurring words, all their attention is freed for decoding and spelling less frequent words and more importantly, for processing meaning” (Cunningham, 2000).

How are High-Frequency Words assessed in *i-Ready Diagnostic*?

Test items in *i-Ready Diagnostic* assess children’s ability to recognize high-frequency words. Some stems, which comprise questions or directions, are read aloud, and children are asked to choose among written answer choices. Other stems are written, and children are asked to choose among answer choices that are read aloud. Children can use an audio icon to hear items and answer choices repeated.

How are High-Frequency Words taught in *i-Ready Instruction*?

In *i-Ready Instruction*, words are taught and practiced both in isolation and in context. Animation and audio throughout the instructional modules engage and motivate students. For example:

- An engaging animated character often establishes a context for a group of words.
- Words are presented on the screen and read aloud.
- A sentence and a visual are often displayed to provide context for the word.
- Students then identify the word in the context sentence or in isolation.

Repetition helps build automatic recognition of high-frequency words. Interactive quizzes assess progress.

Vocabulary

Vocabulary is the set of words we use to speak, listen, write, and read. Since at least 1925, vocabulary has been empirically linked to reading comprehension (Whipple, 1925) and the National Reading Panel confirmed this in its review of scientific research. Knowing what words mean is a critical part of understanding what we read. Reading development requires continued growth in the size of students’ reading vocabularies. Both the Common Core State Standards and the National Reading Panel report emphasize the importance of students acquiring rich and varied vocabulary knowledge.

The Research

- Word knowledge affects reading comprehension, which in turn helps students expand their knowledge bases, which in turn facilitates vocabulary growth and reading comprehension (Cunningham & Stanovich, 1998; Johnson & Rasmussen, 1998).
- In content-area instruction, new vocabulary constitutes both information students must learn and concepts they need to understand to function within the subject (Armbruster & Nagy, 1992; Rekrut, 1996).
- “People with more extensive vocabularies not only know more words but also know more about the words they know” (Curtis & Glaser, 1983).
- A learner’s knowledge of words and what they mean is an important part of the reading process, as knowledge of word meanings affects the extent to which the learner comprehends what he or she reads (National Reading Panel, 2000).
- Oral and written vocabulary instruction is a valuable component of beginning reading, because student understanding of word meanings and how words are used in text contributes significantly to general reading comprehension (Vaughn & Linan-Thompson, 2004).
- Children who enter Kindergarten with low vocabularies tend to encounter reading difficulties (Scarborough, 2001).
- Oral language development is a critical part of addressing vocabulary deficits (Anderson et al., 1985; Barnett, 2001).
- Children who have low vocabularies tend to read less and thus develop less vocabulary and fall further behind their peers (Cunningham & Stanovich, 1998).

- Vocabulary instruction that makes students think about the meaning of a word and demands that they engage with the meaning of the word is more effective than instruction that does not (Beck et al., 2002).
- Teaching prefixes, suffixes, and root words can help students understand a wider range of word meanings (White et al., 1989; Biemiller & Slonim, 2001).

How is Vocabulary assessed in *i-Ready Diagnostic*?

Test items in *i-Ready Diagnostic* assess students' knowledge of both Tier 2 words (academic or literary words) and Tier 3 words (domain-specific or content-area words).

The words assessed were selected by teachers and reading specialists using research-based lists that included:

- Words Worth Teaching (Biemiller, 2010)
- The Living Word Vocabulary (Dale & O'Rourke, 1981)
- The Educator's Word Frequency Guide (Zeno, 1995)
- The Academic Word List (Coxhead, 2000)

The words were selected to reflect the types of words children learn in various disciplines at different grade levels and in various stages of their lives. Test items assess knowledge of these words in context. Test items aimed at early readers include visual support. Because oral vocabulary is a critical part of reading development, test items at Kindergarten through Grade 2 are supported by audio.

How is Vocabulary taught in *i-Ready Instruction*?

A key goal of *i-Ready Instruction* is to expand vocabulary in order to help readers communicate and comprehend effectively. The instructional modules use categories and rich context to teach the words students need to know. Students are encouraged to make connections between words and to connect words with real-world experiences. Words are explained, and students complete interactive exercises that lead them to engage with the meaning of the words. Corrective feedback redirects students and reinforces understanding of the correct meaning of each word. Instruction also focuses on key vocabulary skills such as prefixes and suffixes, synonyms and antonyms, homophones, and multiple-meaning words. Opportunities are provided for practice, and interactive quizzes assess progress on each skill.

Comprehension

Comprehension is the reason why we read; it is the meaning behind words. It is often referred to as “the essence of reading” (Durkin, 1978). The National Reading Panel described it as “the construction of the meaning of a written text through a reciprocal interchange of ideas between the reader and the message in a particular text” (2000). The RAND Reading Study Group defined “reading comprehension as the process of simultaneously extracting and constructing meaning” (2002). It is the last piece of the reading puzzle that *i-Ready Diagnostic & Instruction* assesses and teaches with literature and informational texts of increasing complexity deemed necessary by the Common Core State Standards.

Informational Text: Students' academic success is closely tied to their ability to comprehend informational text. Informational text is the primary source of students' new knowledge and information after the primary grades. It also makes up the majority of reading done outside of school.

Literature: The Common Core State Standards note that as students read literature—stories, dramas, poems, and myths from diverse cultures and different time periods—they gain literary and cultural knowledge as well as familiarity with various text structures and literary elements, which include setting, characters, plot, and theme.

The Research

- “Comprehension should be assessed frequently as a way to track students’ growth and provide useful information that can guide instructional and diagnostic decision-making” (Klinger, Vaughn, & Boardman, 2007).
- Reliable assessment should guide aligned comprehension instruction (Lehr & Osborn, 2005).
- Comprehension instruction should start as soon as students start interacting with text and continue through high school (Duke & Pearson, 2002; Pressley & Block, 2002).
- Students must extend the range and flexibility of their reading comprehension strategies in order to maintain or improve their level of reading proficiency (Duke & Pearson, 2002).
- “Achieving success in subject areas ranging from social studies to science requires that students be able to comprehend the texts of such subjects” (Neufeld, 2003).
- “Strong evidence links readers’ awareness of text structure to successful reading comprehension” (Coyne, Chard, Zipoli, & Ruby, 2007).
- Student awareness and understanding of the similarities or structure across texts gives them a frame of reference for processing and remembering the information and allows them to consider authors’ messages in a broader context of literature and the world (Carnine & Kinder, 1985; Dickson, Simmons, & Kame’enui, 1998).
- “Instruction of the content and organization of stories improves story comprehension, measured by the ability of the reader to answer questions and recall what was read” (National Reading Panel, 2000).

How is Comprehension assessed in *i-Ready Diagnostic*?

Students’ abilities to understand both literary text and informational text are evaluated in *i-Ready Diagnostic*. The focus in Kindergarten is on listening comprehension. At this grade, comprehension items are supported by both audio and art. Reading comprehension is the focus at Grade 1 and above. Students are presented with a passage, and interactive, multiple-choice items are shown next to the passage. When a passage has more than one page, students are able to page back and forth through the passage while still able to view the item, which encourages students to find textual support for the answer they select.

How is Comprehension taught in *i-Ready Instruction*?

The major focus of the comprehension instructional modules in *i-Ready Diagnostic* is on developing various comprehension skills, such as comparing and contrasting, distinguishing fact from opinion, and prediction, to name just a few. Graphic organizers, concept maps, and flow charts are used to aid in the explicit instruction of main ideas and details, story elements, sequencing, and determining cause and effect. Comprehension activities emphasize the ability of the reader to actively construct meaning when reading both narrative and expository text. All of these comprehension skills are presented using a mix of literary and informational passages.

The instructional modules both model and support the use of dialogue to make meaning from text and to apply comprehension skills more effectively. Animated, interactive lessons assist students in navigating the complexities of the comprehension process and develop their understanding of text by introducing several viewpoints. The lessons model a small group discussion format with a main teacher character and three students to simulate how a real student might participate and react in such sessions. Students are then encouraged to discuss concepts and ideas from the lesson in the classroom. Opportunities are provided for practice, and interactive quizzes assess progress on each skill.

Mathematics Skills Assessed and Taught

The Common Core State Standards organize mathematical content within grades by domains—big ideas that connect topics across grades. A major goal of this grouping is to build understanding of mathematical concepts within each domain and how they progress across grades.

i-Ready Diagnostic & Instruction further organizes the Common Core Domains into four major groups.

Number and Operations

- Counting and Cardinality (Grade K)
- Number and Operations in Base Ten (Grades K–5)
- Number and Operations—Fractions (Grades 3–5)
- The Number System (Grades 6–8)

Counting and Cardinality begins with early counting and telling how many in one group of objects. Addition, subtraction, multiplication, and division grow from these early roots.

Number and Operations in Base Ten extends the counting sequence to gain foundations for place value. Students' work in the base-ten system is intertwined with their work on counting and cardinality, and with the meanings and properties of addition, subtraction, multiplication, and division. Work in the base-ten system relies on these meanings and properties, but also contributes to deepening students' understanding of them.

Number and Operations—Fractions develops an understanding of fractions as numbers that operate in similar ways as whole numbers. Students build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. They use equivalent fractions as a strategy to add and subtract fractions and apply and extend previous understandings of multiplication and division to multiply and divide fractions.

The Number System domain builds on two important ideas that have developed throughout earlier grades, leading to an understanding of the rational numbers as a number system. The first is the representation of whole numbers and fractions as points on the number line, and the second is a firm understanding of the properties of operations on whole numbers and fractions. On this foundation, students build new understandings of operations with fractions and negative numbers and complete their growing fluency with algorithms for the four operations.

The Research

- Place value is essential to learning computational algorithms. When working with place value, students need to understand how they name quantities. This means they need to develop a strong conceptual understanding of the base-ten system, positional notation, and how breaking apart numbers can assist them in adding and subtracting (<http://www.achievethecore.org>).
- Some algorithms are harder to understand than others. In all cases, students understand algorithms better if they can learn them with the support of manipulatives and drawings. Scaffolds such as these actually support the development of computational fluency (Fuson, 2003).
- “[E]ven in an intuitive discussion of fractions for beginning students, we should instill the right way to think about them: these are numbers. One advantage of the number line model is that it allows an unambiguous formulation of the basic concepts of “equal,” “smaller,” and “bigger” among fractions. Simple experimentations on the number line will expose students to the phenomenon of equivalent fractions” (H. Wu, 2011).
- The meaning of operations doesn’t change from whole numbers to fractions. The focus of instruction should be on making sense of mathematical ideas, not on remembering procedures or tricks (Z. Wu, 2001; Ott, 1990; Graeber and Tanenhaus, 1993; Gojak, 2013).

- “Decimal fractions are a particularly difficult domain for many students. Complete understanding requires multiplicative thinking, which is not natural but requires a reconceptualization of the relationship of numbers from that required in additive relationships. A comparison of pretest and posttest results revealed that students who worked on contextual problems made significantly more progress in their knowledge of decimals than did those who worked on non-contextual problems” (Irwin, 2001).

How are Number and Operations Skills and Concepts assessed in *i-Ready Diagnostic*?

In *i-Ready Diagnostic*, the items aligned to the Numbers and Operations in grades K–2 allow students to demonstrate proficiency in the skills associated with counting, whole numbers, the algorithms of the operations, and understanding of place value.

In these grades, in the least difficult items, virtual manipulatives are used to help students show conceptual understanding of place value and the algorithms for adding and subtracting. For example, students can utilize a virtual base-ten block tool to help with regrouping for solving subtraction items.

In grades 3–5, the items aligned to the Number and Operations domain allow students to demonstrate a deeper understanding of the concepts they learned in the primary grades, while also demonstrating their understanding of how these concepts expand into other sets of numbers, such as fractions and decimals. In this domain, there are technology-enhanced items where students are able to show conceptual understanding of fractions by plotting the fractions on a number line tool.

In grades 6–8, the items aligned to the Number and Operations domain allow students to demonstrate their understanding of how the concepts they learned earlier in this domain extend to the integers and real numbers. They also demonstrate their facility with converting among different representations of numbers.

How are Number and Operations Skills and Concepts taught in *i-Ready Instruction*?

The instructional modules in *i-Ready Instruction* provide opportunities for students to develop an understanding of whole numbers, fractions, decimals, and rational numbers.

The sequence of instruction begins with whole numbers. Students use virtual counters, models, and manipulatives to develop understanding of the concepts of counting and cardinality and place value. Students then use these understandings to develop strategies and algorithms to add, subtract, multiply, and divide whole numbers.

The instructional modules expand upon students’ understanding of whole numbers to develop the concepts of fractions and decimals. Students continue to use virtual tools and models, including the number line and area models, to develop an understanding of unit fractions, equivalent fractions, comparing fractions, and adding, subtracting, multiplying, and dividing fractions. Decimal numbers are introduced using money and area models, and a connection is made to decimal fractions and the base-ten system.

Students use their understanding of whole numbers and fractions as points on the number line, as well as their understanding of the properties of operations on whole numbers and fractions, to understand positive and negative rational numbers as a number system. Students are provided opportunities to practice all of these skills both in isolation and in real-world contexts, and interactive quizzes assess progress on each skill.

Algebra and Algebraic Thinking

- Operations and Algebraic Thinking (Grades K–5)
- Ratios and Proportional Relationships (Grades 6–7)
- Expressions and Equations (Grades 6–8)
- Functions (Grade 8)

Operations and Algebraic Thinking is about understanding and using numbers. This domain deals with the meanings of the basic operations—the kinds of quantitative relationships they model and the kinds of problems they can be used to solve as well as their mathematical properties and relationships. It describes concepts, properties, and representations that extend to other number systems, to measures, and to algebra.

Ratios and Proportional Relationships extends students' work in measurement and in multiplication and division from earlier grades. Students understand ratio concepts and use ratio reasoning to solve problems. They analyze proportional relationships and use them to solve real-world and mathematical problems.

Expressions and Equations builds on work with the properties of operations in the context of operations with whole numbers, decimals, and fractions and applies and extends these previous understandings to algebraic expressions. Students start to use properties of operations to manipulate algebraic expressions and produce different but equivalent expressions for different purposes.

Functions involves defining, evaluating, and comparing functions and using functions to model relationships between quantities.

The Research

- “Students require help to acquire number-specific computational resources, but these resources must not be thought of as consisting of a collection of isolated “facts” ... [P]ractice will have its greatest effect when “facts” are not treated in isolation, and when practice on number triads follows, and is continually linked to meaningful examination of patterns and strategies” (Sherin & Fuson, 2005).
- “Research has indicated that beginning with problem situations yields greater problem-solving competence. Children who start with problem situations directly model solutions to these problems. They later move to more advanced mathematical approaches as they progress through levels of solutions and problem difficulty. Thus, their development of computational skills and their acquisition of problem-solving skills are intertwined as both develop with understanding” (Fuson, 2003).
- Algebraic thinking in elementary school “can be supported through the use of ‘algebrafied’ tasks that help children look for general relationships. We define these tasks as ones that are transformed from arithmetic problems to opportunities for conjecturing, generalizing, and justifying mathematical relationships” (Soares, Blanton & Kaput, 2006).
- Studies “have shown that instruction that bridges formal algebra instruction to previously grounded representations helps students learn processes such as algebraic modeling of verbally presented relations. The two studies differed in the type of grounded representations used, yet they yielded similar results, suggesting that a crucial feature of success was the role of grounded intermediate representations in students’ learning” (Nathan & Koedinger, 2000).

How are Algebra and Algebraic Thinking assessed in *i-Ready Diagnostic*?

In *i-Ready Diagnostic*, the items aligned to Algebra and Algebraic Thinking in grades K–2 allow students to demonstrate their ability to represent problem situations with number sentences.

As in Number and Operations, in these earlier grades, students use virtual manipulatives to represent these problem situations. For example, a 10-frame with counters can be used to represent what is meant by the equation $5 + 2 = 7$, and how that may be manipulated to show understanding that $7 - 2 = 5$.

In grades 3–5, the items aligned to Algebra and Algebraic Thinking expand to include students’ capabilities of modeling problems using equations. These items allow students to demonstrate their understanding by having them select the equation that best models a mathematical or real-world problem.

In grades 6–8, the items aligned to Algebra and Algebraic expand on students’ understanding of modeling problems to using different representations to solve the problems, including expressions and equations and functions. In these grades, students may represent situations by graphing a line that represents a situation on a coordinate graphing tool.

How are Algebra and Algebraic Thinking taught in *i-Ready Instruction*?

i-Ready Instruction uses a problem-solving approach to develop the meanings and properties of the operations. Lessons provide students with the opportunity to use virtual manipulatives, tape diagrams, arrays, and area models to solve a variety of problem types with unknowns in different places.

The meanings and properties of the operations are then used to solve one-step, two-step, and multi-step problems. As the modules progress through the problem types, the range of numbers used is extended and different problem-solving strategies are developed.

A problem-solving approach and real-world contexts are also used to explore ratios and proportional relationships. The instructional modules guide students to use manipulatives, tables, and graphs to explore these concepts, and the outcomes of their explorations are used to formally define the concepts.

The idea of equations is developed through a progression: representing an unknown amount with a letter, the properties and order of operations, and evaluating and simplifying expressions. A variety of equations are solved using a balance model, tape diagrams, and interactive guided examples.

i-Ready lessons introduce patterns and variables in upper elementary school. These concepts are recalled in middle school to introduce the idea of a function. Various representations of functions are studied, including algebraic, numeric, graphic, and verbal. As with other topics, meaningful real-life contexts are used to motivate learning.

Measurement and Data

Measurement and Data (Grades K–5)

Statistics and Probability (Grades 6–8)

Measurement and Data: Geometric Measurement connects geometry and number. Measurement assigns a number to a magnitude of some attribute shared by a class of objects, such as length. Before learning to measure attributes, students learn to recognize them and distinguish them from other attributes. The purpose of measurement is to allow indirect comparisons of objects’ amount of an attribute using numbers.

Measurement and Data: Measurement Data involves taking measurements. This domain path strengthens and applies what students are learning in arithmetic and builds foundations for the study of Statistics and Probability in middle school. Early work with data involves counting and order relations then moves to solving addition and subtraction problems in a data context. Work with data is also closely related to the number line, fraction concepts, fraction arithmetic, and solving problems that involve the four operations.

Statistics and Probability builds on students’ knowledge and experiences in data analysis developed in earlier grades. Students come to a deeper understanding of variability and more precise descriptions of data distributions. They use random sampling to draw inferences about a population and draw informal comparative inferences about two populations. Students then extend their understanding to investigate patterns of association in bivariate data. This domain also involves investigating chance processes and developing, using, and evaluating probability models.

The Research

- “Children are tacit measurers of nearly everything.... [C]lassroom research points to the importance of helping children go beyond procedural competence to learn about the mathematical underpinnings of measure so that procedures and concepts are mutually bootstrapped” (Lehrer, 2003).
- “Measure of distances requires restructuring space so that one “sees” counts of units as representing an iteration of successive distance.... [S]tudies of children’s development suggest that acquisition of this understanding involves the coordination of multiple constructs, especially those of unit and zero-point.” Lehrer cites the work of Clements, Battista, and Sarama (1998) in which they reported that using computer tools that mediated children’s experience of unit and iteration helped children mentally restructure length into units (Lehrer, 2003).
- “Measurement is the process of assigning a number to a magnitude of some attribute shared by some class of objects, such as length, relative to a unit. Before learning to measure attributes, children need to recognize them, distinguishing them from other attributes. That is, the attribute to be measured has to ‘stand out’ for the student and be discriminated from the undifferentiated sense of amount that young children often have, labeling greater lengths, areas, volumes, and so forth, as ‘big’ or ‘bigger.’ ... [B]oth in measurement and in estimation, the concept of unit is crucial” (Progressions for the Common Core State Standards in Mathematics, 2012).
- “We must help our students to make explicit connections between proportions and data and chance. The ‘leveling’ power of averages when comparing groups of unequal size and the expected center and expected spread across repeated samples are two examples of such opportunities to connect to proportions” (Watson & Shaughnessy, 2004).
- There are “three main components of graph comprehension; these components show a progression of attention from local to global features [of a graph]: (a) To read information directly from a graph, one must understand the conventions of graph design; (b) to manipulate the information read from a graph, one makes comparisons and performs computations; and (c) to generalize, predict, or identify trends, one must relate the information in the graph to the context of the situation” (Friel, Curcio, & Bright, 2001).

How are Measurement and Data assessed in *i-Ready Diagnostic*?

In *i-Ready Diagnostic*, the items aligned to Measurement and Data allow students to observe, collect, display, organize, and interpret measures and data.

In grades K–2, the items are concentrated on measuring using virtual tools such as a ruler, and interpreting data displayed in simple graphs such as picture and bar graphs.

In grades 3–5, the items aligned to Measurement and Data provide opportunities for students to demonstrate their extended understanding of more complex measurements and data sets. The items aligned to this domain in these grades also emphasize conceptual understanding of geometric measurement. For example, there is a tool that allows students to fill a rectangular prism with unit cubes to demonstrate an understanding of volume.

In grades 6–8, the items no longer have any focus on geometric measurement, concentrating solely on the concepts of statistics and probability. Items ensure that students are given the opportunity to demonstrate their conceptual understanding of more complex data sets. Technology-enhanced items allow students to demonstrate their understanding of bivariate data by graphing linear functions that closely represent a data set.

How are Measurement and Data taught in *i-Ready Instruction*?

The instruction modules in *i-Ready* provide students with virtual measurement tools including 1-inch tiles, 1-centimeter tiles, beakers, balance scales, protractors, and area and volume models to explore measurement concepts. The number line is used to develop length concepts, as well as to understand the number systems and their operations.

Real-world contexts are used in conjunction with these various measurement tools to enhance students' conceptual understanding. The contexts provide meaning when working with measurement and data analysis. A variety of application problems are solved using problem-solving strategies, including using a tape diagram, number line, or other model to understand the application problem. Lessons use virtual tools to have students sort objects or data into categories and take measurements. The information collected is used to build and analyze bar graphs and line plots.

i-Ready Instruction takes middle-school students through the process of data analysis. Students ask a question, collect data, organize the data using an appropriate graphical display, and then analyze the data in order to answer the question. This process allows students to gain an understanding of what the data represents when organized in various displays such as dot plots, histograms, and box plots.

The lessons also extend students' understanding of proportional relationships to develop probability concepts and an understanding of statistical inferences. Students use proportional reasoning to make predictions about random events and then test their predictions. They also use proportional reasoning to make inferences about a population based on a random sample from the population. They understand that their predictions and inferences are not going to be exact but close enough to gather useful information. That is the nature of Statistics.

Geometry

Geometry (Grades K–8)

Geometry supports the development of number and arithmetic concepts and skills. This domain addresses a series of levels of geometric and spatial thinking. The goals for elementary geometry fall into three categories: geometric shapes, their components, their properties and their categorization based on those properties; composing and decomposing shapes; and spatial relations and spatial structuring. In middle school, students solve real-world and mathematical problems involving angle measure, area, surface area, and volume. They explore congruence and similarity and understand and apply the Pythagorean Theorem.

The Research

- The Progressions for the Common Core State Standards in Mathematics describes four levels of geometric thinking:
 - **Visual/syncretic:** Students recognize shapes, e.g., a rectangle “looks like a door.”
 - **Descriptive:** Students perceive properties of shapes, e.g., a rectangle has four sides, all its sides are straight, opposite sides have equal length.
 - **Analytic:** Students characterize shapes by their properties, e.g., a rectangle has opposite sides of equal length and four right angles.
 - **Abstract:** Students understand that a rectangle is a parallelogram because it has all the properties of parallelograms.

(Progressions for the Common Core State Standards in Mathematics, 2012)

- “Activities with mosaics and others using paper folding, drawing, and pattern blocks can enrich children’s store of visual structures. They also develop knowledge of shapes and their properties. To promote the transition from one level to the next, instruction should follow a five-phase sequence of activities.” These instructional phases are inquiry, directed orientation, explicitation, free orientation and integration. For van Hiele, geometry begins with play (van Hiele, 1999).
- To avoid misconceptions about geometry, it is important to emphasize the properties and characteristics of a concept, provide many examples and non-examples, even if the child is not ready to specifically name the non-examples, pay close attention to language use and challenge understanding and broaden generalizations (Oberdorf & Taylor-Cox, 1999).

How is Geometry assessed in *i-Ready Diagnostic*?

In *i-Ready Diagnostic*, the items aligned to Geometry allow students to demonstrate proficiency in identifying, analyzing, and reasoning with shapes and figures.

In grades K–2, the items are concentrated on two areas—students are provided the opportunity to demonstrate proficiency with the attributes of different shapes, and they are able to show connections to a conceptual understanding of fractions as part of a whole. Technology-enhanced items allow students to sort or identify shapes that have similar attributes.

In grades 3–5, the items aligned to Geometry expand on students’ understanding of figures and begin to assess student understanding of the attributes in hierarchies. These items also ask students to demonstrate a conceptual understanding of two-dimensional figures in space. Some of the technology-enhanced items have students plot shapes in the first quadrant of a coordinate grid. Other items may have students fill in a two-dimensional space with unit squares to help demonstrate proficiency with a conceptual understanding of area.

In grades 6–8, there is somewhat of a shift in the domain. In grades K–5, the only geometric measurement concepts covered in the Geometry domain are those that deal with conceptual understanding of area. However, in grades 6–8, with the Measurement and Data domain focusing on Statistics and Probability, all of the geometric measurement concepts fall under the Geometry domain. These include area of composite figures, surface area, and volume. In addition to these concepts, higher-level geometric concepts are also assessed in *i-Ready* in grades 6–8. These concepts include relating transformations to congruence and similarity, and analyzing proofs of the Pythagorean Theorem and its converse.

Some of the technology-enhanced items in this domain at these grade levels use a virtual protractor to allow students to demonstrate proficiency with rotations.

How is Geometry taught in *i-Ready Instruction*?

i-Ready Instruction follows the four levels of geometric thinking.

- **Visual/syncretic:** shape recognition
- **Descriptive:** identification of properties of shapes
- **Analytic:** characterize shapes based on their properties
- **Abstract:** classify shapes in hierarchies

Explorative and hands-on activities are used to develop concepts. Activities include using pattern blocks, area tiles, unit cubes, grids, and the coordinate plane to sort, classify, construct, dissect, locate, and transform various geometric objects. The focus is to help students develop understanding of geometric relationships.

Appendix–i-Ready Assessment and Universal Design

All items in *i-Ready* are designed to be accessible for all students regardless of their need for accommodation. In most cases, students who require accommodations (e.g., large print; extra time) will not require additional help during administration.

The thoughtful planning that went into the general design of the assessment ensures that a large percentage of students requiring accommodations will have the necessary adjustments without compromising the interpretation or purpose of the test.

According to the Standards for Educational and Psychological Testing (AERA, APA, NCME, 2014), “Universal Design processes strive to minimize access challenges by taking into account test characteristics that may impede access to the construct for certain test takers.” *i-Ready Diagnostic* was developed with the universal principles of design for assessment in mind.

According to the NCEO Synthesis study (<http://www.cehd.umn.edu/nceo/onlinepubs/Synthesis44.html>), the seven elements of Universal Design as they apply to large-scale assessment are:

1. Inclusive assessment population
2. Precisely defined constructs
3. Accessible, non-biased items
4. Amendable to accommodations
5. Simple, clear, and intuitive instructions and procedures
6. Maximum readability and comprehensibility
7. Maximum legibility

To address these points, *i-Ready* development considered several issues related to accommodations.

Most accommodations may be grouped into the following general categories: timing, scheduling, presentation, setting, and response mode. *i-Ready* addresses each of these accommodations:

- **Timing**—Students may need extra time to complete the task. The diagnostic assessment may be stopped and started as needed to allow students needing extra time to finish. The Diagnostic is untimed and can be administered in multiple test sessions. In fact, to ensure accurate results, a time limit is not recommended for any student, though administration must be completed within a period of no longer than three weeks.
- **Flexible Scheduling**—Students may need multiple days to complete the assessment. *i-Ready* recommends that all students be given multiple days, as necessary, to complete the test (administration must be completed within a period of no longer than three weeks).
- **Accommodated Presentation of Material**—All *i-Ready Diagnostic* items are presented in a large, easily legible format on the computer screen. There is only one item on the screen at a time, the bold font is large and friendly, and nearly all of the items for grades K–5 have audio support. *i-Ready* has an option for audio support for math items at grade levels K–5. Directions are either read aloud, demonstrated visually, or generally self-evident, regardless of language abilities. Use of clear, concise, and chronological directions build student understanding. The program employs physical cues such as pointing or facial expressions, symbolic cues such as pictures and icons, and auditory cues, such as asking a question.

- **Setting**—Students may need to complete the task in a quiet room or with a small group of peers. This can easily be done, as *i-Ready Diagnostic* is available on any computer with internet access that meets the technical requirements. Furthermore, all students are encouraged to use quality headphones in order to hear the audio portion of the items. Headphones also help to cancel out peripheral noise, which can be distracting to students.
- **Response Accommodation**—Students should be able to control a mouse. They only need to be able to move a cursor with the mouse and be able to point, click, and drag. There are some students who do not have this capacity, and they can work with *i-Ready Diagnostic* with third party software, such as TOBII Communicator.

In terms of item bias, Differential Item Functioning was performed on the current items in the assessment bank. Of the nearly 5,000 items for which this analysis was available, only about 1% of the items had large or significant Differential Item Functioning. Items that demonstrate large Differential Item Functioning are targeted for removal.

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